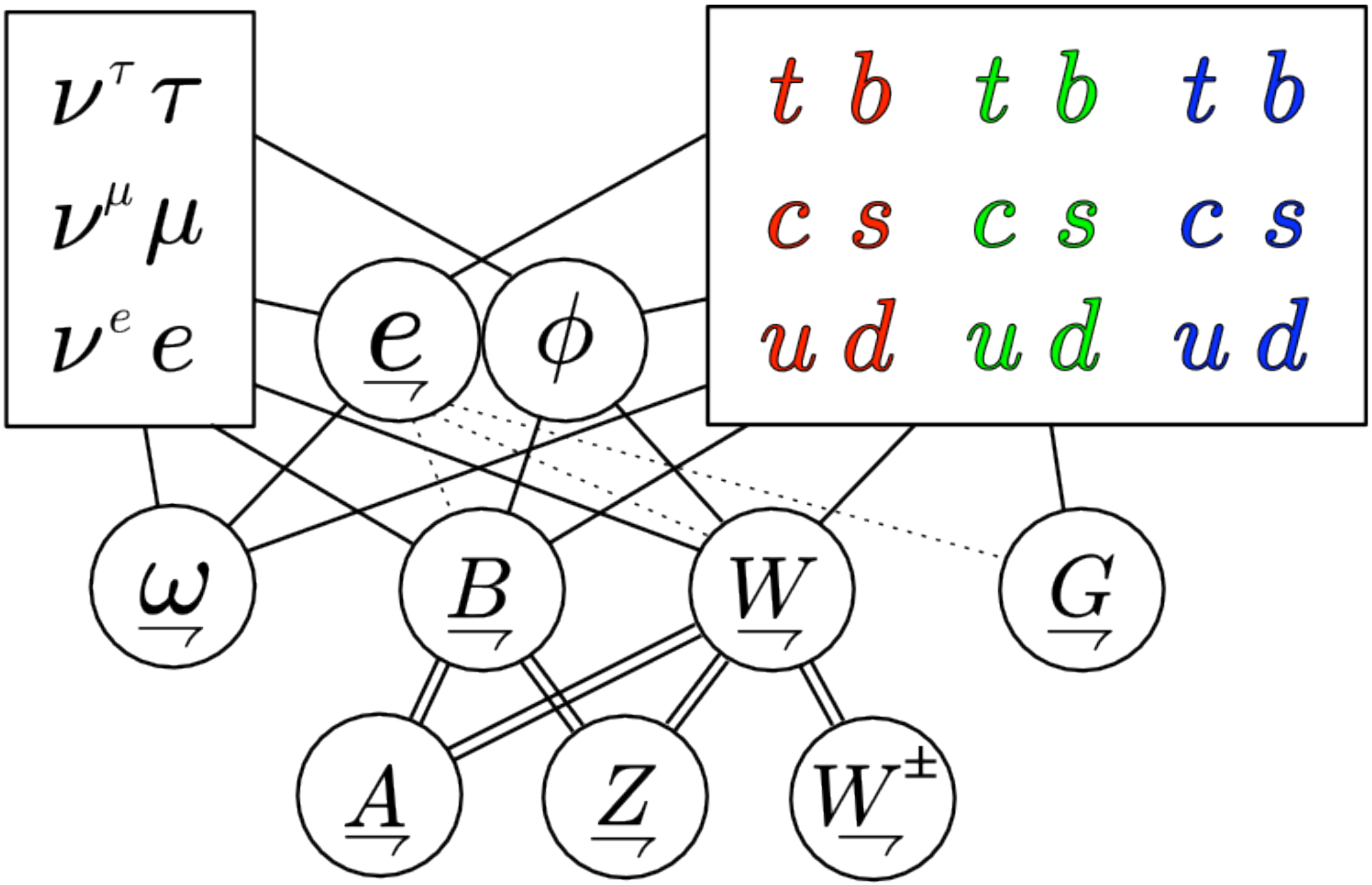
Standard model and gravity



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Standard model and gravity in one big connection

$$A = H + G + \psi = \begin{bmatrix} H^+ & \psi^- \\ & G^- \end{bmatrix} \qquad \in \quad so(1,7) + so(1,7) + \mathbb{C}(8\times 8)$$

$$= \begin{bmatrix}
\frac{1}{2}\omega_L + iW^3 & iW^1 + W^2 & -\frac{1}{4}e_R\phi_0^* & \frac{1}{4}e_R\phi_+ & \nu_L & u_L^r & u_L^g & u_L^b \\
iW^1 - W^2 & \frac{1}{2}\omega_L - iW^3 & \frac{1}{4}e_R\phi_+^* & \frac{1}{4}e_R\phi_0 & e_L & d_L^r & d_L^g & d_L^b \\
-\frac{1}{4}e_L\phi_0 & \frac{1}{4}e_L\phi_+ & \frac{1}{2}\omega_R + iB & & \nu_R & u_R^r & u_R^g & u_R^b \\
\frac{1}{4}e_L\phi_+^* & \frac{1}{4}e_L\phi_0^* & & \frac{1}{2}\omega_R - iB & e_R & d_R^r & d_R^g & d_R^b \\
 & & & & iB & & & \\
 & & & & & \frac{-i}{3}B + iG^{3+8} & iG^1 - G^2 & iG^4 - G^5 \\
 & & & & & iG^1 + G^2 & \frac{-i}{3}B - iG^{3+8} & iG^6 - G^7 \\
 & & & & & iG^4 + G^5 & iG^6 + G^7 & \frac{-i}{3}B - \frac{2i}{\sqrt{3}}G^8
\end{bmatrix}$$

