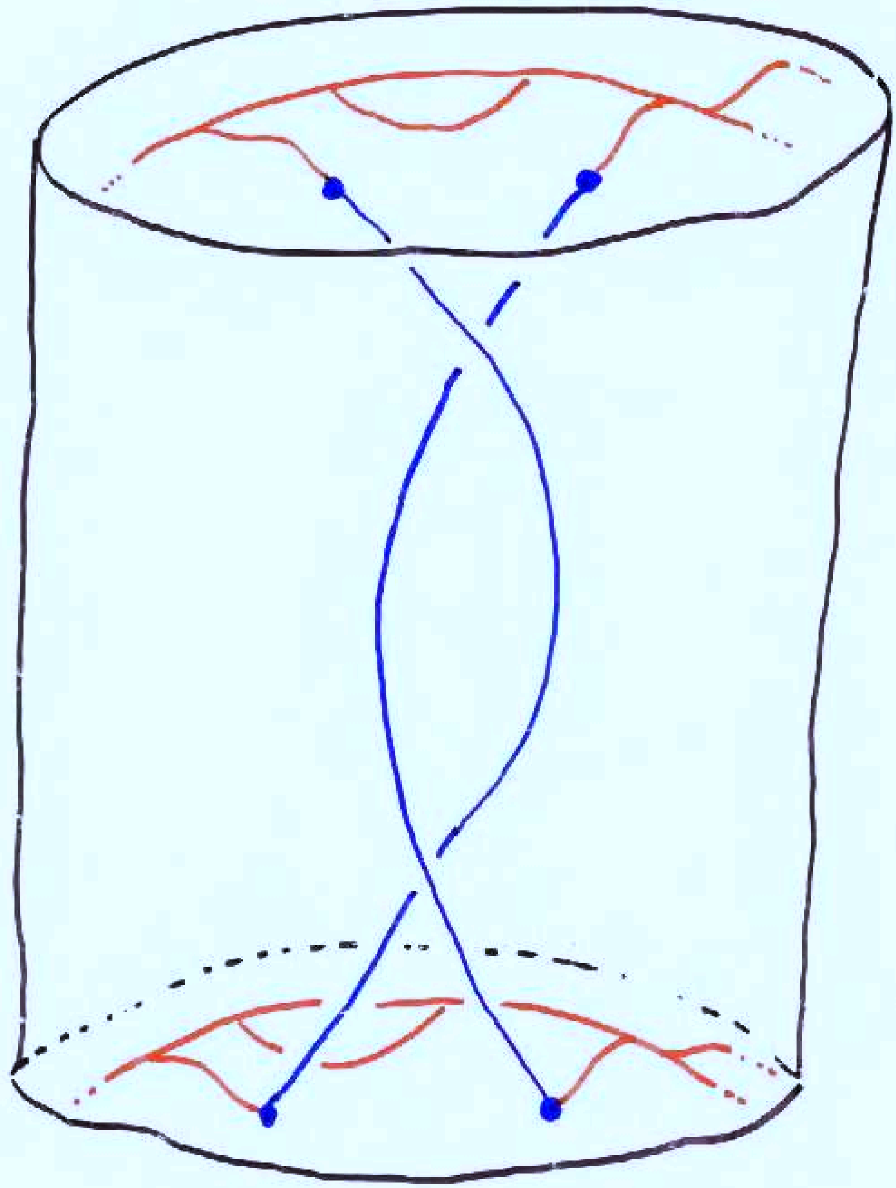
A Causal Spin Foam

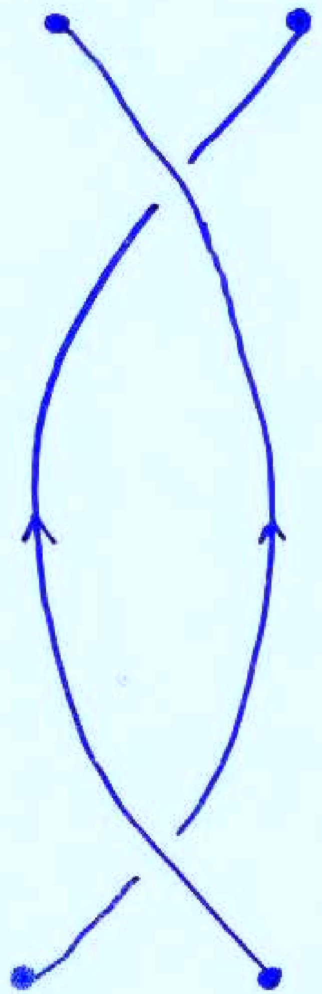
motivation, construction, properties

(D. Oriti , T. T.)



