General covariance and the foundations of quantum theory

Robert Oeckl

Centro de Ciencias Matemáticas
UNAM, Morelia

Quantum Theory: Reconsideration of Foundations – 6
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“That ‘sharp time’ is an anomaly in Q.M. and that besides, so to speak independently of that, the special role of time poses a serious obstacle to adapting Q.M. to the relativity principle, is something that in recent years I have brought up again and again, unfortunately without being able to make the shadow of a useful counterproposal.”

E. Schrödinger, *Die gegenwärtige Situation in der Quantenmechanik*, Naturwissenschaften **23** (1935), 807–812, 823–828, 844–849

The special role of time

Historically, quantum theory was first developed in a non-relativistic context, modeled on an analogy with non-relativistic classical mechanics. This imprinted a special role of time on its very foundations.

The operational meaning of its ingredients is tied to a background time

- States in the Hilbert space $\mathcal{H}$ encode information on a system between measurements.
- The product of observables understood as operators on $\mathcal{H}$ encodes temporal composition of measurements.
- The Hamiltonian operator encodes evolution in time.

While a "work-around" was found to accomodate special relativity, general relativity poses a much more serious challenge.

Advocating a new formulation of the foundations

Can we reformulate what constitutes a quantum theory such that

- there is no reference to time
- the traditional formalism of a Hilbert space with operators as observables is recovered exactly (in the right cirumstances)
- locality is manifest?

Advocating a new formulation of the foundations

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YES, using:

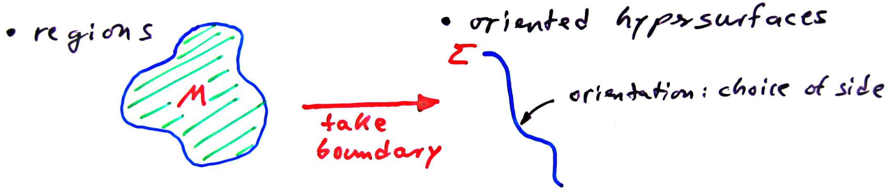
- The mathematical framework of **topological quantum field theory.** (A branch of modern algebraic topology.)
- A generalization of the **Born rule.**

This is called the **general boundary formulation (GBF).**

GBF: Basic ingredients – spacetime

The elimination of an abolsute notion of time comes at the cost of introducing a weak notion of spacetime. This consists of:

- **hypersurfaces**: oriented manifolds of dimension $d - 1$
- **regions**: oriented manifolds of dimension d with boundary



This setting is generally covariant in the sense that the manifolds merely need to carry a topological structure. However, they may carry additonal structure such as a metric, depending on the model to be considered.

GBF: Basic ingredients – algebraic structures

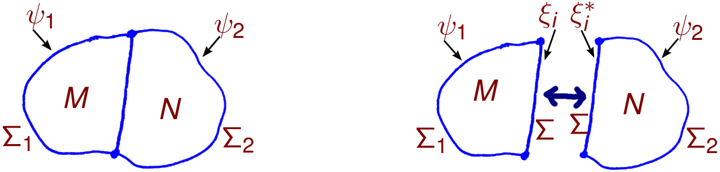
As in **topological quantum field theory** [Witten, Segal, Atiyah etc. 1980's] algebraic structures are associated to the spacetime structures:

- To each hypersurface Σ associate a Hilbert space $\mathcal{H}_\Sigma$ of **states.**
- To each region M with boundary ∂M associate a linear **amplitude map** $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$.

Core axioms

The structures are subject to a number of **axioms**:

- Let $\overline{\Sigma}$ denote Σ with opposite orientation. Then $\mathcal{H}_{\overline{\Sigma}} = \mathcal{H}_{\Sigma}^*$.
- **(Decomposition rule)** Let $\Sigma = \Sigma_1 \cup \Sigma_2$ be a disjoint union of hypersurfaces. Then $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$.
- **(Gluing rule)** If M and N are adjacent regions, then:

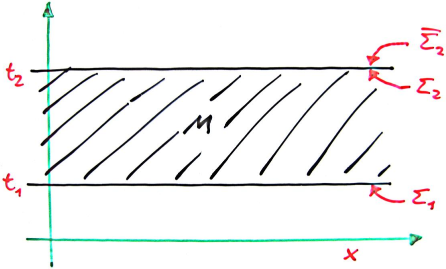


$$\rho_{M\cup N}(\psi_1 \otimes \psi_2) := \sum_{i\in\mathbb{N}} \rho_M(\psi_1 \otimes \xi_i)\rho_N(\xi_i^* \otimes \psi_2)$$

Here, $\psi_1 \in \mathcal{H}_{\Sigma_1}$, $\psi_2 \in \mathcal{H}_{\Sigma_2}$ and $\{\xi_i\}_{i\in\mathbb{N}}$ is an ON-basis of $\mathcal{H}_\Sigma$.

Recovering transition amplitudes

Consider special regions in Minkowski space.



