Numerical Spin Foam Computations of Dual Yang-Mills Theory

Talk prepared for Loops 07
June 26

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University of Western Ontario
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Based upon: arXiv:0705.2629v2

Yang-Mills Theory

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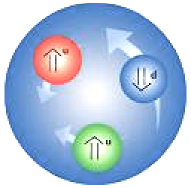
- Yang-Mills quantum field theories are the basic building blocks of the standard model.

$$\mathcal{L}_{QCD} = -\sum_q \bar{q}(\not{D}+m)q - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\alpha}$$



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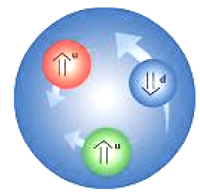

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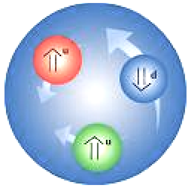

- We shall focus on second term: *pure* Yang-Mills
- Need systematic method for questions with non-perturbative answers, i.e. confinement.

Conventional Lattice Gauge Theory

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- Idea: Regularize space-time on a lattice. Estimate correlation functions using Monte Carlo.

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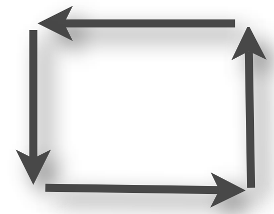
$$\downarrow$$

$$\mathcal{Z} = \int \left(\prod_{e \in E} dg_e \right) e^{-\sum_{p \in P} S(g_p)}$$

$$g_p = g_{e_1(p)} g_{e_2(p)} g^{-1}_{e_3(p)} g^{-1}_{e_4(p)}$$

(Discrete holonomy around a plaquette)



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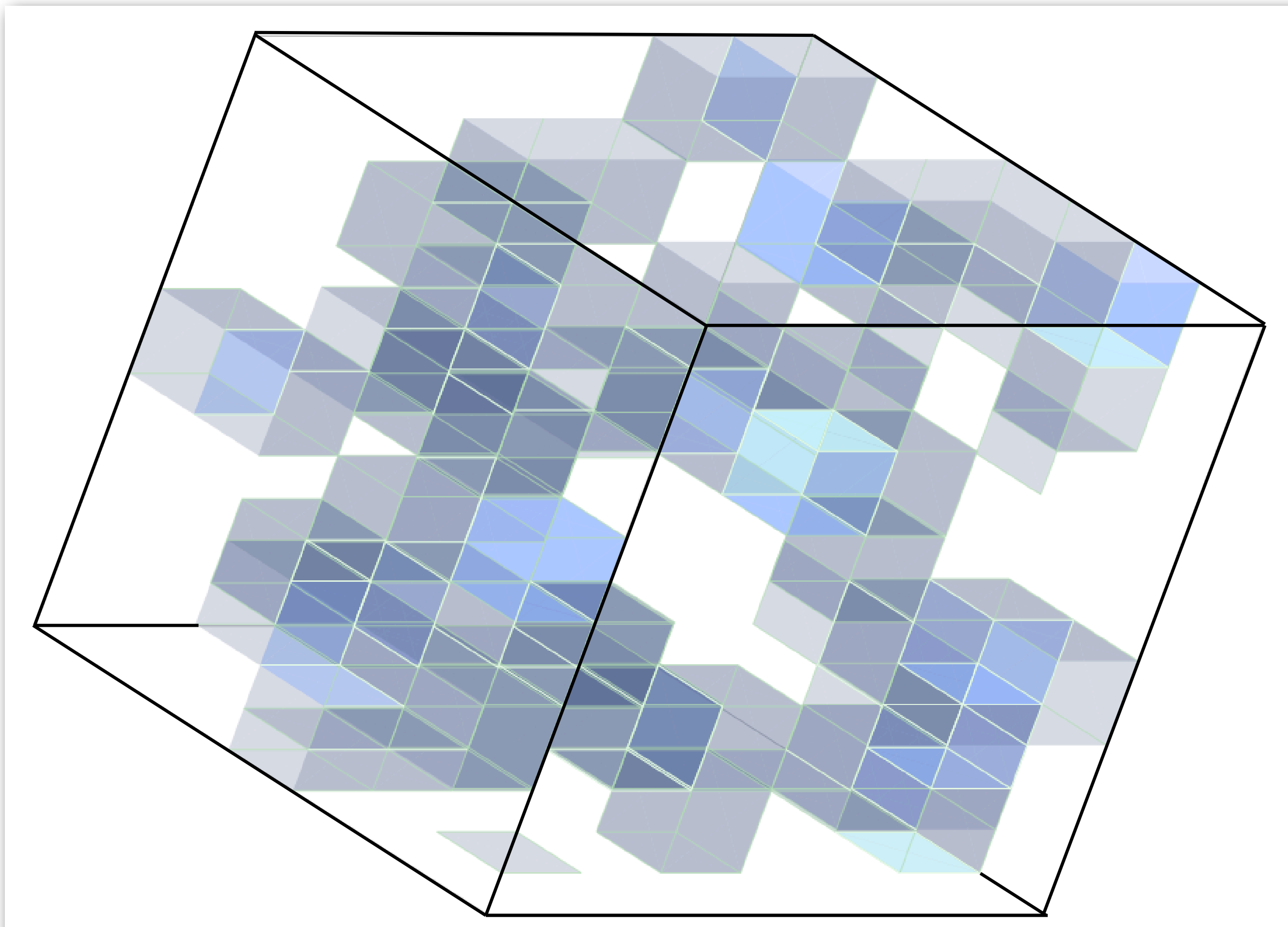
- Many colorings vanish. To ``survive" group integration, a plaquette coloring must satisfy certain conditions...

