RESEARCH STATEMENT

Alejandro Corichi

My current research interests are centered around various aspects of classical and quantum gravitational physics. In quantum gravity, the approach I am most familiar with is the non-perturbative canonical quantization program, also known as *Loop Quantum Gravity* (LQG) (or *Quantum Geometry*) [1]. I also have interests in quantum field theory in curved space and in geometrical and fundamental aspects of quantum theory. In classical gravity I have interests in several aspects of black hole physics and in particular, I have been involved in different studies of the framework known as *Isolated Horizons* [2], with different matter couplings.
The main directions within these lines where I have centered my research are:

Quantum Aspects of Gravity, Cosmology and Black Holes.

I am very interested in the fundamental problem of making quantum theory and relativity compatible in what is known as *quantum gravity*. One of the most conservative approaches to this problem is the implementation of 'conventional' quantization methods to the gravitational field. This non-perturbative canonical quantization program has become in recent years a serious candidate for a quantum theory of geometry (with possibly other matter field), thanks to the introduction of connection and loop variables by Ashtekar [3], Rovelli and Smolin in the late 80's [4].

One of the most remarkable results coming from this approach is the particular picture of the geometry at the Planck scale. Excitations of geometry are one dimensional, polymer-like, and the spectra of geometric operators turn out to be discrete [5]. Furthermore, the resulting quantum geometry happens to be intrinsically non-commutative: the operators associated, for instance, to the area of two intersecting surfaces do not commute [6]. This fact posses very interesting challenges: How is one to recover a commutative, smooth geometry in certain limit?, What are the implications of this non-commutativity in, say, the definition of 'coordinates' on the manifold? Are space coordinates destined to be non-commutative? Thus, part of my current research is in the direction of answering such questions, and to understand the role that the non-commutativity plays in the whole formalism [7].

The other question I am interested in is the macroscopic/semi-classical limit of the theory. Historically, the first attempts were the so called weave states, where the state was defined over a complicated loop that was woven to reproduce a smooth geometry at certain scales [8]. In this direction, in collaboration with M. Reyes we proposed some simple and preliminary "Gaussian" weaves, that might approximate flat space [9]. It is important to understand the relation between this states and other proposals such as the coherent states of Thiemann and Winkler and the Statistical Geometry approach of Bombelli. Still, the basic question of what semi-classical states are and how one should construct them is open. New proposals, in particular in the direction of linking non-perturbative states with Fock excitations have recently appeared. It is important to understand its relevance and interconnection with the other approaches. In particular it is very important to gain insight on how to address the issue of constructing dynamical semi-classical states. I have been involved in the study of the problem of constructing dynamical semi-classical states and comparing them to kinematical 'coherent' states, for simple systems [10, 11]. I plan to continue working on these problems in the near future. Another interesting issue in the task of defining semi-classical states within the theory is how to connect the small scale geometry with a semi-classical large scale geometry. With Bombelli and Winkler, we have proposed to employ a statistical geometry framework for that purpose [12]. Further work is needed to have a complete framework to be employed in phenomenological applications, and in relating microscopic discrete structures with macroscopic geometries.

One of the main features of loop quantization is the use of a non-canonical algebra of observables for the quantization procedure. This means that the holonomies, that are exponentiated versions of the connection, are the fundamental variables that get promoted to well defined operators. The quantization that respects diffeomorphism invariance, recently shown to be unique, is such that the connection is not a well defined operator. A similar quantum representation for an ordinary quantum mechanical system has been used with great success in loop quantum cosmology. The formalism suffers, however, from a small drawback: the dynamics can not be unambiguously implemented without introducing some "lattice like" structure. This introduces an undesired element into the quantization. Recently, in collaboration with Vukašinac and Zapata, we have analyzed this type of quantization, sometimes called "polymeric", within a program tailored for defining the dynamics of loop quantized theories via a –Wilson style– renormalization procedure. The system can indeed be successfully treated and the continuum limit happens to be equivalent to the standard Schrödinger representation [13, 14]. In collaboration with Ashtekar and Singh, we continued this avenue and investigated the issue of the continuum limit in loop quantum cosmology. We stablished that the standard 'Schrödinger like' quantization, known as the Wheeler-DeWitt quantization is *not* recovered in any suitable limit. Thus, LQC is fundamentaly discrete [15].

