Prospectos en Topología
Semester 2024-2

Noé Bárcenas and Carlos Pérez Estrada
barcenas@matmor.unam.mx
cperez@matmor.unam.mx

During the 2024-2 term, the Seminar "Prospectos en Topología" will have the following two thematic blocks:

1. Extreme amenability of topological groups in terms of Structural Ramsey Theory.

2. Introduction to L^2-invariants.

The seminar will also feature a number of individual talks. The schedule for the seminar is as follows:

- Individual Talk: "Dimensión fractal y magnitud persistente"

  - **Speaker:** Haydeé Contreras Peruyero.
  - **Date:** January 29th.
  - **Abstract:** En años recientes han habido varios esfuerzos en tratar de estudiar la autosimilitud de estructuras de bases de datos usando topología. La magnitud persistente surge de la conexión entre la homología persistente y la homología de magnitud, es un invariante numérico de un complejo simplicial filtrado asociado a un conjunto de datos.

    En esta plática vamos a explorar los conceptos de magnitud persistente y como estamos usando estos invariantes para estudiar la autosimilitud de redes complejas y como lo podemos usar para clasificar bases de datos reales con estructura tipo fractal.

    Trabajo en conjunto con Nina Otter, Maria Antonietta Pascali, Sanjukta Krishnagopal, Rayna Andreeva y Elizabeth Thompson.

- Extreme amenability of topological groups in terms of Structural Ramsey Theory

  Objective: To rewiev the characterization of the automorphism groups of countable ultrahomogeneous structures (i.e. closed subgroups of the countable infinite symmetric group S_∞) that are extemely amenable in terms of Ramsey criteria in their respective Fraïssé classes (i.e. the KPT correspondence) following [KPT05]. For those automorphism groups that are not extremely amenable, we will study how to compute their universal minimal flow using precompact expansions in the sense of [NVT13] and [Bar20].

  1. Characterization of the extremely amenable subgroups of S_∞ in terms of Structural Ramsey Theory
     - **Speaker:** Carlos Pérez Estrada.
     - **Date:** February 12th.
     - **References:** [KPT05, Sections 0-4].
  2. Examples of extremely amenable groups using the KPT correspondence
     - **Speaker:** Mario Jardón Santos.
     - **Date:** February 19th.
     - **References:** [KPT05, Sections 5-6].
  3. Universal minimal flows in terms of precompact expansions

- **Speaker:** Luis David Reyes Saenz.
  - **Date:** February 26th.
  - **References:** [KPT05, Section 7], [NVT13], [Kub14] and [Bar20].
4. Computations of universal minimal flows
   - **Speaker:** Carlos López-Callejas.
   - **Date:** March 11th.
   - **References:** [KPT05, Section 8].

- Individual Talk:

  - **Speaker:** Antoine Goldsborough.

  - **Date:** April 1st.

- Individual Talk:

  - **Speaker:** Yesenia Villicaña Molina.

  - **Date:** April 8th.

- Individual Talk: "Characterizing LEF and sofic groups"

  - **Speaker:** Philip Stetson.

  - **Date:** April 15th.

- Introduction to L^2-invariants

  Objective: To discuss the theory of L^2-invariants from several perspectives and study its connections with the Zero-in-the-Spectrum Conjecture and the Strong Novikov Conjecture. We will also see how to define L^2-invariants on condensed sets.

  1. Introduction to L^2-invariants from the algebraic point of view
     - **Speaker:** Noé Bárcenas Torres.
     - **Date:** April 22nd.
     - **References:** [Lö9].
  2. Comparison between the analytic and geometric perspectives of L^2-invariants
     - **Speaker:** Noé Bárcenas Torres.
     - **Date:** April 29th.
     - **References:** [Lö1, Section 1.3].
  3. The Strong Novikov Conjecture implies the Zero-in-the-Spectrum Conjecture
     - **Speaker:** Carlos Pérez Estrada.
     - **Dates:** May 6th and May 13th.
     - **Abstract:** These sessions are devoted to introduce the Strong Novikov Conjecture for groups, the Zero-in-the Spectrum Conjecture for aspherical closed Riemannian manifolds M along with its reformulation by means of L^2-invariants, and to prove that the Strong Novikov Conjecture for $\pi_1(M)$ implies the Zero-in-the-Spectrum Conjecture for M.
     - **References:** [Lö2, Chapter12], [Lot96] and [Gro95].
  4. L^2-invariants and Condensed Mathematics
     - **Speaker:** José Manuel Mendoza.
     - **Date:** May 20th.
     - **Abstract:** An introduction to Condensed Mathematics is given in order to address the construction of L^2-Betti numbers on condensed sets carrying an action of a discrete group given in [KKL23, Chapter 14].
     - **References:** [SC19] and [KKL23].

- Individual Talk: "Condensed Mathematics, extremally disconnected spaces and forcing"

  – **Speaker:** Jorge Antonio Cruz Chapital.
  – **Date:** May 27th.

- Individual Talk:

  – **Speaker:** John Guaschi.
  – **Date:** June 24th.

References

[Bar20] Keegan Dasilva Barbosa. A categorical notion of precompact expansion. Preprint at arXiv: 2002.11751 [math.CO], 2020.

[Gro95] Mikhael Gromov. Geometric reflections on the Novikov conjecture. In *Novikov conjectures, index theorems and rigidity, Vol. 1 (Oberwolfach, 1993)*, volume 226 of *London Math. Soc. Lecture Note Ser.*, pages 164–173. Cambridge Univ. Press, Cambridge, 1995.

[KKL23] Dominik Kirstein, Christian Kremer, and Wolfgang Lück. Some problems and conjectures about L^2-invariants. Preprint at arXiv: 2311.17830 [math.AT], 2023.

[KPT05] A. S. Kechris, V. G. Pestov, and S. Todorcevic. Fraïssé limits, Ramsey theory, and topological dynamics of automorphism groups. *Geom. Funct. Anal.*, 15(1):106–189, 2005.

[Kub14] Wiesław Kubiś. Fraïssé sequences: category-theoretic approach to universal homogeneous structures. *Ann. Pure Appl. Logic*, 165(11):1755–1811, 2014.

[LÖ01] Wolfgang Lück. L^2-invariants and their applications to geometry, group theory and spectral theory. In *Mathematics unlimited—2001 and beyond*, pages 859–871. Springer, Berlin, 2001.

[LÖ02] Wolfgang Lück. L^2-*invariants: theory and applications to geometry and K-theory*, volume 44 of *Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics]*. Springer-Verlag, Berlin, 2002.

[LÖ09] Wolfgang Lück. L^2-invariants from the algebraic point of view. In *Geometric and cohomological methods in group theory*, volume 358 of *London Math. Soc. Lecture Note Ser.*, pages 63–161. Cambridge Univ. Press, Cambridge, 2009.

[Lot96] John Lott. The zero-in-the-spectrum question. *Enseign. Math. (2)*, 42(3-4):341–376, 1996.

[NVT13] L. Nguyen Van Thé. More on the Kechris-Pestov-Todorcevic correspondence: precompact expansions. *Fund. Math.*, 222(1):19–47, 2013.

[SC19] Peter Scholze and Dustin Clausen. Lectures on condensed mathematics. *Lecture Notes* URL: https://people.mpim-bonn.mpg.de/scholze/Condensed.pdf, 2019.