$$\boxed{\sum_{J} \prod_{e \notin \Gamma} \Delta_{J_e} \prod_{e \in \Gamma} \chi_{J_e}(m_e) \prod_{e \in T} \delta_{J_e,0} \prod_{t}}$$

$\in \mathbb{R}$



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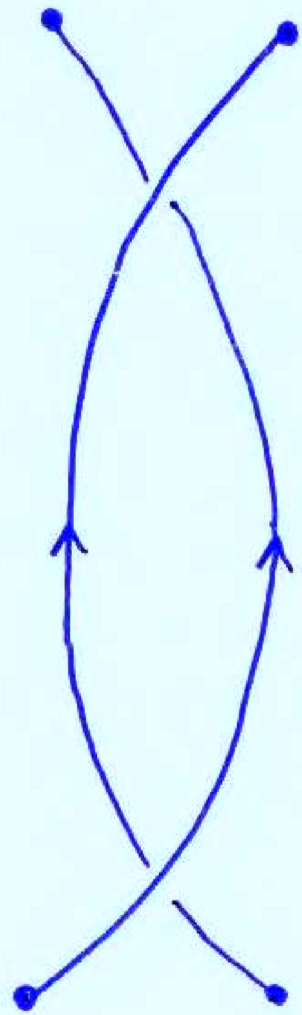

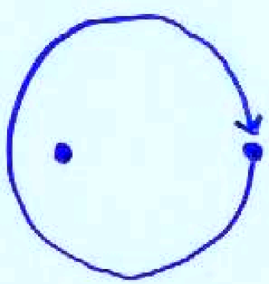

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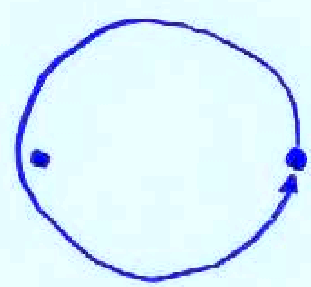
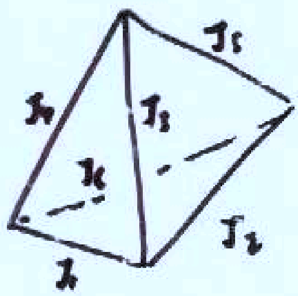


