A universal framework for quantum theory from qubits to quantum gravity

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Gravity in Qubits
Bratislava, Slovakia
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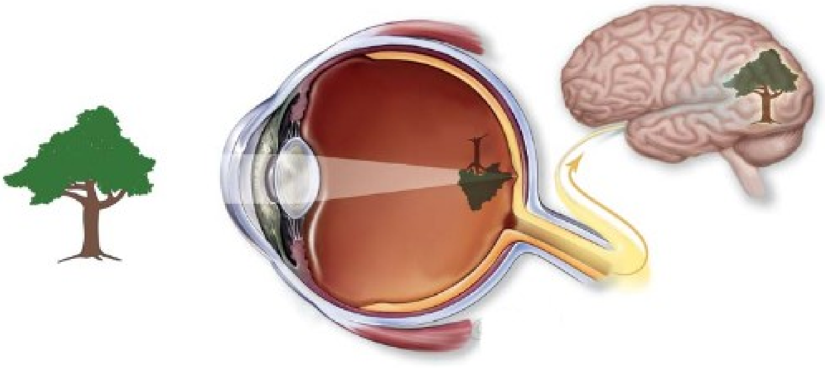
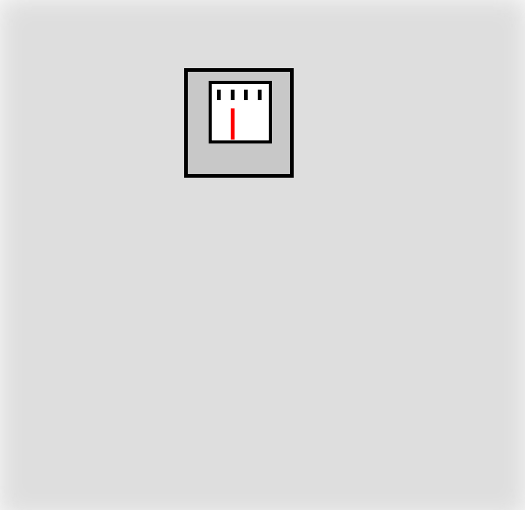
Vision and reconstruction



http://tpe.vision.aveugles.free.fr/vision.php

Processes and interfaces



Operational approach:
Fundamental notions

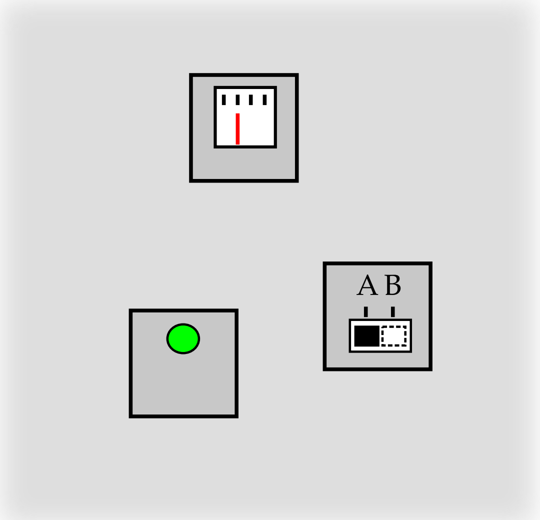
- experiment
- measurement
- observation
- preparation
- intervention

Subsume instance as:

- **process**

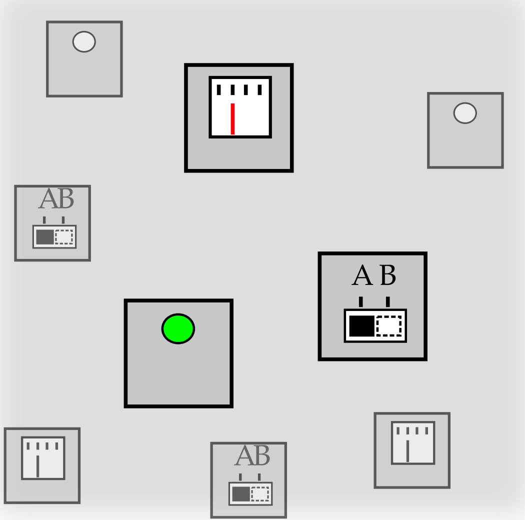
Processes have **outcomes**.
Represent processes as **boxes**.

