PROGRAM FOR THE TRANSCONTINENTAL SEMINAR: "COARSE GEOMETRY, ASSEMBLY AND EXPANDERS"

NOÉ BÁRCENAS, DANIEL JUAN-PINEDA, DAVID ROSENTHAL, AND MARIO VELASQUEZ

1. Introduction

The sixth edition of the Transcontinental Seminar analyzes the relationship between coarse geometry and the Baum-Connes Assembly Map. In the first part of the seminar the basics of operator algebras, K-Theory and K-homology will be discussed, with particular emphasis on their connection to the Baum-Connes conjecture.

The Baum-Connes conjecture [1] predicts an isomorphism of abelian groups

$$\mu_G : K_*^G(\underline{E}G) \longrightarrow K_*^{\text{top}}(C_r^*(G)),$$

where $K_*^G(-)$ denotes equivariant K-homology, a generalized equivariant homology theory. The space $\underline{E}G$ is the classifying space for proper actions, $K_*^{\text{top}}(A)$ denotes *topological* K-theory of the C^*-algebra A, and $C_r^*(G)$ denotes the *reduced* C^*-algebra of the discrete group G.

Given a finite generating set S for a group G, the word-length metric associated to S defines a metric on the Cayley graph $C(G, S)$ that is a quasi-isometry invariant of the group G. This metric space captures the so-called "large scale" geometry of G. The second part of the seminar will introduce the basic methods from coarse geometry and will introduce K-homology associated to a coarse structure.

Coarse geometry associates to a metric space X (or more general structures) a C^*-algebra $C^*(X)$ encoding the large scale geometry of the space X. Methods from C^*-algebra and operator theory, originally developed in index theory for non-compact manifolds, associate to this data an assembly map

$$K_*(X) \to K_*(C(X)).$$

The statement of the Coarse Baum-Connes conjecture [11] is that this map is an isomorphism. The Baum-Connes conjecture is related to several conjectures, like Novikov's conjecture on the homotopy invariance of higher signatures, the Kaplansky conjecture, and many others (see [6] for more on this topic). A generalization of the Baum-Connes conjecture allows for more general coefficients. It predicts the existence of an isomorphism

$$A : K_*(\underline{E}G, A) \to K_*(A \wr_r G)$$

for any G-C^*-algebra A. Counterexamples to this conjecture are known due to the work of Higson-Lafforgue-Skandalis, but it is still studied since it has nice inheritance properties (like passing to subgroups). In the third part of the seminar we will consider a description of the coarse structures that lead to these counterexamples.

The seminar is organized as follows:

(i) Part 1: Basics of topological K-Theory, K-homology and C^*-algebras.
  - First Talk: General Overview of the Baum Connes conjecture, Coarse Geometry and expanders. This talk should give a general introduction

to the diverse topics of the seminar. Introduce some Basics on C^*-algebras, including the Gelfand-Naimark Theorem.
- Second Talk: The definition of K-theory for C^*-algebras. K_0 as Murray-von Neumann equivalence classes of projectors in A, $K_1(A)$ as the quotient of the infinite dimensional general linear group in A modulo the identity component. Strengthened Bott periodicity and homotopy invariance as the crucial difference to algebraic K-theory (without proofs). References:[5], chapter 4, [3], [10], Chapter 1 of [5].
- Third Talk: Classical K-theory and K-homology. This talk should introduce topological K-theory, state the Serre-Swan Theorem relating operator theory to topological K-theory. Also, K-homology should be introduced as an extension group, and as the K-theory of the Paschke dual algebra. Comment on how involved it is to prove the homotopy invariance of K-homology in any of these formulations. Reference: [5], chapter 5. Description of the classical Baum-Connes map.

(ii) Part 2: Coarse geometry and coarse K-homology.
- First talk. Coarse structures. This talk will introduce the notions of coarse structures, controlled and bounded sets, cones and the C^* algebra of a Coarse space. special interest should be devoted to the relationship between coarse structures associated to Cayley graphs and quasi-isometry. References: [5], chapter 6 pages 142- 152, [8] for the emphasis on groups and coarse structures associated to them.
- Second talk. K-homology of coarse structures and the coarse Assembly map. References: [5], pages 152- 163. For the coarse assembly map, [5], chapter 12 section 3.

(iii) Third talk. Scaleable spaces. This talk will deal with sections 4, 5 in [5].

(iv) Part 3: Coarse structures, expanders and assembly maps.
- Expanders, coarse geometry and failure of surjectivity of the coarse assembly map. This talk will deal with the example in section 6 of [4].
- Coarse embeddability and finite asymptotic dimension. This talk will give some indication of how these properties are used to prove positive results on the coarse assembly map. References: [13, 7].

References

[1] P. Baum, A. Connes, and N. Higson, Classifying space for proper actions and K-theory of group C^*-algebras, *C^*-algebras: 1943–1993 (San Antonio, TX, 1993)*, pages 240–291, Amer. Math. Soc., Providence, RI, 1994.

[2] G. Bell and A. Dranishnikov, Asymptotic dimension, *Topology Appl.* **155** (2008), no. 12, 1265–1296. MR 2423966 (2009d:55001)

[3] Blackadar, B. *K-Theory for Operator Algebras, Second edition.* Mathematical Sciences Research Institute Publications, Volume 5, Cambridge University Press, 1998.

[4] N. Higson, V. Lafforgue, and G. Skandalis. Counterexamples to the Baum-Connes conjecture. *Geom. Funct. Anal.*, 12(2):330–354, 2002.

[5] N. Higson and J. Roe. *Analytic K-homology.* Oxford Mathematical Monographs. Oxford University Press, 1998

[6] Wolfgang Lück and Holger Reich. The Baum-Connes and the Farrell-Jones conjectures in K- and L-theory. In *Handbook of K-theory. Vol. 1, 2*, pages 703–842. Springer, Berlin, 2005.

[7] Nowak, P. and Yu, Guoliang. *Large Scale Geometry*, Book project, $http://www.math.tamu.edu/\ pnowak/book_etb/book_etb.pdf$

[8] Roe, J. Notes on *Coarse Geometry*, Book project, available on the author's homepage.

[9] Roe, J. *Lectures on Coarse Geometry*, Volume 31 of University Lecture Series, American Mathematical Society, 2003.

[10] N. E. Wegge-Olsen, *K-theory and C∗-algebras: A Friendly Approach.* The Clarendon Press, Oxford University Press, New York, 1993.

[11] G. Yu, Coarse Baum-Connes conjecture. *K-theory*, 9:199–221, 1995.

[12] G. Yu, The Novikov conjecture for groups with finite asymptotic dimension, *Ann. of Math. (2)* **147** (1998), no. 2, 325–355. MR 1626745 (99k:57072)

[13] G. Yu, Large scale geometry and its applications. *Noncommutative geometry and global analysis*, 305–315, *Contemp. Math.*, **546**, Amer. Math. Soc., Providence, RI, 2011.

E-mail address: `barcenas@math.uni-bonn.de`

E-mail address: `daniel@matmor.unam.mx`

E-mail address: `rosenthd@stjohns.edu`

E-mail address: `mavelasquezm@gmail.com`