Correct interactions and charges from curvature:

$$F = dA + AA$$

$$= (dH + HH) + (dG + GG) + (d\psi + H\psi + \psi G)$$

Gravitational part of bosonic connection

Using **chiral** (*Weyl*) $\mathbb{C}(4 \times 4)$ representation of **Cl(1,3) Dirac matrices**:

$$\gamma_0 = \sigma_1 \otimes 1 = \begin{bmatrix} & 1 \\ 1 & \end{bmatrix} \qquad \gamma_\pi = i\sigma_2 \otimes \sigma_\pi = \begin{bmatrix} & \sigma_\pi \\ -\sigma_\pi & \end{bmatrix}$$

$$\gamma_{0\varepsilon} = \gamma_0 \gamma_\varepsilon = \begin{bmatrix} -\sigma_\varepsilon & \\ & \sigma_\varepsilon \end{bmatrix} \qquad \gamma_{\varepsilon\pi} = \gamma_\varepsilon \gamma_\pi = \begin{bmatrix} -i\epsilon_{\varepsilon\pi\tau}\sigma_\tau & \\ & -i\epsilon_{\varepsilon\pi\tau}\sigma_\tau \end{bmatrix}$$

Spacetime frame and **spin connection**:

$$\begin{aligned} \underrightarrow{\omega} + \underrightarrow{e} &= \underrightarrow{dx}^a \tfrac{1}{2} \omega_a{}^{\mu\nu} \gamma_{\mu\nu} + \underrightarrow{dx}^a (e_a)^\mu \gamma_\mu \\ &= \begin{bmatrix} (-\underrightarrow{\omega}^{0\varepsilon}\sigma_\varepsilon - \frac{i}{2}\underrightarrow{\omega}^{\varepsilon\pi}\epsilon_{\varepsilon\pi\tau}\sigma_\tau) & (\underrightarrow{e}^0 + \underrightarrow{e}^\pi \sigma_\pi) \\ (\underrightarrow{e}^0 - \underrightarrow{e}^\pi \sigma_\pi) & (\underrightarrow{\omega}^{0\varepsilon}\sigma_\varepsilon - \frac{i}{2}\underrightarrow{\omega}^{\varepsilon\pi}\epsilon_{\varepsilon\pi\tau}\sigma_\tau) \end{bmatrix} \\ &= \begin{bmatrix} \underrightarrow{\omega}_L & \underrightarrow{e}_R \\ \underrightarrow{e}_L & \underrightarrow{\omega}_R \end{bmatrix} \quad \in \quad \underrightarrow{Cl}^{1+2}(1,3) \end{aligned}$$

Note algebraic equivalence: $\quad Cl^{1+2}(1,3) = Cl^2(1,4) = so(1,4)$

Bosonic connection

$$\underrightarrow{H} = \tfrac{1}{2}\underrightarrow{\omega} + \tfrac{1}{4}\underrightarrow{e}\phi + \underrightarrow{B} + \underrightarrow{W} = \begin{bmatrix} \frac{1}{2}\underrightarrow{\omega_L} + i\underrightarrow{W}^3 & i\underrightarrow{W}^1 + \underrightarrow{W}^2 & -\frac{1}{4}\underrightarrow{e_R}\phi_0^* & \frac{1}{4}\underrightarrow{e_R}\phi_+ \\ i\underrightarrow{W}^1 - \underrightarrow{W}^2 & \frac{1}{2}\underrightarrow{\omega_L} - i\underrightarrow{W}^3 & \frac{1}{4}\underrightarrow{e_R}\phi_+^* & \frac{1}{4}\underrightarrow{e_R}\phi_0 \\ -\frac{1}{4}\underrightarrow{e_L}\phi_0 & \frac{1}{4}\underrightarrow{e_L}\phi_+ & \frac{1}{2}\underrightarrow{\omega_R} + i\underrightarrow{B} & \\ \frac{1}{4}\underrightarrow{e_L}\phi_+^* & \frac{1}{4}\underrightarrow{e_L}\phi_0^* & & \frac{1}{2}\underrightarrow{\omega_R} - i\underrightarrow{B} \end{bmatrix}$$

$$= d\underrightarrow{x}^a \tfrac{1}{2} h_a{}^{\alpha\beta}\gamma_{\alpha\beta} \in \underrightarrow{so}(1,7) = \underrightarrow{Cl}^2(1,7) \subset \underrightarrow{\mathbb{C}}(8\times 8)$$

Clifford bivector parts:

$$\underrightarrow{\omega} = d\underrightarrow{x}^a \tfrac{1}{2}\omega_a{}^{\mu\nu}\gamma_{\mu\nu} \qquad \leftarrow \text{spin connection}$$

$$\underrightarrow{e}\phi = d\underrightarrow{x}^a (e_a)^\mu \phi^\phi \gamma_{\mu\phi} \begin{cases} \underrightarrow{e} = d\underrightarrow{x}^a (e_a)^\mu \gamma_\mu & \leftarrow \text{frame (vierbein)} \\ \phi = \phi^\phi \gamma_\phi \left\{ \begin{array}{l} \phi_+ = (-\phi^5 + i\phi^6) \\ \phi_0 = (\phi^7 + i\phi^8) \end{array} \right\} & \leftarrow \text{Higgs} \quad \phi\phi = -M^2 \end{cases}$$

$$\underrightarrow{B} = -d\underrightarrow{x}^a \tfrac{1}{2} B_a (\gamma_{56} - \gamma_{78}) \qquad \leftarrow \downarrow \text{electroweak gauge fields}$$

$$\underrightarrow{W} = -\tfrac{1}{2}\underrightarrow{W}^1(\gamma_{67} + \gamma_{58}) - \tfrac{1}{2}\underrightarrow{W}^2(-\gamma_{57} + \gamma_{68}) - \tfrac{1}{2}\underrightarrow{W}^3(\gamma_{56} + \gamma_{78})$$

indices: $0 \le a,b \le 3 \qquad 0 \le \mu,\nu \le 3 \qquad 5 \le \phi,\psi \le 8$