Processes and interfaces



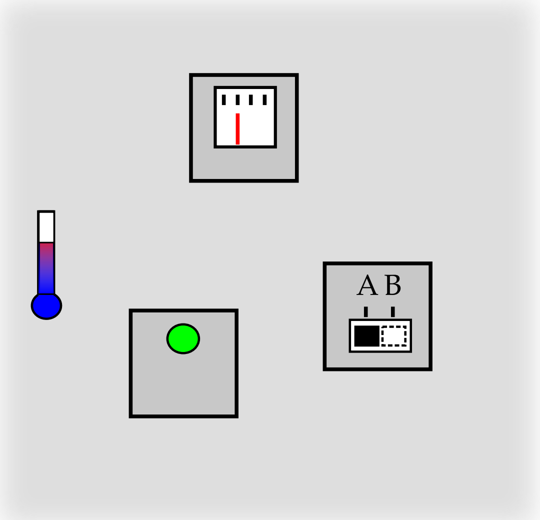
Processes are not isolated. Outcomes depend on other processes. We want to predict **correlations**.

Processes and interfaces



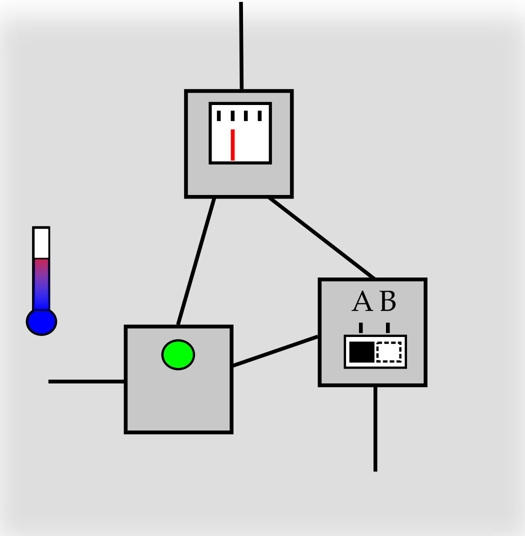
The outcome of a given set of processes depends generally on a large number of other processes.

Processes and interfaces



We treat these external processes collectively. We call this a **boundary condition**.

Processes and interfaces

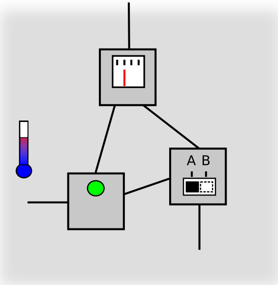


We introduce the notion of **interface** to model interaction between processes. An interface encodes communication or information exchange between processes. we depict this as a **link**.

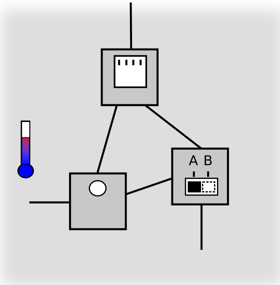
Open links carry boundary conditions.

Processes and interfaces form networks, represented as **labeled graphs**.

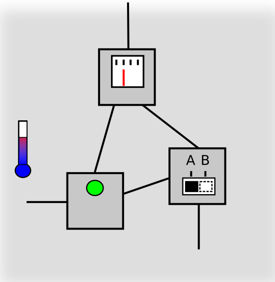
Probabilities



To obtain a probability we need to ask for ⟵ this conditional on this ⟶



Probabilities



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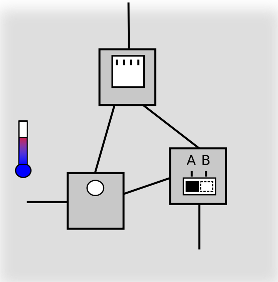
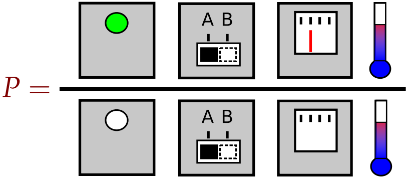
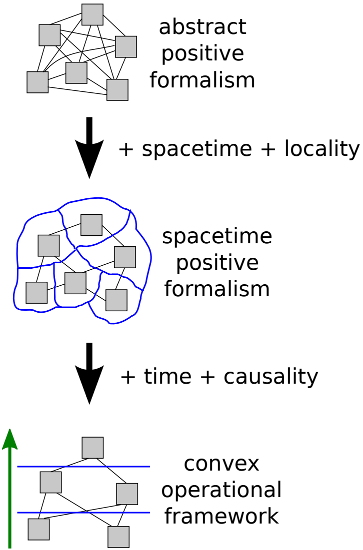
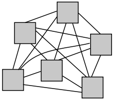
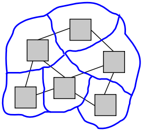
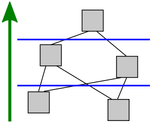


