A new gauge invariant framework for cosmological perturbations

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Background independent Quantum Gravity

gauge invariant observables

⇙ **large scale limit**

minisuperspace approximation

global observables

⇘ **perturb around b'ground**

quantum fields on curved spacetime

local fields, gauge problem

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⇄

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local fields, gauge problem

- (quantum) dynamic of background influences dynamics of perturbations
- backreaction of the perturbations onto background

What are the gauge invariant observables reducing to local (quantum) field observables?

A gauge invariant Hamiltonian framework for cosmological perturbations to arbitrary high order

perturbations around symmetry reduced sectors (of gr)

- **gauge invariant:**
  unambigious results, allows characterization of symmetric physical states
- **to arbitrary high order:**
  backreaction, scattering, embedding in full theory
- **canonical:**
  quantization, space–time algebra of observables: locality
- **“background” fully dynamical:**
  definition of physical time parameter, backreaction, effective approach

Do not perturb around fixed background metric but around phase space sector describing configurations with high symmetry.

Overview

1. Perturbation variables
2. Complete observables
3. Perturbative complete observables
4. Locality properties
5. Outlook and Conclusions

Subdivide phase space

phase space $=$
space of fields on Σ

$\mathcal{P} =$ projection acting on
phase space functions

$\swarrow$ $\searrow$

$\mathcal{P}$[functions]	$(\mathrm{Id} - \mathcal{P})$[functions]
averaging $$E = \frac{1}{3}\int_\Sigma E_j^a \delta_a^j \, d\sigma$$	inhomogeneities $$e_j^a = E_j^a - E\,\delta_j^a$$
evaluation on a "background" phase space point $$\delta_j^a = E_j^a(\text{flat space})$$	perturbations $$e'^a_j = E_j^a - \delta_j^a$$

Homogeneous and inhomogeneous variables

- subdivision consistent with symplectic structure
- homogeneous variables arise through averaging from full phase space
- are fully dynamical $\rightarrow$ provide global clocks, important for invariance to higher order

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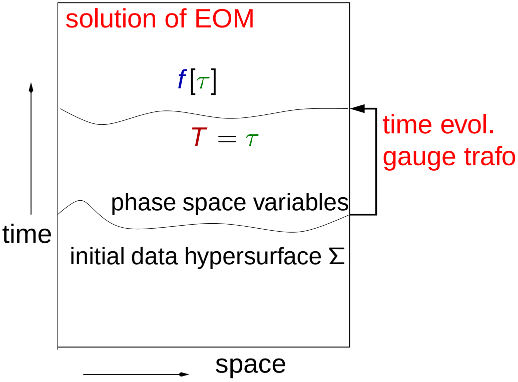
- homogeneous variables (■ A, E) are zeroth order
- inhomogeneous (canonical) variables (■ a^j_a, e^b_k) are first order
- no higher order variables
- **higher order terms** are polynomials of first order variables

How to compute gauge invariant observables?

Complete Observables C.Rovelli ’90s, B.D. 04,05



- clocks T: specify position and shape of hypersurfaces by $T = \tau$
- $f[\tau]$ gives value of phase space function f on hypersurface $T = \tau$
- $f[\tau]$ is invariant under changes of initial data hypersurface Σ (i.e. gauge tranformations)

Compute complete observables B.D. 04,05

- use clock adapted gauge generators $\tilde{C}_K$: generate weakly Abelian gauge flows
- this allows for a power series expansion of the complete observable

$$f[\tau] \simeq \sum_{r=0}^{\infty} \frac{1}{r!} \{\cdots\{f, \tilde{C}_{K_1}\}, \cdots\}\tilde{C}_{K_r}\} \, (\tau^{K_1} - T^{K_1}) \cdots (\tau^{K_r} - T^{K_r})$$

- for a certain set of clocks this power series expansion can be used to expand the complet observable **order by order**
- results in **gauge invariant observables of order m**
- contact with standard perturbation theory:
  terms can be interpreted as
  - **free propagation**
  - **interaction terms**
  - **gauge invariant extension terms**

Clocks

- **first order clocks** (no zeroth order term) describe shape of hypersurface $T^K = \tau^K = 0$
  - longitudinal clocks: $T^0 = \Delta^{-1}(\frac{1}{2}{}^T a^d{}_d - {}^{LL}a^d{}_d)$ related to longitudinal gauge, non–local
  - scalar field (inhomogeneities) as clocks: $T^0 = \psi$ local description of hypersurface
- **global zeroth order clock** (no first order term) describes position of hypersurface, provide global time parameter, associated generator $\tilde{C}$ $T = \tau$
  - volume of hypersurface $T = \mathrm{Vol}_\Sigma$
  - averaged scalar field $T = \Psi$

These clocks allow for a consistent expansion of the complete observables for arbitrary values of τ.