Curvature of bosonic connection

$$\underrightarrow{\underrightarrow{F}} = \underrightarrow{d}\underrightarrow{H} + \underrightarrow{H}\underrightarrow{H} \qquad\qquad \underrightarrow{H} = \tfrac{1}{2}\underrightarrow{\omega} + \tfrac{1}{4}\underrightarrow{e}\phi + \underrightarrow{B} + \underrightarrow{W}$$

$$= \left(\tfrac{1}{2}(\underrightarrow{d}\underrightarrow{\omega} + \tfrac{1}{2}\underrightarrow{\omega}\underrightarrow{\omega}) + \tfrac{1}{16}M^2\underrightarrow{e}\,\underrightarrow{e}\right) \qquad \leftarrow \text{spacetime } \gamma_{\mu\nu}$$

$$+ \left(\tfrac{1}{4}\big(\underrightarrow{d}\,\underrightarrow{e} + \tfrac{1}{2}[\underrightarrow{\omega}, \underrightarrow{e}]\big)\phi - \tfrac{1}{4}\underrightarrow{e}\big(\underrightarrow{d}\phi + [\underrightarrow{B}+\underrightarrow{W}, \phi]\big)\right) \qquad \leftarrow \text{mixed } \gamma_{\mu\phi}$$

$$+ \left(\underrightarrow{d}\underrightarrow{B} + \underrightarrow{d}\underrightarrow{W} + \underrightarrow{W}\underrightarrow{W}\right) \qquad \leftarrow \text{higher } \gamma_{\phi\psi}$$

$$= \tfrac{1}{2}\big(\underrightarrow{\underrightarrow{R}} + \tfrac{1}{8}M^2\underrightarrow{e}\,\underrightarrow{e}\big) + \tfrac{1}{4}\big(\underrightarrow{\underrightarrow{T}}\phi - \underrightarrow{e}\,\underrightarrow{D}\phi\big) + \big(\underrightarrow{\underrightarrow{F}}_B + \underrightarrow{\underrightarrow{F}}_W\big)$$

$$= \underrightarrow{\underrightarrow{F}}_s + \underrightarrow{\underrightarrow{F}}_m + \underrightarrow{\underrightarrow{F}}_h$$

Modified BF action over 4D base manifold:

$$S = \int \left\langle \underrightarrow{\underrightarrow{B}}\,\underrightarrow{\underrightarrow{F}} + \Phi(\underrightarrow{H}, \underrightarrow{\underrightarrow{B}})\right\rangle = \int \left\langle \underrightarrow{\underrightarrow{B}}\,\underrightarrow{\underrightarrow{F}} - \frac{1}{4}\underrightarrow{\underrightarrow{B}}_s\underrightarrow{\underrightarrow{B}}_s\gamma + \underrightarrow{\underrightarrow{B}}_m * \underrightarrow{\underrightarrow{B}}_m + \underrightarrow{\underrightarrow{B}}_h * \underrightarrow{\underrightarrow{B}}_h\right\rangle$$

$$= \int \left\langle \underrightarrow{\underrightarrow{F}}_s\,\underrightarrow{\underrightarrow{F}}_s\gamma^- + \frac{1}{4}\underrightarrow{\underrightarrow{F}}_m * \underrightarrow{\underrightarrow{F}}_m + \frac{1}{4}\underrightarrow{\underrightarrow{F}}_h * \underrightarrow{\underrightarrow{F}}_h\right\rangle$$

Gravitational action

$$S_s = \int \left\langle \underline{\underline{B}}_s \underline{\underline{F}}_s + \Phi_s(\underline{\underline{B}}_s) \right\rangle = \int \left\langle \underline{\underline{B}}_s \frac{1}{2}\left(\underline{\underline{R}} + \frac{1}{8} M^2 \underline{e}\,\underline{e}\right) - \frac{1}{4} \underline{\underline{B}}_s \underline{\underline{B}}_s \gamma \right\rangle$$

$$\delta \underline{\underline{B}}_s \rightarrow \underline{\underline{B}}_s = \left(\underline{\underline{R}} + \frac{1}{8} M^2 \underline{e}\,\underline{e}\right)\gamma^- \qquad \text{pseudoscalar:} \;\; \gamma = \gamma_0\gamma_1\gamma_2\gamma_3$$

$$S_s = \frac{1}{4} \int \left\langle \left(\underline{\underline{R}} + \frac{1}{8} M^2 \underline{e}\,\underline{e}\right)\left(\underline{\underline{R}} + \frac{1}{8} M^2 \underline{e}\,\underline{e}\right)\gamma^- \right\rangle = \int \left\langle \underline{\underline{F}}_s \underline{\underline{F}}_s \gamma^- \right\rangle$$