Diagrams translate to **positive real numbers**. Probability is a **quotient**.



$$P = \frac{P(g) \diamond Q(A) \diamond R([x_1, x_2]) \diamond b}{P(*) \diamond Q(A) \diamond R(*) \diamond b}$$

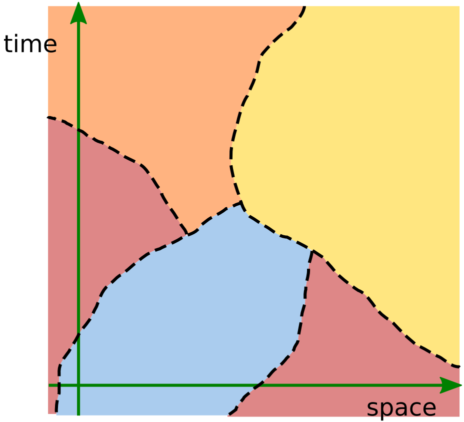

abstract positive formalism
+ spacetime + locality
spacetime positive formalism
+ time + causality
convex operational framework

classical (lattices)		**quantum** (anti-lattices)
abstract classical statistical theory	 abstract positive formalism	abstract quantum theory
	↓ + spacetime + locality	
spacetime statistical field theory	 spacetime positive formalism	general boundary formulation / axiomatic QFT
	↓ + time + causality	
statistical mechanics	 convex operational framework	standard formulation of quantum theory

The local positive formalism

Spacetime locality provides a powerful organizing principle. Processes only interface with adjacent processes. This decreases considerably the inter-connectivity of the graph.

The local positive formalism



For a **local** description cut up spacetime into pieces, called **regions**. These are in contact with each other through **hypersurfaces**.

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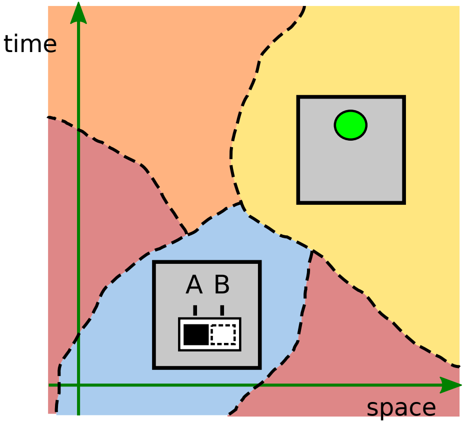

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Associate **processes** to regions to encode experiments, observations, interventions etc.

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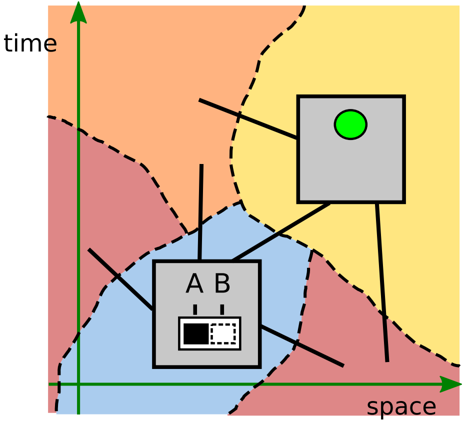

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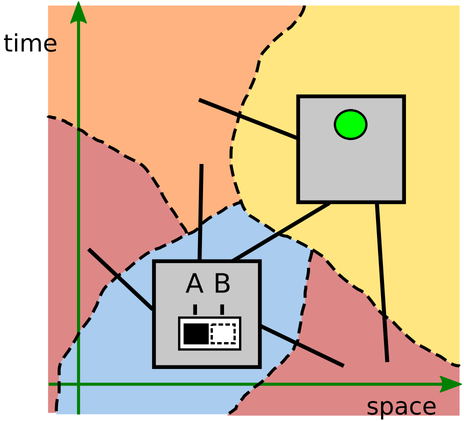