It has generally been regarded that one of the main successes of the loop quantum gravity program is the computation of the Bekenstein-Hawking entropy associated to the horizon of a black hole [16]. I was involved in a project to do that with A. Ashtekar, J. Baez and K. Krasnov [17], where a careful treatment of the boundary conditions was done. Later, in collaboration with A. Ashtekar, the calculation was extended to include non-minimally coupled scalar fields [18]. This later case posses special challenges to the formalism since classically the entropy of a black hole depends also on the scalar field at the horizon, and quantum mechanically the geometric operators need to be modified. It was shown that the nontrivial consistency checks needed to have a coherent description for quantum horizons continue to be met.

Furthermore, a careful treatment of the counting shows that there are several possibilities for the choice of states that can be counted. In a recent study, in a collaboration with students in Valencia, Spain, a direct counting of the number of states was done for small Planck size horizons, using both possibilities [19, 20]. It has been shown that the counting is consistent with an asymptotic linear relation between entropy and area (with a logarithmic correction), but also that the relation shows some oscillatory behavior for small black holes [21]. Furthermore, the spectrum of the quantum black hole was studied and found to possess an unexpected feature: the entropy seems to take only a discrete set of values, becoming equidistant for large black holes, making it compatible with the expectations of Bekenstein for black hole entropy (in the large area limit). This phenomenon of 'entropy quantization' is robust and independent of the counting method employed [22]. This intriguing behavior certainly calls for some explanation. Preliminary proposals in this direction have been put forward in the literature [23], but a deeper understanding is still missing.

In order to gain a full understanding of the conceptual problems that are common to the quantum theory of gravity, it is sometimes useful to consider simpler models, with the hope of learning important lessons for the complete theory. Usually, this implies performing a symmetry reduction or considering theories in less than four dimensions. In this spirit, I have been involved in the study of the aplication of loop quantization techniques to cosmological models, known as *loop quantum cosmology*, and in the midi-superspace quantization of Gowdy models . The Gowdy model is the simplest, inhomogeneous, closed cosmological model and has been extensively studied at the quantum level. Even when the symmetry reduction makes the model 'solvable', one is still treating a field theory system with an infinite number of degrees of freedom. We have shown, in collaboration with J. Cortez and H. Quevedo that a very natural quantization, involving a choice of internal time for the evolution does not admit time evolution as a unitary process [24]. This results opened up the way to the study of the physical implications of choosing different representations of the CCR. Within these studies, in collaboration with Cortez and Mena-Marugán we have shown that, by performing a canonical transformation on the original system before quantization, the resulting description *does* admit time evolution as a unitary process [25]. Furthermore, in collaboration with Velhinho, we have shown that the requirement of unitary time evolution is sufficient to prove uniqueness of the quantum representation [26]. This result is, to the best of my knowledge, the first one involving uniqueness of a quantum representation on time dependent backgrounds.

For the simplest case of homogeneous and isotropic models, within the approach known as loop quantum cosmology, there have been very impressive recent results. It has been shown that, for a massless-scalar field with and without a cosmological constant, the big bang gets replaced by a *Big Bounce*. The singularity is resolved [27]. For the case of a flat FRW, we showed analytically that this bounce is generic and the matter density is bounded for all states of the theory [15]. Furthermore, we showed that a semiclassical state at late times had to come from a semiclassical states [28]. These results are encouraging and one should try to apply them to more general systems. But before that, it is important to critically analyze the lessons from the isotropic models [29].

The study of the Gowdy model was a motivation to study the formulation of the Schrödinger representation on a curved spacetime. We have constructed such representation —that had not been done before— and found that the resulting quantum theory had some unexpected features [30]. This in turn has lead to an example of a quantization ambiguity in field theory [31]. An application of the quantization procedure for a self-dual decomposition of an Abelian gauge theory yields the Kodama state as the vacuum of the theory, and in particular, for the free Maxwell field [32]. This result has put in proper perspective previous results that involved self-duality and link invariants. Still to be understood are the more mathematical aspects of the Schrödinger representation on curved space such as the nature of the quantum configurations space in the general case. An understanding in the case of Gowdy was reported in [33].

