"Prospectos en Topología"
Semester 2023-1

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During the term 2023-1, the Seminar "Prospectos en Topología" will have the following three thematic blocks:

1. Applications of Asymptotic Cones in Geometric Group Theory.
2. Big Mapping Class Groups.
3. Measurable Group Theory.

A schedule and description of the thematic blocks is as follows.

1 Applications of Asymptotic Cones in Geometric Group Theory

Objective: To give some applications of asymptotic cones in Geometric Group Theory. Concretely, it will be showed how asymptotic cones simplify Gromov's Polynomial Growth Theorem's proof and how they exhibit the non-relative hyperbolicity of Mapping Class Groups.

1. Gromov's Polynomial Growth Theorem I
   - **Speaker:** Carlos Adrián Pérez Estrada.
   - **Date** August 8th.
   - **Abstract:** We will give an introduction to the growth of finitely generated groups with focus on the polynomial growth of nilpotent groups. Then we will start to prove Gromov's Theorem about the virtual nilpotency of finitely generated groups with polynomial growth.
   - **References:** [DK18] and [vdDW84]
2. Gromov's Polynomial Growth Theorem II
   - **Speaker:** Carlos Adrián Pérez Estrada.
   - **Date** August 15th.
   - **Abstract:** We will continue to prove Gromov's Polynomial Growth Theorem about the virtual nilpotency of finitely generated groups with polynomial growth using asymptotic cones.
   - **References:** [DK18], [vdDW84] and [Sap15].

3. Non-Relative Hyperbolicity of Mapping Class Groups

   - **Speaker:** Sandy Aguilar.
   - **Date:** August 22nd.
   - **Abstract** The notion of a relative hyperbolic group will be discussed in terms of asymptotic cones in order to sketch the proof of Theorem 1.11 of [DS05] and Corollary 7.9 of [BDM09]; which state that a thick metric space is not asymptotically tree-graded. This will be used to prove that mapping class groups are not generally relatively hyperbolic.
   - **References:** [Beh06], [BDM09] and [DS05].

2 Big Mapping Class Groups

Objective: To introduce mapping class groups of infinite-type surfaces (BMCGS) and discuss their principal properties. In particular, we will focus in their structure as topological groups and coarse spaces, and we will study their cohomology as abstract groups.

1. Preliminaries of BMCGS

   - **Speaker:** Rogelio Niño.
   - **Date** August 29th.
   - **References:** [AV20]

2. Rosendal‘s Coarse Geometry and Mann and Rafi´s Classification of Locally Coarsely Bounded BMCGS.

   - **Speaker:** Anschel Schaffer-Cohen.
   - **Date** September 5th
   - **References:** [Ros22] and [MR20]

3. BMCGS of Random Infinite-type Surfaces

   - **Speaker:** Anschel Schaffer-Cohen.
   - **Date** September 19th

4. Graphs of Curves and Arcs Quasi-isometric to BMCGS

   - **Speaker:** Anschel Schaffer-Cohen.
   - **Date** September 26th
   - **References:** [SC20]

5. Benjamin-Schramm Convergence And Spectrum of Random Surfaces of High Genus

   - **Speaker:** Noé Bárcenas Torres
   - **Date** October 17th

6. Cohomological Properties of BMCGS

   - **Speaker:** Israel Morales.
   - **Date** October 31st

3 Measurable Group Theory

Objective: To study large-scale properties of finitely generated groups by means of their actions on Banach Spaces.

1. Geometric Actions on Banach Spaces
    - **Speaker:** Carlos Adrián Pérez Estrada
    - **Date** November 7th
    - **Abstract:** We will discuss metrically proper actions on infinite-dimensional Banach spaces and a-T-menability for finitely generated groups. Finally, we will review group-theoretical consequences of the previous notions.
    - **References:** [NY12]
2. Property A and Coarse Embeddability in Hilbert Spaces
    - **Speaker:** Noé Bárcenas Torres
    - **Date** November 14th

References

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