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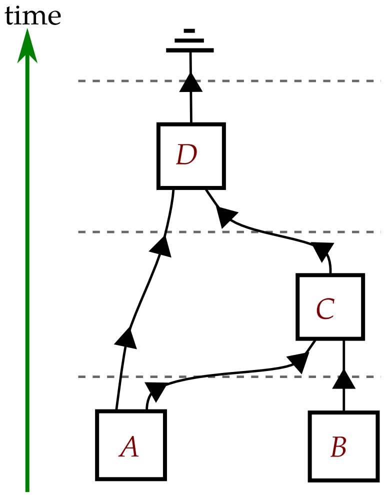
Glue regions along hypersurfaces ⟷ Compose processes along links

Time-evolution positive formalism



Suppose that we have a **fixed notion of time**. The arrangement of processes in time then yields a **partial order**.

We allow links only between processes that are comparable. Links carry **arrows** directed to the future. We obtain a **directed acyclic graph**.

Ingredients

$\mathcal{B}$ real **partially ordered vector space** with inner product $(\!|\cdot,\cdot|\!)$ and **order unit** $\mathbf{e} \in \mathcal{B}$.

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In classical theory: **statistical distributions**

- $\mathcal{B}$ space of square-integrable functions on probability space (L, μ)
- $(\!| \sigma, \tau |\!) = \int \sigma(x)\tau(x) \mathrm{d}\mu(x)$
- $\mathbf{e} = 1$ the unit constant function on L

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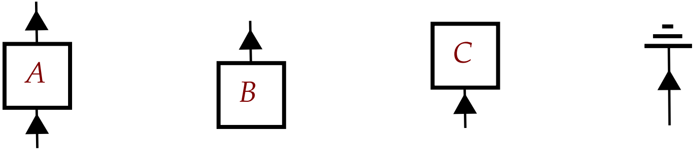
In quantum theory: **density matrices**

- $\mathcal{B}$ space of self-adjoint operators on the Hilbert space $\mathcal{H}$
- $(\!|\sigma, \tau|\!) = \mathrm{tr}(\sigma\tau)$
- $\mathbf{e} = \mathrm{id}$ the identity operator on $\mathcal{H}$

Elementary diagrams

Diagrams represent linear maps.



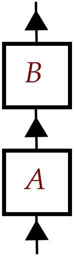
$A : \mathcal{B} \to \mathcal{B}$
operation

$B : \mathbb{R} \to \mathcal{B}$
preparation

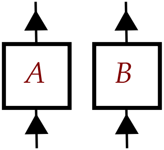
$C : \mathcal{B} \to \mathbb{R}$
effect

$\sigma \mapsto (\!|\mathbf{e}, \sigma|\!)$
discard

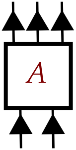
Composition



$B \circ A : \mathcal{B} \to \mathcal{B}$

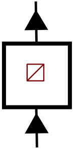
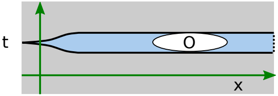
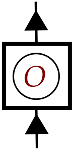


$A \otimes B :$
$\mathcal{B}_1 \otimes \mathcal{B}_2 \to \mathcal{B}_1 \otimes \mathcal{B}_2$



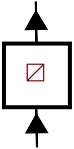
$A : \mathcal{B}_1 \otimes \mathcal{B}_2$
$\to \mathcal{B}_3 \otimes \mathcal{B}_4 \otimes \mathcal{B}_5$

Classical operations

spacetime	diag.	phase space	(stat.) state space
		$U : L \to L$ preserve measure	$\hat{U} : \mathcal{B} \to \mathcal{B}$ $(\hat{U}(\sigma))(x) = \sigma(U^{-1}(x))$
		$O : L \to \mathbb{R}$	$\hat{O} : \mathcal{B} \to \mathcal{B}$ $(\hat{O}(\sigma))(x) = O(x)\sigma(x)$
			$A : \mathcal{B} \to \mathcal{B}$ positive

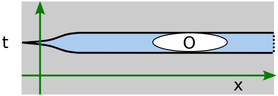
Quantum operations

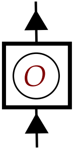
spacetime | diag. | Hilbert space | (stat.) state space