- region: $M = [t_1, t_2] \times \mathbb{R}^3$
- boundary: $\partial M = \Sigma_1 \cup \overline{\Sigma}_2$
- state space: $\mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\overline{\Sigma}_2} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$

Via time-translation symmetry identify $\mathcal{H}_{\Sigma_1} \cong \mathcal{H}_{\Sigma_2} \cong \mathcal{H}$. Then:

$$\boxed{\rho_{[t_1,t_2]}(\psi_1 \otimes \psi_2^*) = \langle \psi_2, U(t_1, t_2)\psi_1 \rangle}$$

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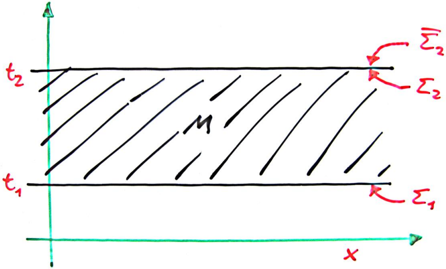

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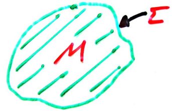
- But, does it make sense do go beyond this example?
- Does the boundary Hilbert space $\mathcal{H}_{\partial M}$ have a useful physical interpretation in general?

Amplitudes and probabilities

Generalizing the Born rule

Consider the context of a general spacetime region M with boundary Σ.



Probabilities in quantum theory are generally conditional probabilities. They depend on two pieces of information. Here these are:

- $\mathcal{S} \subset \mathcal{H}_\Sigma$ representing **preparation** or **knowledge**
- $\mathcal{A} \subset \mathcal{H}_\Sigma$ representing **observation** or the **question**

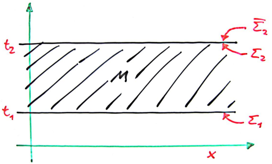
The probability that the system is described by $\mathcal{A}$ given that it is described by $\mathcal{S}$ is:

$$\boxed{P(\mathcal{A}|\mathcal{S}) = \frac{\|\rho_M \circ P_\mathcal{S} \circ P_\mathcal{A}\|^2}{\|\rho_M \circ P_\mathcal{S}\|^2}} = \frac{\sum_{i\in I} |\rho_M\left(P_\mathcal{S}\left(P_\mathcal{A}(\xi_i)\right)\right)|^2}{\sum_{i\in I} |\rho_M\left(P_\mathcal{S}(\xi_i)\right)|^2}$$

- $P_\mathcal{S}$ and $P_\mathcal{A}$ are the orthogonal projectors onto the subspaces, $\{\xi_i\}_{i\in I}$ an ON-basis of $\mathcal{H}_\Sigma$.

Recovering standard probabilities

Generalizing the Born rule



To compute the probability of measuring ψ_2 at t_2 given that we prepared ψ_1 at t_1 we set

$$\mathcal{S} = \psi_1 \otimes \mathcal{H}^*, \quad \mathcal{A} = \mathcal{H} \otimes \psi_2^*.$$

The resulting expression yields correctly

$$P(\mathcal{A}|\mathcal{S}) = \frac{\|\rho_{[t_1,t_2]} \circ P_{\mathcal{S}} \circ P_{\mathcal{A}}\|^2}{\|\rho_{[t_1,t_2]} \circ P_{\mathcal{S}}\|^2} = \frac{|\rho_{[t_1,t_2]}(\psi_1 \otimes \psi_2^*)|^2}{1} = |\langle \psi_2^*, U(t_1,t_2)\psi_1 \rangle|^2.$$

Status and recent developments

- By restricting to special spacetime regions (time-intervals) the traditional formalism is reproduced exactly. [RO 2005; RO 2010]
- Three dimensional quantum gravity is already formulated as a TQFT and fits thus "automatically" into the GBF. Also the GBF is extensively used in spin foam quantum gravity. [C. Rovelli et al.]
- There is a powerful and compelling concept of observable. [RO 2010; RO 2012]
- A natural testing ground for the GBF is quantum field theory.
  - State spaces on timelike hypersurfaces and "evolution" in spacelike directions. [RO 2005]
  - New S-matrix type asymptotic amplitudes in Minkowski space, DeSitter space, Anti-deSitter space. [D. Colosi, RO 2008; D. Colosi 2009; M. Dohse 2011]
  - Quantum Yang-Mills Theory in 2 dimensions for arbitrary regions and hypersurfaces with corners. [RO 2006]
  - Rigorous (holomorphic) quantization of linear and affine field theories without need for metric background. [RO 2010; RO 2011]

Selected references

Short overview:

R. O., *Probabilities in the general boundary formulation*, J. Phys.: Conf. Ser. **67** (2007) 012049, hep-th/0612076.

Basics:

R. O., *General boundary quantum field theory: Foundations and probability interpretation*, Adv. Theor. Math. Phys. **12** (2008) 319-352, hep-th/0509122.

Observables:

R. O., *Observables in the general boundary formulation*, Quantum Field Theory and Gravity (Regensburg, 2010), Birkhäuser, Basel, 2012, pp. 137–156, arXiv:1101.0367.

R. O., *Schrödinger-Feynman quantization and composition of observables in general boundary quantum field theory*, arXiv:1201.1877.