Dual states: Closed, branched, colored surfaces



We specialize now to SU(2), D=3

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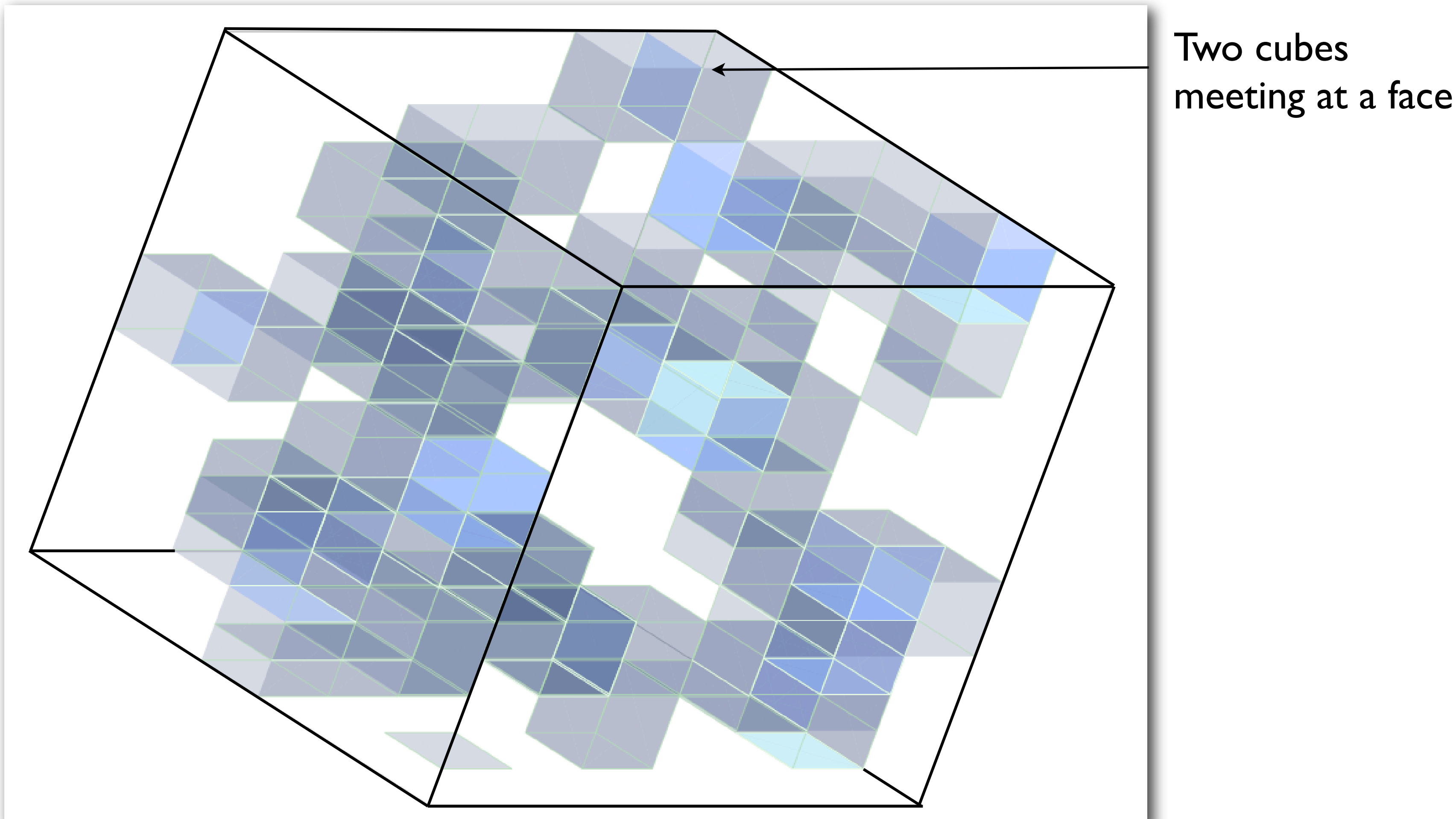