$U : \mathcal{H} \to \mathcal{H}$
unitary

$\hat{U} : \mathcal{B} \to \mathcal{B}$
$\hat{U}(\sigma) = U \sigma U^\dagger$





$O : \mathcal{H} \to \mathcal{H}$
hermitian
$O = \sum_j o_j P_j$

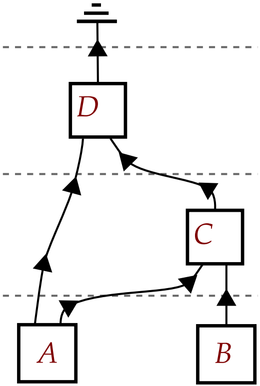
$\hat{O} : \mathcal{B} \to \mathcal{B}$
$\hat{O}[i](\sigma) = P_i \sigma P_i^\dagger$
$\hat{O}[*](\sigma) = \sum_j \hat{O}[j]$
$\hat{O}(\sigma) = \sum_j o_j \hat{O}[j]$

$A : \mathcal{B} \to \mathcal{B}$
completely positive
$A(\sigma) = \sum_j K_j \sigma K_j^\dagger$

Evaluation and probability

Example:

- preparations A, B
- measurements C, D
- a discard effect $\mathbf{e}$



Evaluation and probability

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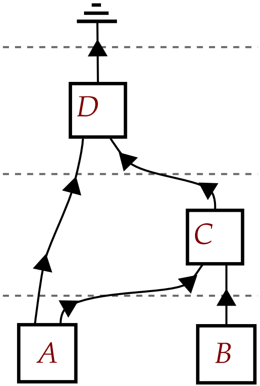
What is the probability for outcome (i, j) in measurements (C, D)?



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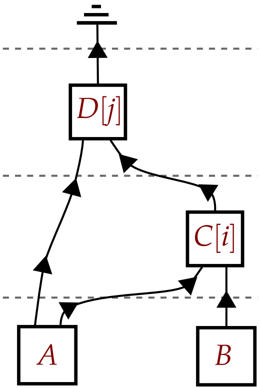

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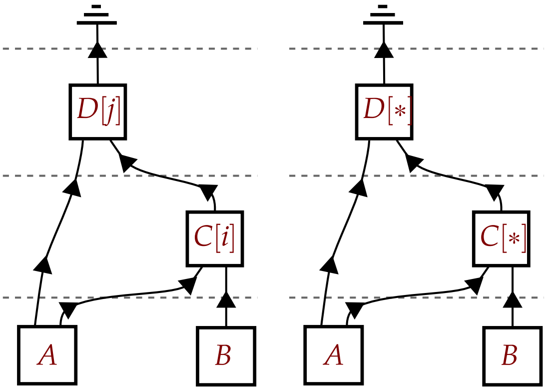

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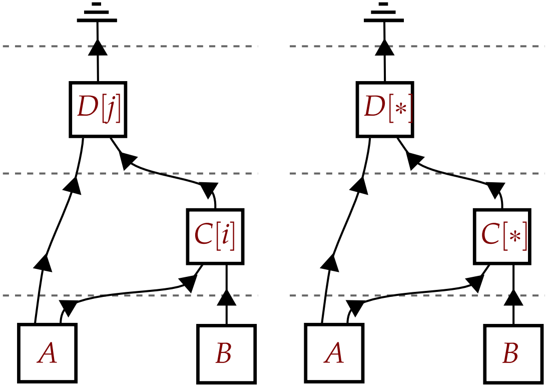

$A \diamond B \diamond C[i] \diamond D[j] \diamond \mathbf{e}$ $\quad$ $A \diamond B \diamond C[*] \diamond D[*] \diamond \mathbf{e}$

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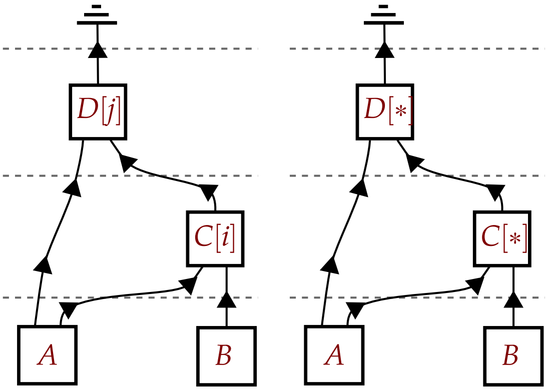

$A \diamond B \diamond C[i] \diamond D[j] \diamond \mathbf{e}$ $\quad$ $A \diamond B \diamond C[*] \diamond D[*] \diamond \mathbf{e}$

Probability: $$P(i,j) = \frac{A \diamond B \diamond C[i] \diamond D[j] \diamond \mathbf{e}}{A \diamond B \diamond C[*] \diamond D[*] \diamond \mathbf{e}}$$

Causality

Causality is implemented through a normalization condition on operations. This is **time-asymmetric**, requires directionality.