On a more speculative front, I have proposed together with Sudarsky and Ryan an outline of several problems that current approaches to quantum gravity possess, and possible avenues for its resolution [34]. This outline needs, of course, of further work in order to make concrete proposals. We have also proposed a possible new approach to quantum gravity phenomenology, outside the current paradigms of Lorentz invariance violations and DSR [35]. These, in particular imply new couplings between polarized matter and gravity that are absent in Minkowski space. This proposal was recently extended in Ref.[36]. Concrete experiments could be devised in order to put bounds on these couplings.

Classical General Relativity

In the search for a quantum of a black hole within the loop approach, it was necessary to restrict attention to the 'black hole' sector of phase space. This lead to what has become known as the *Isolated Horizons* formalism [37]. The idea is to give geometrically motivated boundary conditions on the theory which allow for a well defined action principle and a Hamiltonian description. This formalism has in particular, generalized the laws of black hole mechanics to situations in which the horizon is isolated, but the exterior region is allowed to be dynamical. I have been particularly interested in the formalism when non-linear matter fields are considered. The application of the formalism to stationary situations has lead to an understanding of the rich interplay between the solitons in, say, the Einstein Yang Mills system and the colored black holes [38]. The formalism is also well suited for posing uniqueness ('no-hair') conjectures for the existence of solutions [39], where the charges defining the solution would be defined at the horizon and not at infinity. More recently, we have considered isolated horizons for theories where the matter is non-minimally coupled to gravity, where the treatment based on Noether charges tells us that the Black Hole entropy is not proportional to the area but it also depends on the matter fields at the horizon [40]. The IH formalism is able to incorporate these situations. I have also been involved in the study of the isolated horizons formalism when there are two horizons present such as in a Schwarzschild-de Sitter spacetime where there are both a BH and a cosmological horizons [41]. The IH formalism is well defined and gives rise to new mass formulae and entropy bounds that can be compared with the Bekenstein and the D-bound.

One of the potential applications of dynamical and isolated horizons formalisms is given by the numerical simulations of collapsing systems. In this regard I have begun numerical investigations of the collapse of a minimally coupled scalar soliton into a black hole [42], and the dynamical horizon that forms. Numerical simulations indicate that the theoretical formula for the flux of matter through the horizon matches well with the finite mass difference of the horizon [43] at different stages of its growth. The next step is to start with a hairy black hole slightly perturbed as initial state and evolve to see what the final configuration is. Certain conjectures suggest that the final state might look like a AdS black hole surrounded by a moving domain wall. Second, I have been involved in developing gauge conditions for both lapse and shift functions that, when coupled to some version of the Einstein equations's render a hyperbolic system [44].

Open problems I am interested in include a precise definition of what are called 'horizon charges' and of the uniqueness and completeness conjectures. This has important consequences in the IH formalism, within the context of the recently introduced multipoles, and should be explored.

Quantum Theory and Geometry

I am also interested in other related problems involving geometry and quantum theory, and in particular in geometric phases (including gravity [45]) and the geometrical formulation of quantum mechanics. More precisely, the incorporation of the *superposition principle* and constraints into the geometric formalism [46]. This is, in my view, a very important issue since the superposition principle lies at the forefront of the quantum theory but is normally excluded from the geometric formulation of the theory, where the space of states is a non-linear manifold [47]. In this direction I have proposed to view the standard superposition principle as consisting of two different principles, depending on the properties of the physical system under consideration. More recently, I have proposed the first step to analyze quantum constrained systems from the geometric perspective. Interestingly, an unforseen connection with the so called Master constraint program arises [48]. Further work is needed to generalize these results to systems where the physical Hilbert space is not a subspace of the kinematical one, and to study the geometry of the group average procedure. Another particular feature of quantum theory that interests me is the application of the geometric formulation to entanglement, and in particular in a possible (geometrical) definition of distance for entangled states.

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