$$\left\langle \underline{\underline{R}}\,\underline{\underline{R}}\gamma^- \right\rangle = \underline{d}\left\langle \left(\underline{\omega}\,\underline{d}\,\underline{\omega} + \frac{1}{3}\underline{\omega}\,\underline{\omega}\,\underline{\omega}\right)\gamma^- \right\rangle \qquad \leftarrow \text{Chern-Simons}$$

$$\frac{1}{4!}\left\langle \underline{e}\,\underline{e}\,\underline{e}\,\underline{e}\gamma^- \right\rangle = \underline{e} \qquad \leftarrow \text{volume element}$$

$$\left\langle \underline{e}\,\underline{e}\,\underline{\underline{R}}\gamma^- \right\rangle = \underline{e} R \qquad \leftarrow \text{curvature scalar}$$

$$S_s = \frac{\Lambda}{12} \int \underline{e}\,(R + 2\Lambda) \qquad \text{cosmological constant:} \quad \Lambda = \frac{3}{4} M^2$$

Why this Lie algebra

$$\underset{\cdot}{\underrightarrow{A}} = \underrightarrow{H} + \underrightarrow{G} + \underset{\cdot}{\psi} = \begin{bmatrix} \underrightarrow{H}^{+} & \underset{\cdot}{\psi}^{-} \\ & \underrightarrow{G}^{-} \end{bmatrix}$$

$$= \begin{bmatrix}
\frac{1}{2}\underrightarrow{\omega}_L + i\underrightarrow{W}^3 & i\underrightarrow{W}^1 + \underrightarrow{W}^2 & -\frac{1}{4}\underrightarrow{e}_R\phi_0^* & \frac{1}{4}\underrightarrow{e}_R\phi_+ & \underset{\cdot}{\nu}_L & \underset{\cdot}{u}^r_L & \underset{\cdot}{u}^g_L & \underset{\cdot}{u}^b_L \\
i\underrightarrow{W}^1 - \underrightarrow{W}^2 & \frac{1}{2}\underrightarrow{\omega}_L - i\underrightarrow{W}^3 & \frac{1}{4}\underrightarrow{e}_R\phi_+^* & \frac{1}{4}\underrightarrow{e}_R\phi_0 & \underset{\cdot}{e}_L & \underset{\cdot}{d}^r_L & \underset{\cdot}{d}^g_L & \underset{\cdot}{d}^b_L \\
-\frac{1}{4}\underrightarrow{e}_L\phi_0 & \frac{1}{4}\underrightarrow{e}_L\phi_+ & \frac{1}{2}\underrightarrow{\omega}_R + i\underrightarrow{B} & & \underset{\cdot}{\nu}_R & \underset{\cdot}{u}^r_R & \underset{\cdot}{u}^g_R & \underset{\cdot}{u}^b_R \\
\frac{1}{4}\underrightarrow{e}_L\phi_+^* & \frac{1}{4}\underrightarrow{e}_L\phi_0^* & & \frac{1}{2}\underrightarrow{\omega}_R - i\underrightarrow{B} & \underset{\cdot}{e}_R & \underset{\cdot}{d}^r_R & \underset{\cdot}{d}^g_R & \underset{\cdot}{d}^b_R \\
 & & & & i\underrightarrow{B} & & & \\
 & & & & & \frac{-i}{3}\underrightarrow{B} + i\underrightarrow{G}^{3+8} & i\underrightarrow{G}^1 - \underrightarrow{G}^2 & i\underrightarrow{G}^4 - \underrightarrow{G}^5 \\
 & & & & & i\underrightarrow{G}^1 + \underrightarrow{G}^2 & \frac{-i}{3}\underrightarrow{B} - i\underrightarrow{G}^{3+8} & i\underrightarrow{G}^6 - \underrightarrow{G}^7 \\
 & & & & & i\underrightarrow{G}^4 + \underrightarrow{G}^5 & i\underrightarrow{G}^6 + \underrightarrow{G}^7 & \frac{-i}{3}\underrightarrow{B} - \frac{2i}{\sqrt{3}}\underrightarrow{G}^8
\end{bmatrix}$$

Note: Only one generation, and fermion masses not quite right.

For three generations: $\underset{\cdot}{\underrightarrow{A}} \in \underrightarrow{so}(1,7) + \underrightarrow{so}(1,7) + 3 * \underset{\cdot}{\mathbb{R}}(8 \times 8) = ?$

BIG Lie algebra: $n = 28 + 28 + 3 * 64 = 248$

Real simple compact Lie groups

rank	group	a.k.a.	dim	name
r	A_r	$SU(r+1)$	$r(r+2)$	**special unitary group**
r	B_r	$SO(2r+1)$	$r(2r+1)$	odd **special orthogonal group**
r	C_r	$Sp(2r)$	$r(2r+1)$	symplectic group
$r>2$	D_r	$SO(2r)$	$r(2r-1)$	even **special orthogonal group**
2	G_2		14	G2
4	F_4		52	F4
6	E_6		78	E6
7	E_7		133	E7
8	E_8		248	**E8**

"E8 is perhaps the most beautiful structure in all of mathematics, but it's very complex."

— Hermann Nicolai

Triality decomposition of E8

John Baez in TWF90:

> ... we now look at the vector space
>
> so(8) + so(8) + end(S+) + end(S-) + end(V)
>
> ...Since so(8) has a representation as linear transformations of V, it has two representations on end(V), corresponding to left and right matrix multiplication; glomming these two together we get a representation of so(8) + so(8) on end(V). Similarly we have representations of so(8) + so(8) on end(S+) and end(S-). Putting all this stuff together we get a Lie algebra, if we do it right - and it's E8.