$$e^{i\theta} + e^{-i\theta} \in \mathbb{R}$$

$$\boxed{Z_{PR} = \sum_{J} \prod_{e} \Delta_{J_e} \prod_{t}}$$

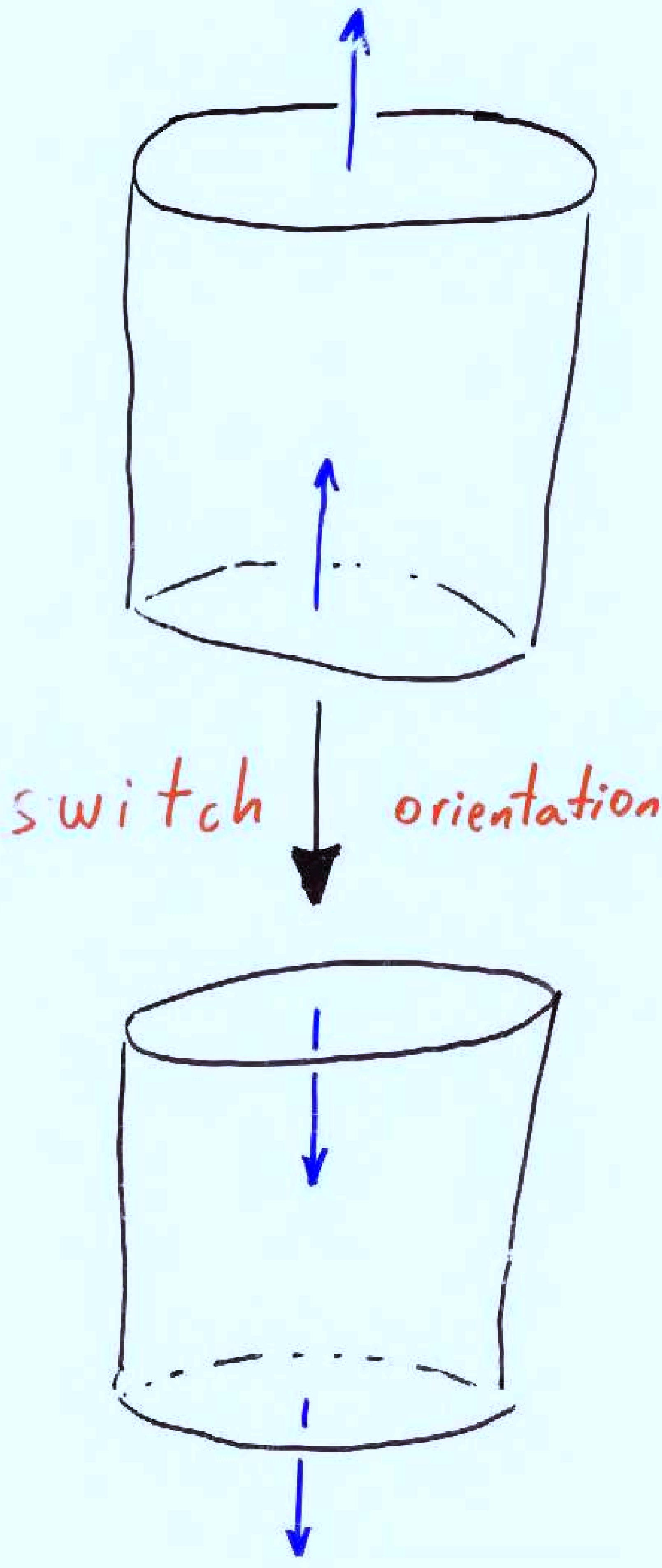
$$\sim \int \mathcal{D}g \; e^{i S_{EH}}$$



$$\xrightarrow{J\text{'s} \gg 1} \quad e^{i S_R} + \overline{e^{i S_R}}$$

$$Z_{PR} \sim \int \mathcal{D}g \, e^{i S_{EH}} + \overline{\int \mathcal{D}g \, e^{i S_{EH}}}$$

$$Z_{GR} + \overline{Z_{GR}}$$


switch orientation

$$S = \int_M \mathrm{Tr}(E \wedge F)$$

E su(2)-valued 1-form

F su(2)-valued 2-form

Discretize:

$M \longrightarrow$ Triangulation + Dual Triangulation

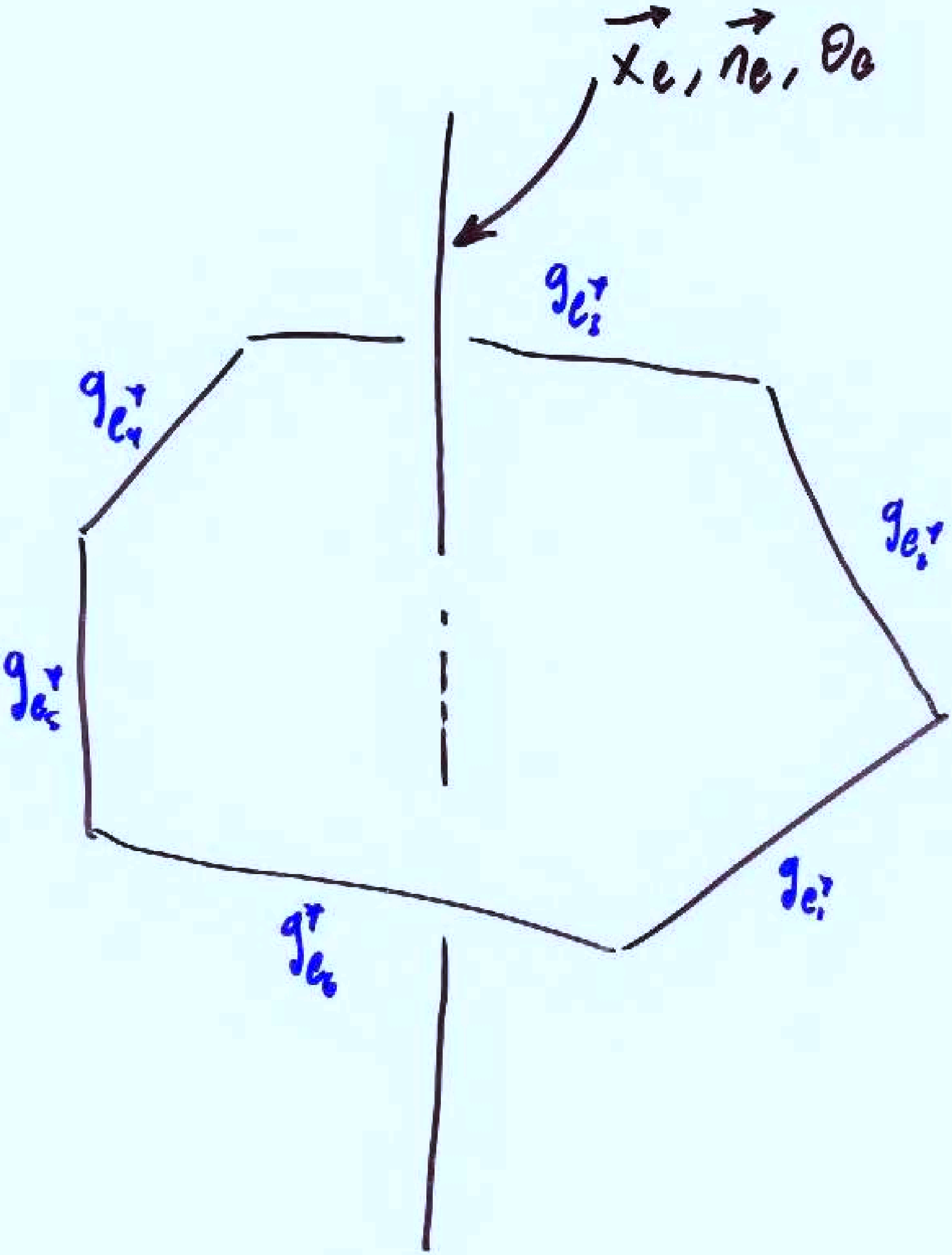
$$E \longrightarrow \int_{\text{edges}} E = X_e = \vec{x}_e \cdot \vec{J} \quad \in su(2)$$

$$F \longrightarrow \int_{\text{dual faces}} F = G_e = g_{e_1^*} \cdots g_{e_n^*} \in SU(2)$$

$$= \mathrm{Exp}(\theta_e \, \vec{n}_e \cdot \vec{J})$$

$$\boxed{\vec{x}_e \in \mathbb{R}^3 \quad , \quad \vec{n}_e \in S^2 \quad , \quad \theta_e \in [0, \pi]}$$

$$S = \sum_{\text{edges}} \mathrm{Tr}(X_e G_e) = \sum_{\text{edges}} \vec{x}_e \cdot \vec{n}_e \, \sin(\theta_e)$$



$\vec{x}_e, \vec{n}_e, \theta_e$
$g_{e_1}^{\dagger}$
$g_{e_2}^{\dagger}$
$g_{e_3}^{\dagger}$
$g_{e_4}^{\dagger}$
$g_{e_5}^{\dagger}$
$g_{e_6}^{\dagger}$

$$Z_{GR} = \int_{\text{volume} \geqslant 0} \mathcal{D}_E \mathcal{D}_A \; e^{iS}$$

$$\text{Volume} = \det(E)$$

$$Z_{GR} = \int_{? \geqslant 0} \prod_{e} d\vec{x}_e \prod_{e^*} dg^*_e \; e^{iS}$$

$$\boxed{\begin{aligned} S &= \sum_e \vec{x}_e \cdot \vec{n}_e \, \sin(\theta_e) \\ S &= \sum_h (\text{Volume})_h \, \sin(\theta_h) \end{aligned}}$$