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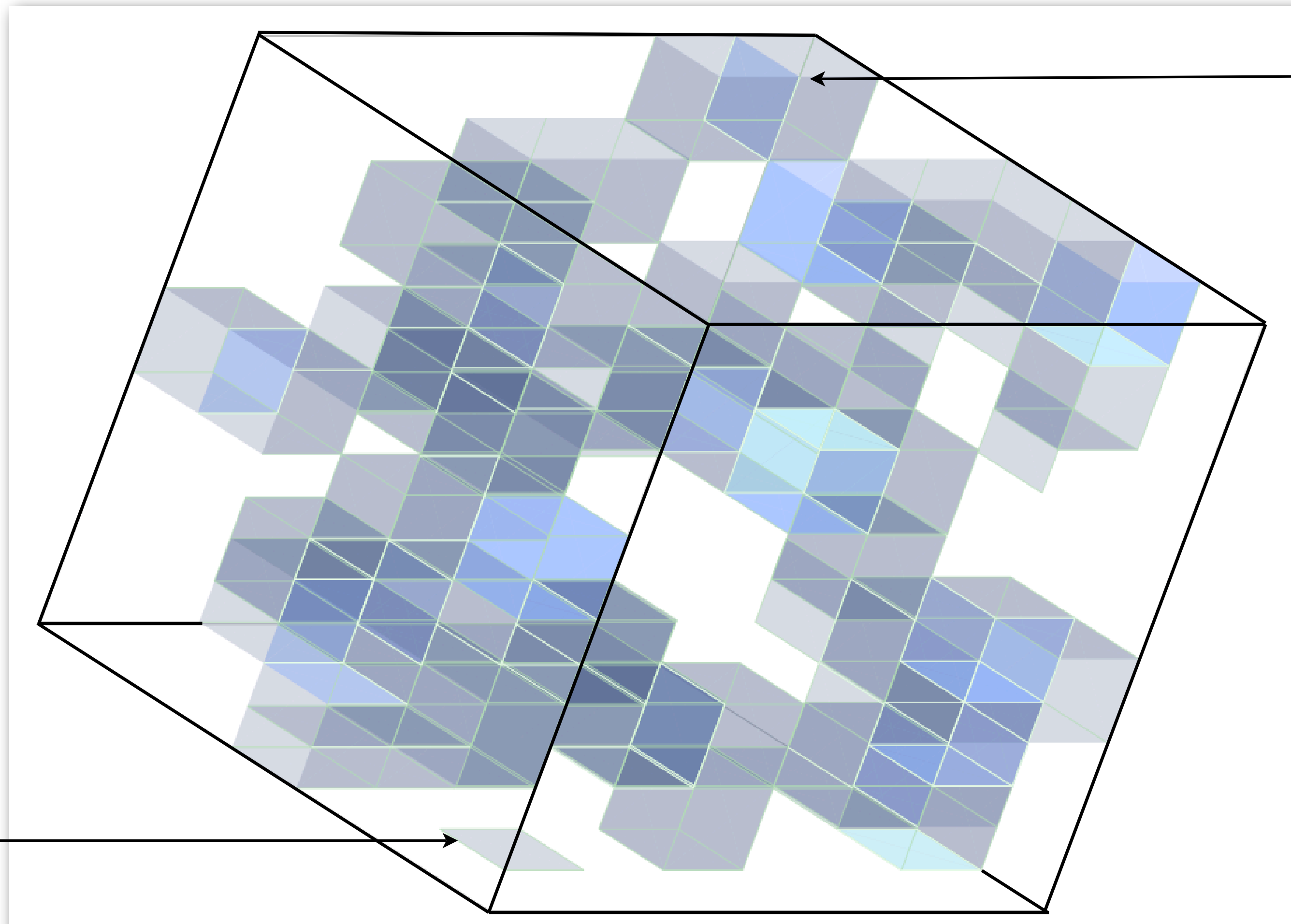

