Spin networks and Simplicial Quantum Gravity

John Swain, Northeastern University, Boston

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3+1 D Regge Calculus

Friedman and Jack, JMP27 (1986) 2973

- Details for flat tetrahedra with R, K and g, as well as the supermetric G, all in terms of volume and faces areas
- Well-defined Hamiltonian constraints and conserved momentum
- But this is a *classical approximation scheme* – can one hope to get *quantized* areas and volumes out of this?

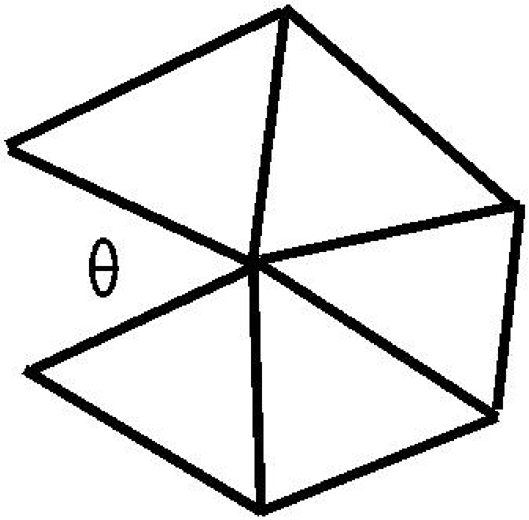
Traditional Approaches to Simplicial Geometry

- Regge Calculus (Regge, Nuovo Cim. 19 (1961) 558).

- 4D: $$\frac{1}{2}\int R\, d\Omega \approx \sum A_i \theta_i$$

- 3D: $$\frac{1}{2}\int R\, d\Omega \approx \sum l_i \theta_i$$

- 2D: $$\frac{1}{2}\int R\, d\Omega \approx \sum \theta_i$$



- Vary edge lengths for dynamics

Dynamical Triangulations

- A variant of Regge calculus with non-dynamical edge-lengths – dynamics specified by connectivity
- Questions of *continuum* limit – but is this what we want if volumes and areas are discrete?
- Causal DT: 2D “small” and 4D “large”? (Ambjorn et al.)

Quantization

- Can try the usual path integral approach (ducking questions of measure):

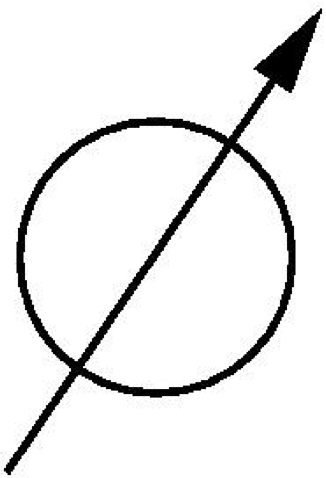
$$< g_2^{(3)}, \phi_2 \mid g_1^{(3)}, \phi_1 >= \int Dg \; e^{-iS[g,\phi]}$$

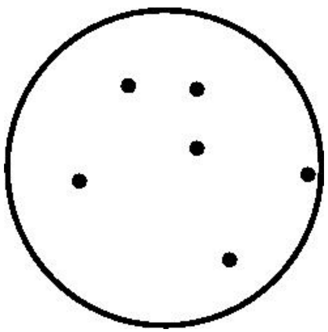
- …but it might be difficult to see how quantized geometrical quantities emerge
- …what are allowed 3-geometries and what does one include in the sum? (what category?)

Analogy with Angular Momentum

- Consider mechanics of rotating rigid bodies :

Classical vs. Quantum





Continuous phase space vs. n spin-1/2 punctures

Path integrals vs. Sums for Angular Momentum

1) straightforward path integral – physical states (the ones we know are |jm>) are not obvious! For transition amplitudes one sums over zillions of classical states which *mostly cancel*:

2) get the discretization of angular momentum from some other approach. With the physical states |jm> in hand, the transition amplitudes are easier to handle – initial and final states are well-defined now and the sums are *finite and discrete*

In other words, we can trade a path integral sum over all classical states for a finite sum over quantum-allowed states.