Pieces of E8

Pirated from GS&W, Superstring Theory:

$$E = B + \Psi = \frac{1}{2} b^{\alpha\beta} \gamma_{\alpha\beta}^{(16)+} + \psi^a Q_a^+$$
$$\in so(16) + S^{(16)+} = \mathrm{Lie}(E8)$$

Lie brackets between generators (structure constants):

$$\begin{aligned}
\left[\gamma_{\alpha\beta}^{(16)+}, \gamma_{\gamma\delta}^{(16)+}\right] &= 2\left\{ -\eta_{\alpha\gamma}\gamma_{\beta\delta}^{(16)+} + \eta_{\alpha\delta}\gamma_{\beta\gamma}^{(16)+} + \eta_{\beta\gamma}\gamma_{\alpha\delta}^{(16)+} - \eta_{\beta\delta}\gamma_{\alpha\gamma}^{(16)+} \right\} \\
\left[\gamma_{\alpha\beta}^{(16)+}, Q_a^+\right] &= \left(\gamma_{\alpha\beta}^{(16)+}\right)^b{}_c \left(Q_a^+\right)^c Q_b^+ = \gamma_{\alpha\beta}^{(16)+} Q_a^+ \\
\left[Q_a^+, Q_b^+\right] &= -\left(\gamma^{(16)+\,\alpha\beta}\right)_{ab} \gamma_{\alpha\beta}^{(16)+}
\end{aligned}$$

$\mathrm{Lie}(E8)$ brackets act as multiplication between 120 dimensional **Cl(16) Clifford bivectors**, B, and positive **chiral**, 128 dim column **spinors**, Ψ:

$$\begin{aligned}
[B_1, B_2] &= B_1 B_2 - B_2 B_1 && \in so(16) \\
[B, \Psi] &= B^+ \Psi && \in S^{(16)+} \\
[\Psi_1, \Psi_2] &= -\Psi_1^\dagger \Gamma^+ \Psi_2 && \in so(16)
\end{aligned}$$

E8 generator conversion

Build new $\mathrm{Lie}(E8)$ generators from old ones:

$$
\begin{array}{lllllll}
H_{\alpha\beta} & = \gamma^{(16)+}_{\alpha\beta} & = \gamma^{(8)+}_{\alpha\beta} \otimes 1 & \in & so(8)^+ \otimes 1 & = & so(8)^H \\
G_{\alpha\beta} & = \gamma^{(16)+}_{(\alpha+8)(\beta+8)} & = P^{(8)}_+ \otimes \gamma^{(8)}_{\alpha\beta} & \in & 1 \otimes so(8) & = & so(8)^G \\
\Psi^I_{\alpha\beta} & = \gamma^{(16)+}_{\alpha(\beta+8)} & = \gamma^{(8)+}_{\alpha} \otimes \gamma^{(8)}_{\beta} & \in & v^{(8)+} \otimes v^{(8)} & = & S^I \\
\Psi^{II}_{ab} & = Q^+_{16(a-1)+b} & = q^+_a \otimes q^+_b & \in & S^{(8)+} \otimes S^{(8)+} & = & S^{II} \\
\Psi^{III}_{ab} & = Q^+_{16(a-1)+b+8} & = q^+_a \otimes q^-_b & \in & S^{(8)+} \otimes S^{(8)-} & = & S^{III}
\end{array}
$$

With these basis generators, the $\mathrm{Lie}(E8)$ elements are:

$$
\begin{aligned}
E &= H + G + \Psi_I + \Psi_{II} + \Psi_{III} \\
&= \frac{1}{2} h^{\alpha\beta} H_{\alpha\beta} + \frac{1}{2} g^{\alpha\beta} G_{\alpha\beta} + \psi_I^{\alpha\beta} \Psi^I_{\alpha\beta} + \psi_{II}^{ab} \Psi^{II}_{ab} + \psi_{III}^{ab} \Psi^{III}_{ab} \\
&\in so(8)^H + so(8)^G + S^I + S^{II} + S^{III}
\end{aligned}
$$

E8 triality structure

The $\mathrm{Lie}(E8)$ brackets between elements in the various parts:

$$\begin{aligned} \left[H_1, H_2\right] &= H_1 H_2 - H_2 H_1 \\ \left[G_1, G_2\right] &= G_1 G_2 - G_2 G_1 \end{aligned}$$

$$\begin{aligned} \left[H, \Psi_I\right] &= H\,\Psi_I \\ \left[H, \Psi_{II}\right] &= H^+\,\Psi_{II} \\ \left[H, \Psi_{III}\right] &= H^+\,\Psi_{III} \end{aligned}$$

$$\begin{aligned} \left[G, \Psi_I\right] &= \Psi_I\,G \\ \left[G, \Psi_{II}\right] &= -\,\Psi_{II}\,G^+ \\ \left[G, \Psi_{III}\right] &= -\,\Psi_{III}\,G^- \end{aligned}$$

$$\begin{aligned} \left[\Psi_I^1, \Psi_I^2\right] &= -\,2\left(\Psi_I^1\,\Psi_I^{2\,T}\right)_H \\ &\quad -2\left(\Psi_I^{1\,T}\Psi_I^2\right)_G \\ \left[\Psi_{II}^1, \Psi_{II}^2\right] &= -\,\left(\Psi_{II}^1\Gamma^+\Psi_{II}^{2\;T}\right)_H \\ &\quad -\,\left(\Psi_{II}^{1\;T}\Gamma^+\Psi_{II}^2\right)_G \\ \left[\Psi_{III}^1, \Psi_{III}^2\right] &= -\,\left(\Psi_{II}^1\Gamma^+\Psi_{II}^{2\;T}\right)_H \\ &\quad -\,\left(\Psi_{II}^{1\;T}\Gamma^-\Psi_{II}^2\right)_G \end{aligned}$$

$$\begin{aligned} \left[\Psi_I, \Psi_{II}\right] &= -\,\left(\Psi_I\Gamma^{++}\Psi_{II}\right)_{III} \\ \left[\Psi_I, \Psi_{III}\right] &= -\,\left(\Psi_I\Gamma^{+-}\Psi_{III}\right)_{II} \\ \left[\Psi_{II}, \Psi_{III}\right] &= -\,\left(\Psi_{II}\Gamma^{++}\Psi_{III}\right)_I \end{aligned}$$