Perturbative complete observables

reorder terms in power series expansion for $f[\tau]$

Perturbative complete observables

$$^{(2)}f[\tau] \quad = \quad \alpha_{free}^{(\tau - T)}(f)$$ "free" propagation

Perturbative complete observables

$$^{(2)}f[\tau] = \alpha_{free}^{(\tau-T)}(f) \qquad \text{“free” propagation}$$

$$-\sum_K {}^{(0)}\{\alpha_{free}^{(\tau-T)}(f), {}^{(1)}\tilde{C}_K\}T^K \qquad \text{gauge invariant extension}$$

Perturbative complete observables

$$
\begin{aligned}
{}^{(2)}f[\tau] \;=\;& \alpha_{free}^{(\tau-T)}(f) && \text{“free” propagation} \\
& - \sum_K {}^{(0)}\{\alpha_{free}^{(\tau-T)}(f), {}^{(1)}\tilde{C}_K\} T^K && \text{gauge invariant extension} \\
& + \int_0^{(\tau-T)} ds\;\; \alpha_{free}^{(\tau-T-s)} \left({}^{(2)}\{\alpha_{free}^{s}(f), \tilde{C}\}\right) && \text{interaction term}
\end{aligned}
$$

Perturbative complete observables

$$
\begin{aligned}
{}^{(2)}f[\tau] &= \alpha_{free}^{(\tau - T)}(f) && \text{“free” propagation} \\
&- \sum_K {}^{(0)}\{\alpha_{free}^{(\tau - T)}(f), {}^{(1)}\tilde{C}_K\} T^K && \text{gauge invariant extension} \\
&+ \int_0^{(\tau - T)} ds \;\; \alpha_{free}^{(\tau - T - s)} \left({}^{(2)}\{\alpha_{free}^{s}(f), \tilde{C}\} \right) && \text{interaction term} \\
&+ \text{gauge invariant extension of interaction term}
\end{aligned}
$$

Perturbative complete observables

$$
\begin{aligned}
{}^{(2)}f[\tau] \;=\;& \alpha_{free}^{(\tau-T)}(f) && \text{“free” propagation} \\
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& +\int_0^{(\tau-T)} ds\;\; \alpha_{free}^{(\tau-T-s)}\left({}^{(2)}\{\alpha_{free}^{s}(f), \tilde{C}\}\right) && \text{interaction term} \\
& + \text{gauge invariant extension of interaction term}
\end{aligned}
$$

Can be extended to any order.

- **We can compute gauge invariant observables order by order.**
- **These observables encode local information.**
- **Moreover time evolution with respect to a physical clock can also be implemented perturbatively.**

Locality properties

scalar fields as clocks

$\{f[\tau], f[\tau']\} = 0$
for τ, τ' space–like related

$\{f[\tau], f[\tau + \epsilon]\}$

$= G(\tau, \tau + \epsilon)\left(1 + \frac{\text{Energy}(f)}{\text{Energy(clocks)}}\right)$

cannot make Energy(clocks)
arbitrarily large

fundamental resolution limit?

Giddings, Marolf, Hartle '05

B.D., Tambornino '06

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longitudinal clocks

$\{f[\tau], f[\tau']\} = 0$
for τ, τ' space–like related
holds only to second order

$\{f[\tau], f[\tau + \epsilon]\}$

$= G(\tau, \tau + \epsilon)$

non–local measurement but

local in flat space limit to 2nd order

B.D., Tambornino '06

Conclusion and Outlook

- central object of the perturbative scheme are observables
- divide phase space into homogeneous and inhomogeneous variables, keep both completely dynamical
- first canonical formalism for cosmological perturbations to any order
- explicit calculation of second order gauge invariants are now possible
- backreaction effects
- hints towards fundamental restrictions of locality

Conclusion and Outlook

- extendible to perturbations around midisuperspaces
- embed approximations into each other
- allows for characterization of physical symmetric states
- altering the constraints (effective approach):
  consistent equations of motion