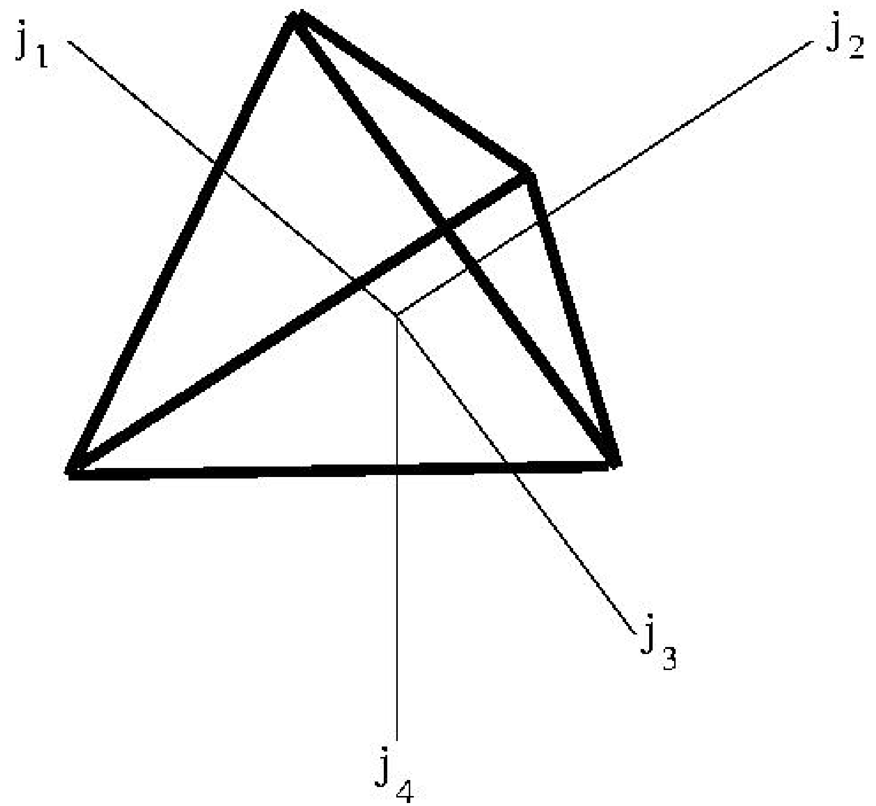
$$< state2 \,|\, state1 >= \int D[path] e^{iS[path]}$$

Associating a Simplicial Geometry with a Spin Network

The idea is simple:

- Simplices are dual to spin networks, with face areas and volumes quantized

- Analogous to starting off angular momentum calculations with known |jm> states



Informing Regge/DT with LQG

- The suggestion then is to keep discrete approaches like Regge, DT, Causal DT, but put in quantization of geometrical quantities from the start
- *i.e. Use* LQG to get kinematic Hilbert space, then go to simplicial quantum gravity to do the dynamics
- This is like doing quantum mechanics (in continuous or discrete time) starting off knowing about discrete |jm> -- *we do this all the time in QM (in SWE we know to use spherical harmonics, in matrix mechanics we know to use a finite basis for spin)*

What does this get one?

- A visualizable simplicial geometry for each spin-network
- Intuitive understanding of why volume is zero for (<4)-valent nodes (need 4 flat faces to enclose a volume)
- Massive reduction in number of degrees of freedom for a Feynman-type sum
- Time evolution can be studied (continuous as we do with angular momentum? -- but here K is constructed from the simplicial quantities!)

An interesting question…

- Implicit in all this is that volumes and areas are all consistent for flat tetrahedra (and higher polytopes) – this is not completely obvious!
- If one has LQG quantization results inconsistent with flat spacetime, perhaps flat spacetime is not consistent – suggests Snyder-Yang-Mendes Algebra (see D. Ahluwahlia, Chryssomalakos and Okon Chryssomalakos and Okon)
  - → Could there be a calculable cosmological constant?
- i.e. 2-length parameter stable deformation of Poincare and Heisenberg algebras

Locality Issues

- Markopoulou and Smolin, 2007 (gr-qc/0702044)

- Physically these "nonlocal" links correspond to large 3-curvatures → expect dynamical suppression
- Is this related to +ve energy/wormholes/time machines etc.?

Models for Flat(?) Quantum Space

- One can find solutions to ADM equations by putting extrinsic and intrinsic curvature equal to zero

$$ {}^{(3)}R + (trK)^2 + K_{ab}K^{ab} = 0 \quad \nabla_b\left(K^{ab} - \gamma^{ab} tr(K)\right) = 0 $$

- Recall that

$$ \frac{1}{2}{}^{(3)}R + (\kappa_1\kappa_2 + \kappa_1\kappa_3 + \kappa_2\kappa_3) = 8\pi G\rho $$

- …but there may yet be issues with vol/area.
- Question: what would defects (nonquadrivalent) nodes in an otherwise flat space mean?
- Particles?

A Connection to Strings?

- Going back to Regge in 2D, 3D, 4D:

In 2D, defects are at *pointlike* singularities

In 3D, defects are along 1D (*stringlike*) singularities

In 4D, defects are along areas (worldines of strings?)

Could this mean something interesting? What restrictions are placed on the dynamics of the hinges themselves? Can they be interpreted as stringy matter? Connection to BF theory (A. Perez Wednesday) – stringy matter natural….

Summary

1) It might make sense to start with what one learns about quantized geometry from LQG and use that as input to simplicial approaches to QG.
2) In a sense, LQG and simplicial approaches would be "dual" to each other.
3) Some hints that a nonzero cosmological constant or a deformed Poincare (or Poincare-Heisenberg) algebra might arise
4) If you do this, you even see something stringy that ought to be associated with curvature – could this have something to do with strings?

Is it possible that everyone doing different things is (at least partially) right? …y, !Gracias!