Two cubes meeting at a face

Closed, due to periodic b.c.'s

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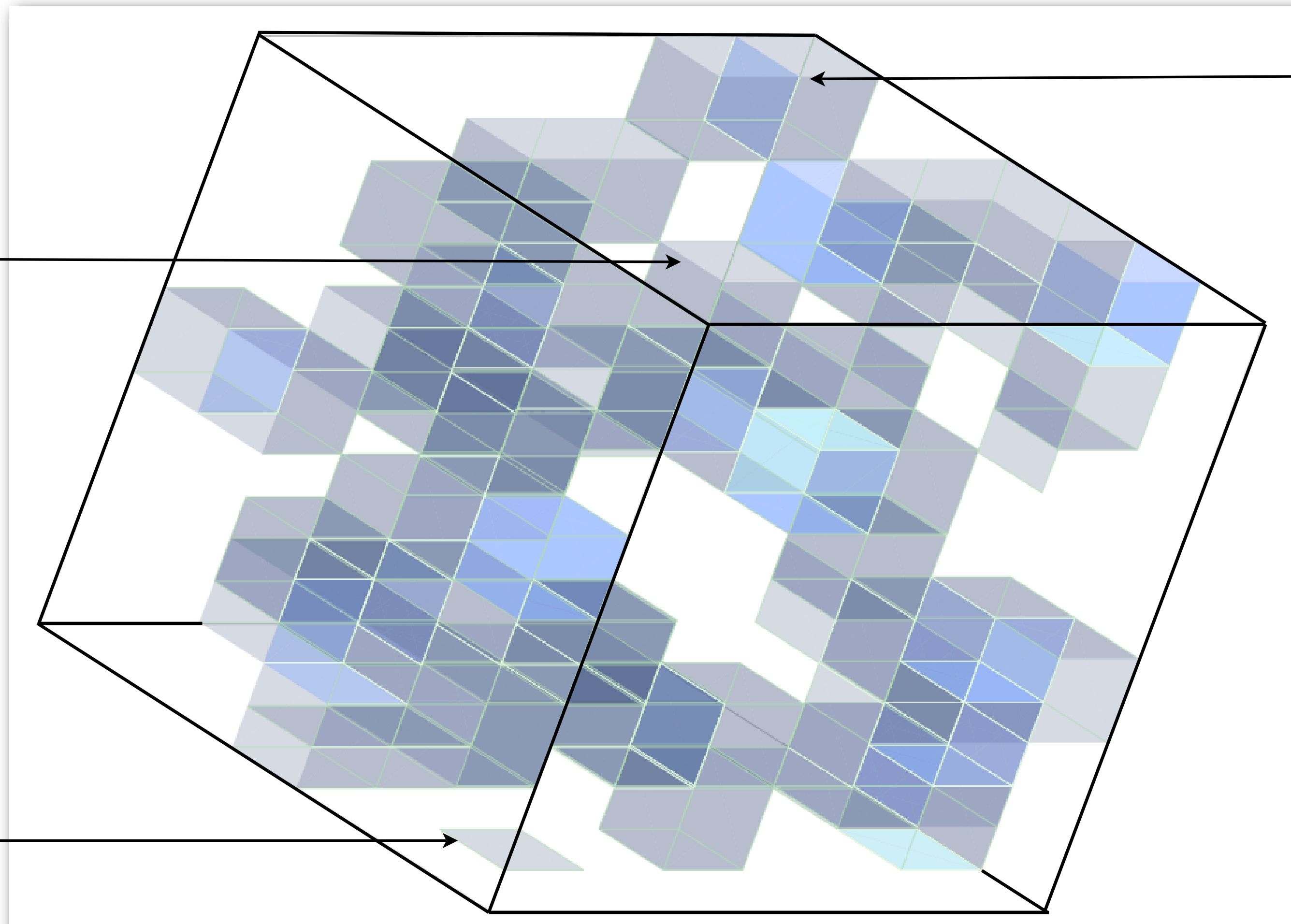

Two cubes meeting at a face

Cube with cap at bottom

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- For small diagrams, these were worked out by hand exactly
- In strong coupling limit, expansions in small diagrams become good a description of the physics.