Note: H acts on Ψ's from the left and G acts from the right.

E8 TOE

Build a real form of complex E8 by using $Cl^2(1,7) = so(1,7)$ instead of $Cl^2(8) = so(8)$. Then **E8 TOE connection** is:

$$\underset{\cdot\rightharpoondown}{A} = \underset{\rightharpoondown}{H} + \underset{\rightharpoondown}{G} + \underset{\cdot}{\Psi}_{I} + \underset{\cdot}{\Psi}_{II} + \underset{\cdot}{\Psi}_{III} =$$

something like

$$\begin{bmatrix} \frac{1}{2}\underset{\rightharpoondown}{\omega_L} + i\underset{\rightharpoondown}{W}^3 & i\underset{\rightharpoondown}{W}^1 + \underset{\rightharpoondown}{W}^2 & -\frac{1}{4}\underset{\rightharpoondown}{e_R}\phi_0^* & \frac{1}{4}\underset{\rightharpoondown}{e_R}\phi_+ \\ i\underset{\rightharpoondown}{W}^1 - \underset{\rightharpoondown}{W}^2 & \frac{1}{2}\underset{\rightharpoondown}{\omega_L} - i\underset{\rightharpoondown}{W}^3 & \frac{1}{4}\underset{\rightharpoondown}{e_R}\phi_+^* & \frac{1}{4}\underset{\rightharpoondown}{e_R}\phi_0 \\ -\frac{1}{4}\underset{\rightharpoondown}{e_L}\phi_0 & \frac{1}{4}\underset{\rightharpoondown}{e_L}\phi_+ & \frac{1}{2}\underset{\rightharpoondown}{\omega_R} + i\underset{\rightharpoondown}{B} & \\ \frac{1}{4}\underset{\rightharpoondown}{e_L}\phi_+^* & \frac{1}{4}\underset{\rightharpoondown}{e_L}\phi_0^* & & \frac{1}{2}\underset{\rightharpoondown}{\omega_R} - i\underset{\rightharpoondown}{B} \end{bmatrix} + \begin{bmatrix} i\underset{\rightharpoondown}{B} & & & \\ & \frac{-i}{3}\underset{\rightharpoondown}{B} + i\underset{\rightharpoondown}{G}^{3+8} & i\underset{\rightharpoondown}{G}^1 - \underset{\rightharpoondown}{G}^2 & i\underset{\rightharpoondown}{G}^4 - \underset{\rightharpoondown}{G}^5 \\ & i\underset{\rightharpoondown}{G}^1 + \underset{\rightharpoondown}{G}^2 & \frac{-i}{3}\underset{\rightharpoondown}{B} - i\underset{\rightharpoondown}{G}^{3+8} & i\underset{\rightharpoondown}{G}^6 - \underset{\rightharpoondown}{G}^7 \\ & i\underset{\rightharpoondown}{G}^4 + \underset{\rightharpoondown}{G}^5 & i\underset{\rightharpoondown}{G}^6 + \underset{\rightharpoondown}{G}^7 & \frac{-i}{3}\underset{\rightharpoondown}{B} - \frac{2i}{\sqrt{3}}\underset{\rightharpoondown}{G}^8 \end{bmatrix}$$

$$+ \begin{bmatrix} \underset{\cdot}{\nu}{}^{e}_{L} & \underset{\cdot}{u}{}^{r}_{L} & \underset{\cdot}{u}{}^{g}_{L} & \underset{\cdot}{u}{}^{b}_{L} \\ \underset{\cdot}{e}_{L} & \underset{\cdot}{d}{}^{r}_{L} & \underset{\cdot}{d}{}^{g}_{L} & \underset{\cdot}{d}{}^{b}_{L} \\ \underset{\cdot}{\nu}{}^{e}_{R} & \underset{\cdot}{u}{}^{r}_{R} & \underset{\cdot}{u}{}^{g}_{R} & \underset{\cdot}{u}{}^{b}_{R} \\ \underset{\cdot}{e}_{R} & \underset{\cdot}{d}{}^{r}_{R} & \underset{\cdot}{d}{}^{g}_{R} & \underset{\cdot}{d}{}^{b}_{R} \end{bmatrix} + \begin{bmatrix} \underset{\cdot}{\nu}{}^{\mu}_{L} & \underset{\cdot}{c}{}^{r}_{L} & \underset{\cdot}{c}{}^{g}_{L} & \underset{\cdot}{c}{}^{b}_{L} \\ \underset{\cdot}{\mu}_{L} & \underset{\cdot}{s}{}^{r}_{L} & \underset{\cdot}{s}{}^{g}_{L} & \underset{\cdot}{s}{}^{b}_{L} \\ \underset{\cdot}{\nu}{}^{\mu}_{R} & \underset{\cdot}{c}{}^{r}_{R} & \underset{\cdot}{c}{}^{g}_{R} & \underset{\cdot}{c}{}^{b}_{R} \\ \underset{\cdot}{\mu}_{R} & \underset{\cdot}{s}{}^{r}_{R} & \underset{\cdot}{s}{}^{g}_{R} & \underset{\cdot}{s}{}^{b}_{R} \end{bmatrix} + \begin{bmatrix} \underset{\cdot}{\nu}{}^{\tau}_{L} & \underset{\cdot}{t}{}^{r}_{L} & \underset{\cdot}{t}{}^{g}_{L} & \underset{\cdot}{t}{}^{b}_{L} \\ \underset{\cdot}{\tau}_{L} & \underset{\cdot}{b}{}^{r}_{L} & \underset{\cdot}{b}{}^{g}_{L} & \underset{\cdot}{b}{}^{b}_{L} \\ \underset{\cdot}{\nu}{}^{\tau}_{R} & \underset{\cdot}{t}{}^{r}_{R} & \underset{\cdot}{t}{}^{g}_{R} & \underset{\cdot}{t}{}^{b}_{R} \\ \underset{\cdot}{\tau}_{R} & \underset{\cdot}{b}{}^{r}_{R} & \underset{\cdot}{b}{}^{g}_{R} & \underset{\cdot}{b}{}^{b}_{R} \end{bmatrix}$$