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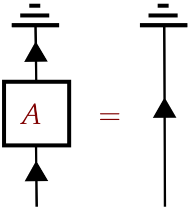
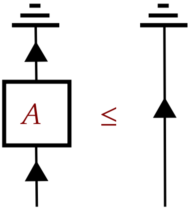
$A : \mathcal{B} \to \mathcal{B}$ **non-selective**



$(\!|\mathbf{e}, A\sigma|\!) = (\!|\mathbf{e}, \sigma|\!)$

$A : \mathcal{B} \to \mathcal{B}$ **selective**



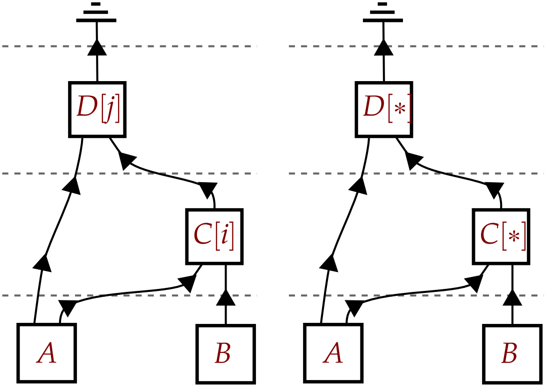
$(\!|\mathbf{e}, A\sigma|\!) \leq (\!|\mathbf{e}, \sigma|\!)$

Evaluation and probability (II)

Example:

- preparations A, B
- measurements C, D
- a discard effect $\mathbf{e}$

What is the probability for outcome (i, j) in measurements (C, D)?



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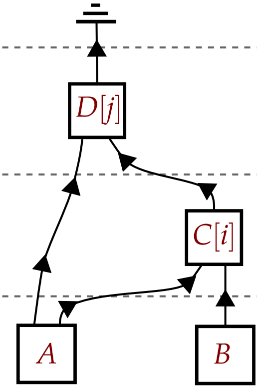

$A \diamond B \diamond C[i] \diamond D[j] \diamond \mathbf{e}$

Causality implies:

Composites of **non-selective operations** (including preparations and discard) evaluate to 1.

$A \diamond B \diamond C[*] \diamond D[*] \diamond \mathbf{e} = 1$

Probability: $\boxed{P(i, j) = A \diamond B \diamond C[i] \diamond D[j] \diamond \mathbf{e}}$

A lesson for QG:

Absent a fixed notion of time, there is no way to consistently implement the standard causality condition. Hence, diagrams do not in general evaluate to probabilities, only quotients do.

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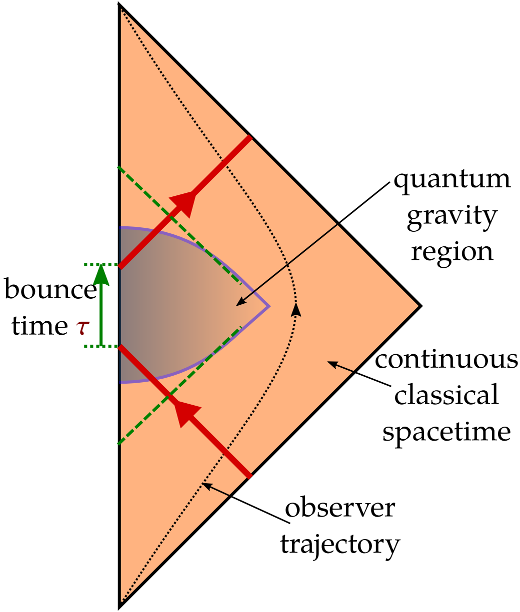
This also applies to the modulus squares of amplitudes!

Example: Black hole bounce



Black hole to white hole bounce model.