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- Amplitude is *local* on configurations - critical for a practical algorithm.
- General theory of dual non-abelian spin foams (Wilson observables,etc):

arXiv:hep-lat/0110034 (R. Oeckl, H. Pfeiffer)

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Use recoupling theory to find efficient, computable formulae for 18j symbols

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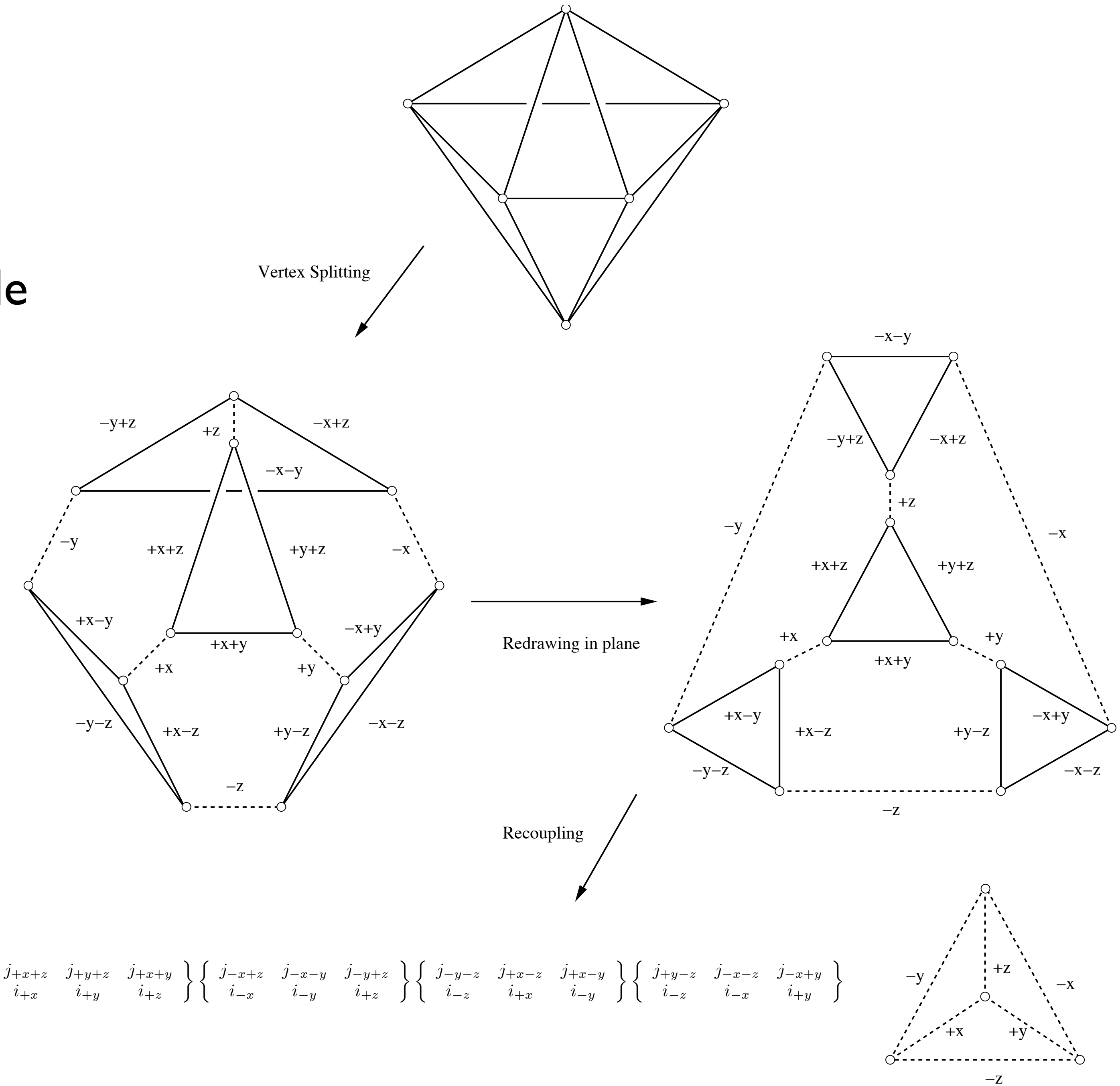

$$\left\{ \begin{matrix} j_{+x+z} & j_{+y+z} & j_{+x+y} \\ i_{+x} & i_{+y} & i_{+z} \end{matrix} \right\} \left\{ \begin{matrix} j_{-x+z} & j_{-x-y} & j_{-y+z} \\ i_{-x} & i_{-y} & i_{+z} \end{matrix} \right\} \left\{ \begin{matrix} j_{-y-z} & j_{+x-z} & j_{+x-y} \\ i_{-z} & i_{+x} & i_{-y} \end{matrix} \right\} \left\{ \begin{matrix} j_{+y-z} & j_{-x-z} & j_{-x+y} \\ i_{-z} & i_{-x} & i_{+y} \end{matrix} \right\}$$

