Quantum of Area and Its Spectroscopy

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- 2 new properties of quantum of area:
  - Ladder Symmetry
  - Degeneracy.
- Fluctuations of a black hole horizon.

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Isolated horizon strategy:

Firstly:

Constructing a classical sector that behaves black hole mechanics.

Then:

Pulling-back canonical variables to the sector

Then:

Quantization the geometry by promoting variables to operators on a floating lattice

Consequence:

A finite independent degrees of freedom appears on quantum isolated horizon.

Major limitations:

1- Classically the metric field must extends through horizon, but the spin network does not!

2- Quantum horizon must be localized as a quantum boundary of its interior states. Isolated horizon is localized classically.

3- No SU(2)-valued tangential edges are allowed on the horizon for no physical reason.

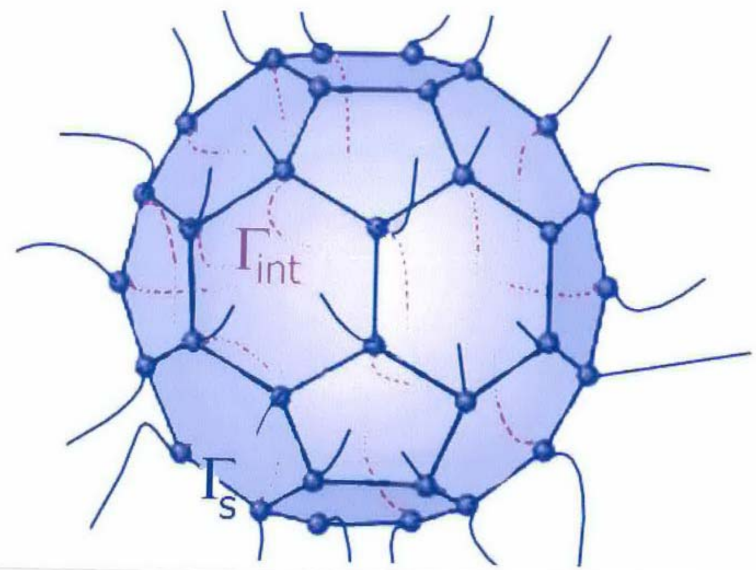
A paradigm for defining a quantum black hole

→ A black hole in the underlying manifold splits the embedding graph into the inside, outside and horizon sub-graphs.

→ Vertices and SU(2)-valued tangential edges reside on the horizon

→ The horizon is described by a SU(2) wave function.

→ All excluded quanta of area are now included.



$$\Psi_{\gamma_s} = \psi\left(h(e_1), \cdots, h(e_N)\right)$$

is $[SO(2)]^N$-valued generalized connection.

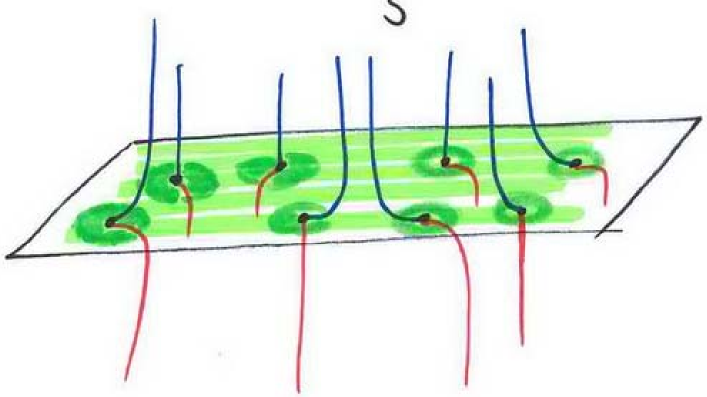
physics of black hole appears after:

Kinematics
- gauge-invariance
- Surface diffeo-invariance

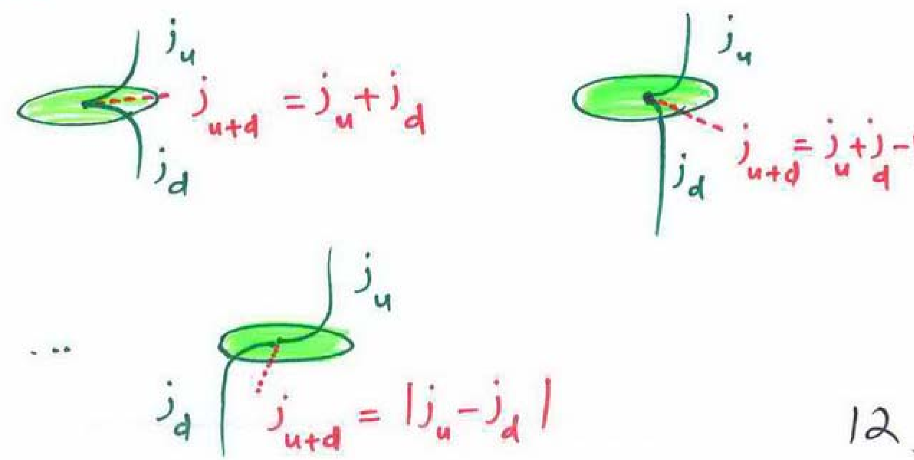
null non-expanding dynamics ⋮
- Hamiltonian-invariance or Hamiltonian eigenvector?