Two parameters:

- shell mass m
- bounce time τ

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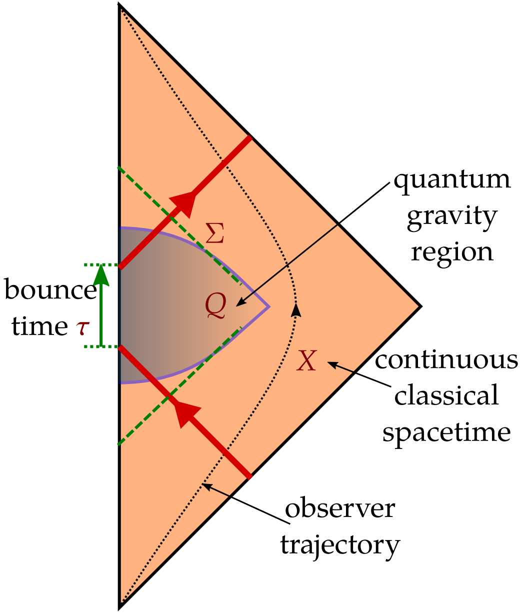

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Distinguish spacetime regions Q and X, with interfacing hypersurface Σ.

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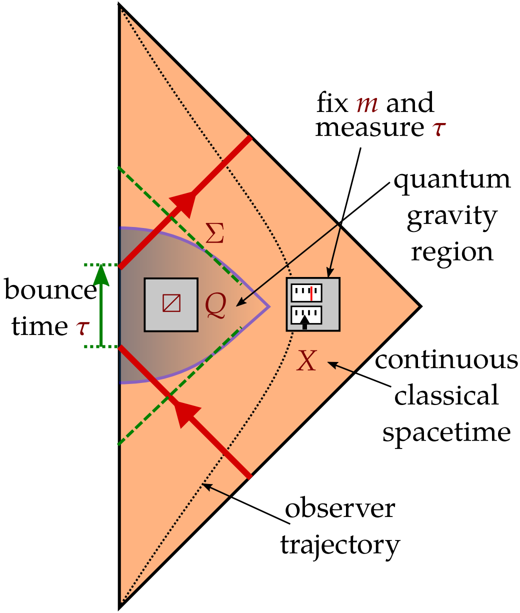


Black hole to white hole bounce model.

Two parameters:

- shell mass m
- bounce time τ

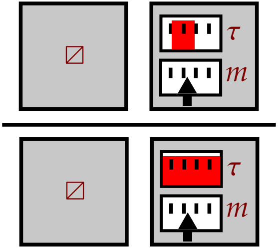
Distinguish spacetime regions Q and X, with interfacing hypersurface Σ.

Introduce two processes with one link.

Example: Black hole bounce

Given a fixed shell mass m, we wish to determine the **probability** for the bounce time τ to lie in a given interval $[\tau_1, \tau_2]$. This is,