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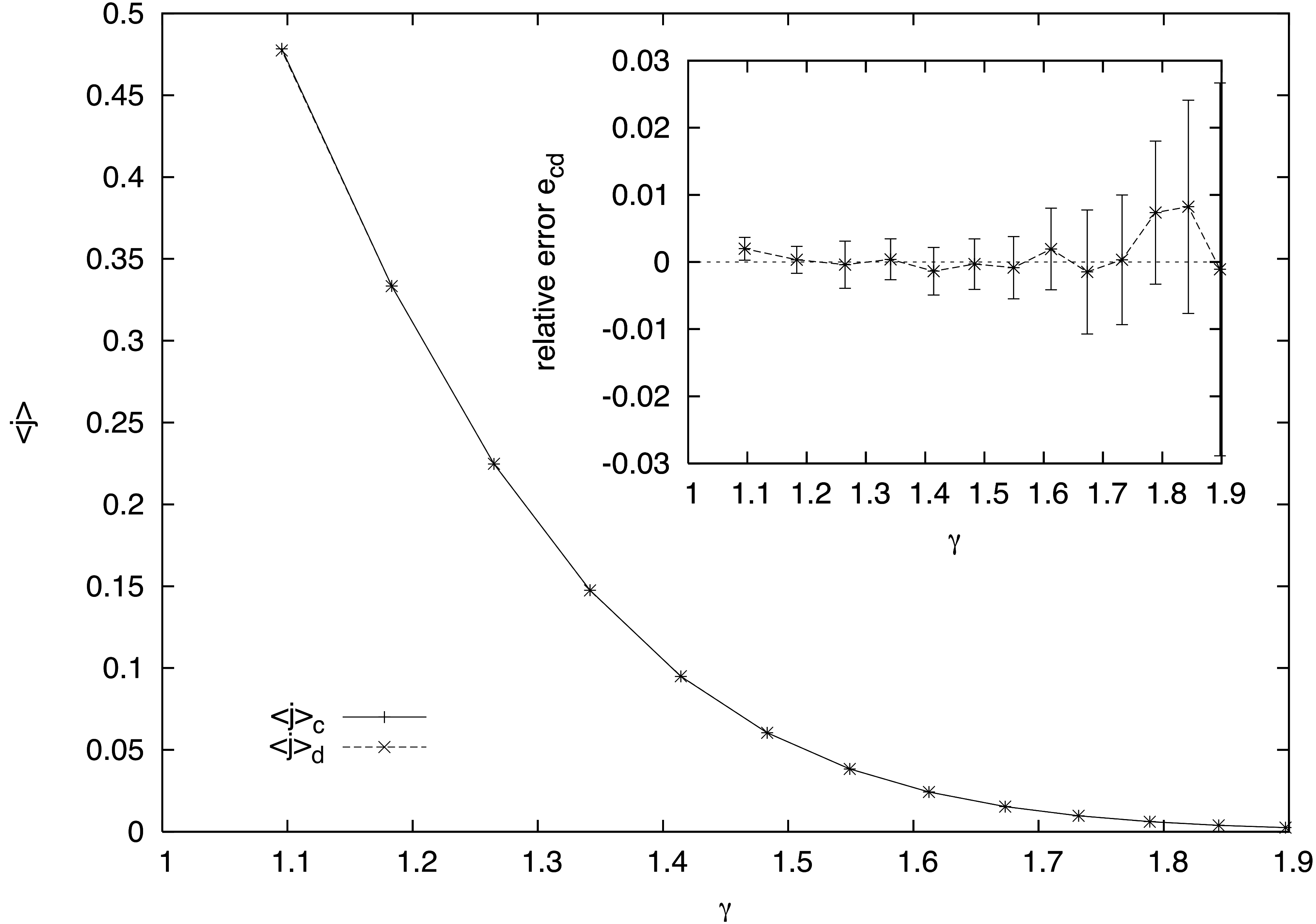
- Single plaquette changes won’t work due to parity constraint
- Find moves as local as possible (efficient updating)

Results:

(8x8x8 lattice)



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- Possibly faster in some contexts

Challenges

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  arXiv: gr-qc/0207041 (D. Oriti and H. Pfeiffer)
  arXiv:0706.1534 (S. Speziale)
  Others?

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