Discussion

What I just did:

- All **gauge fields**, **gravity**, and Higgs in an E8 **connection**, with fermions as **BRST ghosts**.

To do:

- Particle assignments not perfect yet.
- Get the CKM matrix. Might just not work.
- Where does the action come from?
- Symmetry breaking.
- Natural explanation for QM.

What this E8 theory means for LQG:

- Modified BF gravity (MM) is favored — intimate frame and Higgs.
- Flexible as to what (or how) spacetime base manifold happens.
- Keep up the good work!
- Extending LQG methods to E8 gives a TOE.
- E8 $\{10j\}$ symbols...

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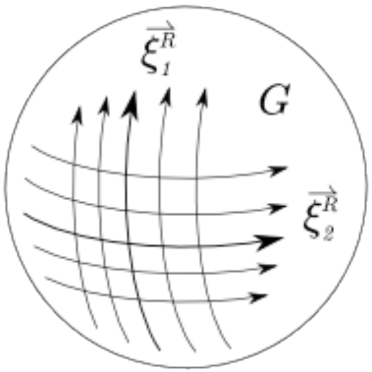
Geometry of Yang-Mills theory

Start with a **Lie group manifold** (*torsor*), G, coordinatized by y^p.
Two sets of invariant vector fields (*symmetries*, ***Killing vector*** *fields*):

$$\vec{\xi}^{L}_{A}(y)\, \underset{\rightharpoondown}{d} g = T_A\, g(y) \qquad \vec{\xi}^{R}_{A}(y)\, \underset{\rightharpoondown}{d} g = g(y)\, T_A$$



Lie derivative: $[\vec{\xi}^R_A, \vec{\xi}^R_B] = C_{AB}{}^{C} \vec{\xi}^R_C$

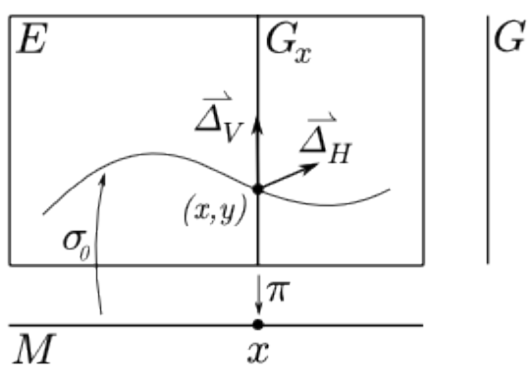
Lie bracket: $[T_A, T_B] = C_{AB}{}^{C} T_C$

Killing form (***Minkowski metric***): $g_{AB} = C_{AC}{}^{D} C_{BD}{}^{C}$

Maurer-Cartan form (***frame***): $\underset{\rightharpoondown}{\mathcal{I}} = \underset{\rightharpoondown}{d} y^p (\xi^R_p)^A T_A$

Entire space of a **principal bundle**: $E \sim M \times G$
Ehresmann principal bundle connection over patches of E:

$$\underset{\rightharpoondown}{\vec{\mathcal{E}}}(x,y) = \underset{\rightharpoondown}{d} x^a A_a{}^B(x)\, \vec{\xi}^L_B(y) + \underset{\rightharpoondown}{d} y^p \vec{\partial}_p$$



Gauge field **connection** over M:

$$\underset{\rightharpoondown}{A}(x) = \sigma_0^* \underset{\rightharpoondown}{\vec{\mathcal{E}}} \underset{\rightharpoondown}{\mathcal{I}} = \underset{\rightharpoondown}{d} x^a A_a{}^B(x)\, T_B$$

BRST gauge fixing

$\delta \underline{L} = 0$ under **gauge transformation**: $\quad \delta \underrightarrow{A} = -\underrightarrow{\nabla} C = -\underrightarrow{d} C - \left[\underrightarrow{A}, C \right]$

Account for gauge part of $\underrightarrow{A}$ by introducing **Grassmann** valued **ghosts**,