(Caselle, D'adda, Magnea) 1989

$$\boxed{\det(E) \geqslant 0 \iff \vec{x}_e \cdot \vec{n}_e \geqslant 0}$$

$$Z_{GR} = \int_{(\vec{x}_e \cdot \vec{n}_e \geq 0)} \prod_e d\vec{x}_e \prod_{e^*} dg_{e^*} \; e^{iS}$$

$$= \int \prod_{e^*} dg_{e^*} \frac{i\,C}{(\sin(\theta_e) + i\varepsilon)^3}$$

Properties:

- Peaked around classical configurations

$$\theta_e \sim 0$$

- The real part is $\delta(\sin(\theta_e))$

$$\frac{i}{p^2 - m^2 + i\varepsilon} \qquad\qquad \frac{i}{(\sin(\theta) + i\varepsilon)^3}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

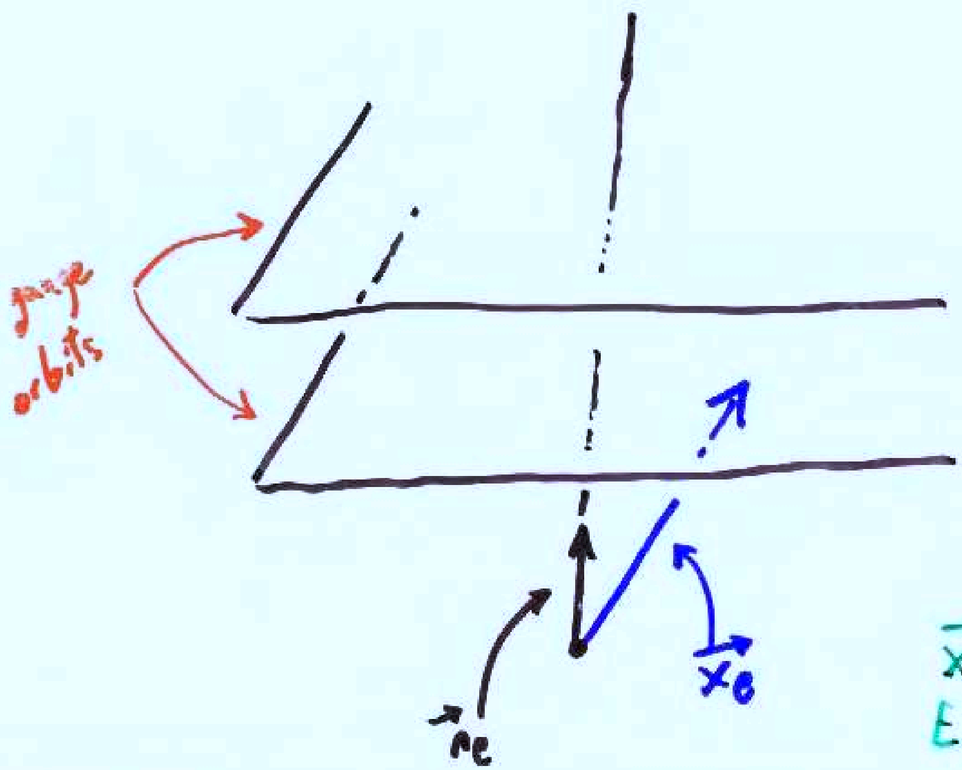
$$\delta(p^2 - m^2) \qquad\qquad \delta(\sin(\theta_e))$$

- Not triangulation independent

- Naive calculation gives $C = \infty$

This divergence points towards a new gauge symmetry of the discrete model

$$\boxed{\begin{array}{l} g_{e^+} \longrightarrow g_{e^+} \\[1em] \vec{x}_e \longrightarrow \vec{x}_e + \vec{\varphi}_e \\ \qquad\qquad \vec{\varphi}_e \cdot \vec{n}_e = 0 \end{array}}$$



$$\vec{x} \longrightarrow \vec{x} + \vec{\varphi} \times \vec{n}$$
$$E \longrightarrow E + [\varphi, F]$$

This 'transversal' symmetry has no continuum analogue, nor does it affect the a-causal model

To Be Addressed

- Is the sharp cut-off the right one?
  - It seems to be causing divergences
  - The character expansion coefficients are rather complicated

- What is the triangulation-independent picture of this, i.e. what is the GFT that generates these Spin Foams?