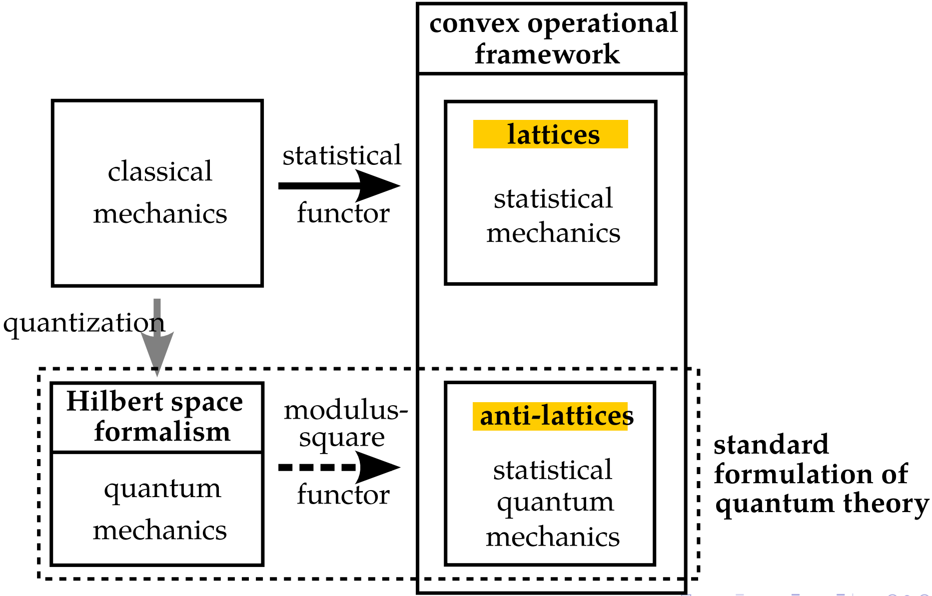
$P(m, [\tau_1, \tau_2]) =$



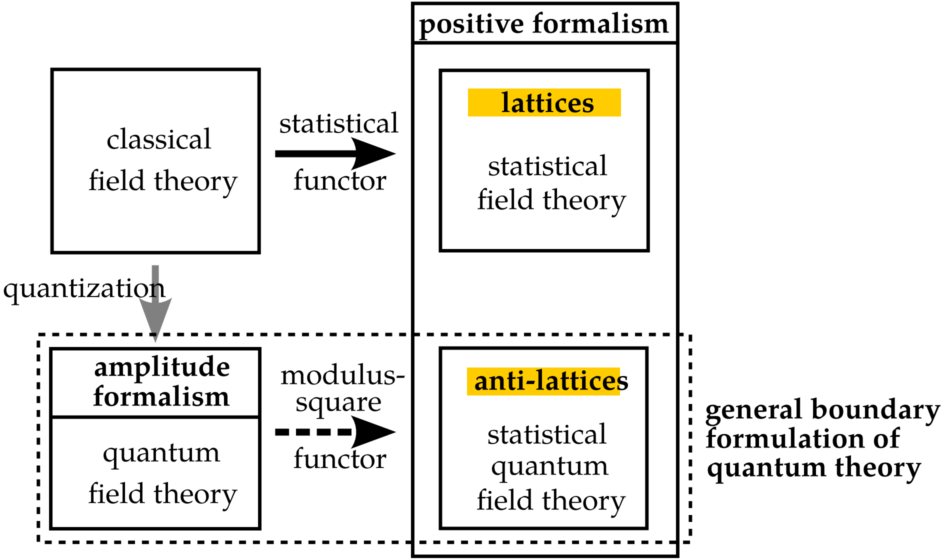
$$P(m, [\tau_1, \tau_2]) = \frac{A_Q \diamond A_X[m, [\tau_1, \tau_2]]}{A_Q \diamond A_X[m, [0, \infty]]}$$

R. O., *A predictive framework for quantum gravity and black hole to white hole transition*, Phys. Lett. A **82** (2018) 2622–2625, `arXiv:1804.02428`.

Fundamental physics: Time-evolution frameworks



Fundamental physics: Spacetime frameworks



Some applications in QFT

- Non-linear models:
  - **Three dimensional quantum gravity** is a TQFT and fits "automatically". [Witten 1988;…]
  - **Quantum Yang-Mills theory** in 2 dimensions for arbitrary regions and hypersurfaces with corners. [RO 2006]
  - **abelian Yang-Mills theory** in higher dimensions. [Díaz 2014–; Díaz, RO 2017]
- New **S-matrix** type asymptotic amplitudes [Colosi, RO 2008; Colosi 2009; Dohse 2011; 2012]
- QFT in **curved spacetime**: dS, AdS and more [Colosi, Dohse 2009–]
- Rigorous and functorial quantization of linear and affine field theories without metric background. [RO 2010; 2011; 2012]
- **Unruh effect**. [Colosi, Rätzel 2012; Bianchi, Haggard, Rovelli 2013]
- Striking results for **fermions**: Hilbert spaces become **Krein spaces** and an **emergent notion of time**. [RO 2012]
- Partial solution of **state-locality** in fermionic QFT [RO 2013]

Conclusions

- The positive formalism **extends the scope of quantum theory** to situations where no metric background is present. It is thus a suitable **basis for formulating quantum theories of gravity**.
- **Loop quantum gravity** already heavily uses the (pure state version) of the positive formalism.
- The positive formalism **unifies** the essentials of **various formalisms** that have been developed for quantum foundations in recent years (causaloid formalism, quantum combs, operator-tensor formalism, process matrix, categorical approaches, ...)
- It **has been successfully applied** in a number of QG and QFT contexts beyond the reach of the standard formulation.

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→ The **positive formalism / GBF** appears ideally suited as a basic ingredient for diverse contributions to the QSS project.

Main reference

R. O., *A local and operational framework for the foundations of physics*, to appear in Adv. Theor. Math. Phys., arXiv:1610.09052.