$\underset{\cdot}{C} \in \mathrm{Lie}(G)_g$, **anti-ghosts**, $\underline{\dot{B}}$, **partners**, $\underline{\lambda}$, and **BRST transformation**:

$$
\begin{aligned}
\underset{\cdot}{\delta} \underrightarrow{A} &= -\underrightarrow{\nabla} \underset{\cdot}{C} & \qquad \underset{\cdot}{\delta} \underset{\cdot}{C} &= -\tfrac{1}{2} \left[\underset{\cdot}{C}, \underset{\cdot}{C} \right] \\
\underset{\cdot}{\delta} \underrightarrow{\underrightarrow{B}} &= \left[\underrightarrow{\underrightarrow{B}}, \underset{\cdot}{C} \right] & \qquad \underset{\cdot}{\delta} \underline{\dot{B}} &= \underline{\lambda} \\
\underset{\cdot}{\delta} \underline{\lambda} &= 0
\end{aligned}
$$

This satisfies $\underset{\cdot}{\delta} \underline{L} = 0$ and $\underset{\cdot}{\delta} \underset{\cdot}{\delta} = 0$.

Choose a **BRST potential**, $\underline{\dot{\Psi}} = \left\langle \underline{\dot{B}} \underrightarrow{A} \right\rangle$, to get new Lagrangian:

$$
\underline{L}' = \underline{L} + \underset{\cdot}{\delta} \underline{\dot{\Psi}} = \underline{L} + \left\langle \underline{\lambda} \underrightarrow{A}_g \right\rangle + \left\langle \underline{\dot{B}} \underrightarrow{\nabla} \underset{\cdot}{C} \right\rangle
$$

BRST partners act as Lagrange multipliers; **effective Lagrangian**:

$$
\underline{L}^{\mathrm{eff}} = \underline{L}[\underrightarrow{\underrightarrow{B}}', \underrightarrow{A}'] + \left\langle \underline{\dot{B}} \underrightarrow{\nabla}' \underset{\cdot}{C} \right\rangle
$$

BRST extended connection

Replace pure gauge part of connection with ghosts:

$$\underset{\cdot}{\underrightarrow{A}} = \underrightarrow{A}' + \underset{\cdot}{C}$$

BRST extended curvature:

$$\underset{\cdot}{\underrightarrow{\underset{\cdot}{\underrightarrow{F}}}} = \underrightarrow{d}\,\underset{\cdot}{\underrightarrow{A}} + \frac{1}{2}\left[\underset{\cdot}{\underrightarrow{A}}, \underset{\cdot}{\underrightarrow{A}}\right] = \underrightarrow{\underrightarrow{F}}' + \underrightarrow{\nabla}'\underset{\cdot}{C} + \frac{1}{2}\left[\underset{\cdot}{C}, \underset{\cdot}{C}\right]$$

$$= \left(\underrightarrow{d}\,\underrightarrow{A}' + \underrightarrow{A}'\underrightarrow{A}'\right) + \left(\underrightarrow{d}\,\underset{\cdot}{C} + \left[\underrightarrow{A}', \underset{\cdot}{C}\right]\right) + \frac{1}{2}\left[\underset{\cdot}{C}, \underset{\cdot}{C}\right]$$

Effective Lagrangian, with $\dot{\underline{B}}' = \underline{B}' + \dot{\underline{B}}$:

$$\underline{L}^{\mathrm{eff}} = \left\langle \dot{\underline{B}}'\,\underset{\cdot}{\underrightarrow{\underset{\cdot}{\underrightarrow{F}}}} + \underline{\Phi}(\underrightarrow{A}', \underline{B}')\right\rangle$$

Crazy idea: **Fermions are gauge ghosts**

$$\underrightarrow{A}' = \underrightarrow{H} + \underrightarrow{G} = \left(\frac{1}{2}\underrightarrow{\omega} + \frac{1}{4}\underrightarrow{e}\,\phi + \underrightarrow{B} + \underrightarrow{W}\right) + \underrightarrow{G}$$

$$\underset{\cdot}{C} = \underset{\cdot}{\psi} = \left(\underset{\cdot}{\nu} + \underset{\cdot}{e} + \underset{\cdot}{u}^{r,b,g} + \underset{\cdot}{d}^{r,b,g}\right)$$

Massive Dirac operator in curved spacetime

$$\left(\not{D} + \phi\right) \underset{\cdot}{\psi} = \gamma^{\mu} \left(e_{\mu}\right)^{a} \left(\partial_{a} + \frac{1}{4} {\omega_{a}}^{\nu\rho} \gamma_{\nu\rho} + B, W, {G_{a}}^{A} T_{A}\right) \underset{\cdot}{\psi} + \phi \underset{\cdot}{\psi}$$

γ_{μ}	**Clifford basis vectors** for **Cl(1,3)**
$\gamma_{\mu\nu} = \gamma_{\mu}\gamma_{\nu}$	**Clifford basis bivectors**
T_{A}	**Lie algebra** basis elements (*generators*)
$\left(e_{\mu}\right)^{a}$	**orthonormal basis vector** components (*frame, vierbein*)
${\omega_{a}}^{\nu\rho}$	**spin connection** components
${B_{a}}^{A}, {W_{a}}^{A}, {G_{a}}^{A}$	Yang-Mills **gauge field** components (*connections*)
ϕ	Higgs scalar field multiplet
$\underset{\cdot}{\psi}$	**Grassmann** valued **spinor** field multiplet

$$\begin{array}{ccc} \text{Clifford algebra} & \longleftrightarrow & \text{Lie algebra} \\ & \nwarrow\!\!\searrow \quad \swarrow\!\!\nearrow & \\ & \text{Matrices} & \end{array}$$