quantum of area

$$a_S = \frac{l_p^2 \gamma}{2} \sum_{\substack{\text{all} \\ \text{vertices } \alpha \\ \text{residing} \\ \text{on} \\ S}} \sqrt{2 j_d^{(\alpha)}(j_d^{(\alpha)}+1) + 2 j_u^{(\alpha)}(j_u^{(\alpha)}+1) - j_{u+d}^{(\alpha)}(j_{u+d}^{(\alpha)}+1)}$$



$$j_{d+u} \in \{ |j_d - j_u|, \ldots, (j_d + j_u) \}$$



Two new properties of quantum of area:

I) Ladder Symmetry

It is proven that the formula generates all quanta of area:

$$a_{\varsigma,n} = a_o \chi \sqrt{\varsigma}\; n$$

SO(3): $\chi = \sqrt{2}$ — Square-free numbers — $n \in \mathbf{N}$

SU(2): $\chi = \frac{1}{2}$ — Discriminantes of all quadratic positive definite forms — $n \in \mathbf{N}$

Square-free numbers = { 1,2, 3, 5, 6, 7,10,11,13,…}

The discriminants = { 3, 4, 7, 8, 11, 15, …}

χ := group characteristic parameter

ς := generation representative

→ A generation = a subset of fixed value of ς

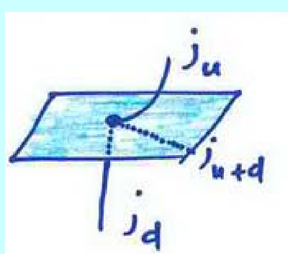
II) Degeneracy

If a horizon is described by SU(2)-wave functions, what is its kinematics degeneracy?

The answer is hidden in the area operator.

$$A \,|\, j_u, j_d, j_{u+d} \rangle = a \,|\, j_u, j_d, j_{u+d} \rangle$$

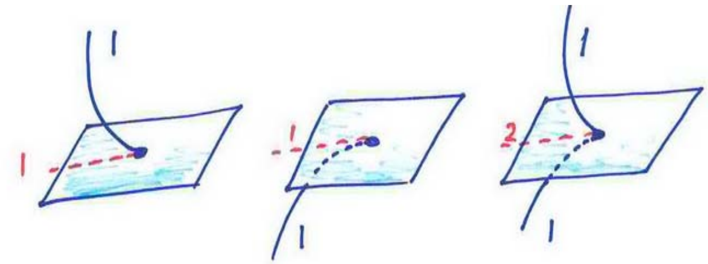
$$a = a_o \sqrt{2 j_u (j_u + 1) + 2 j_d (j_d + 1) - j_{u+d} (j_{u+d} + 1)}$$



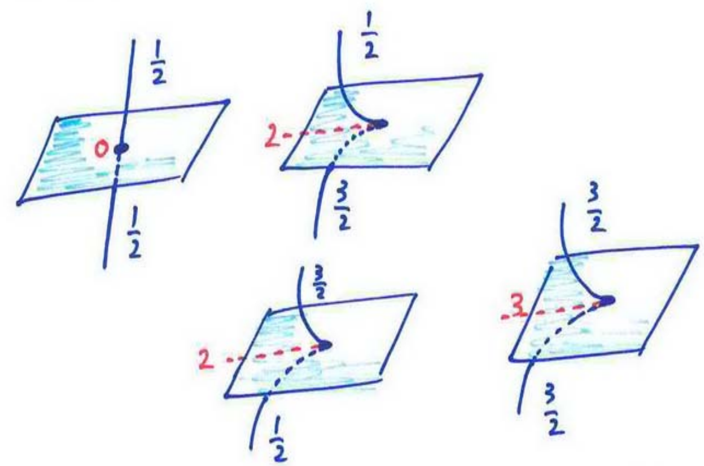
Example:

|0,1,1>, |1,0,1>, and |1,1,2> correspond all to $a = \sqrt{2}\, a_o$,

Pictorially:

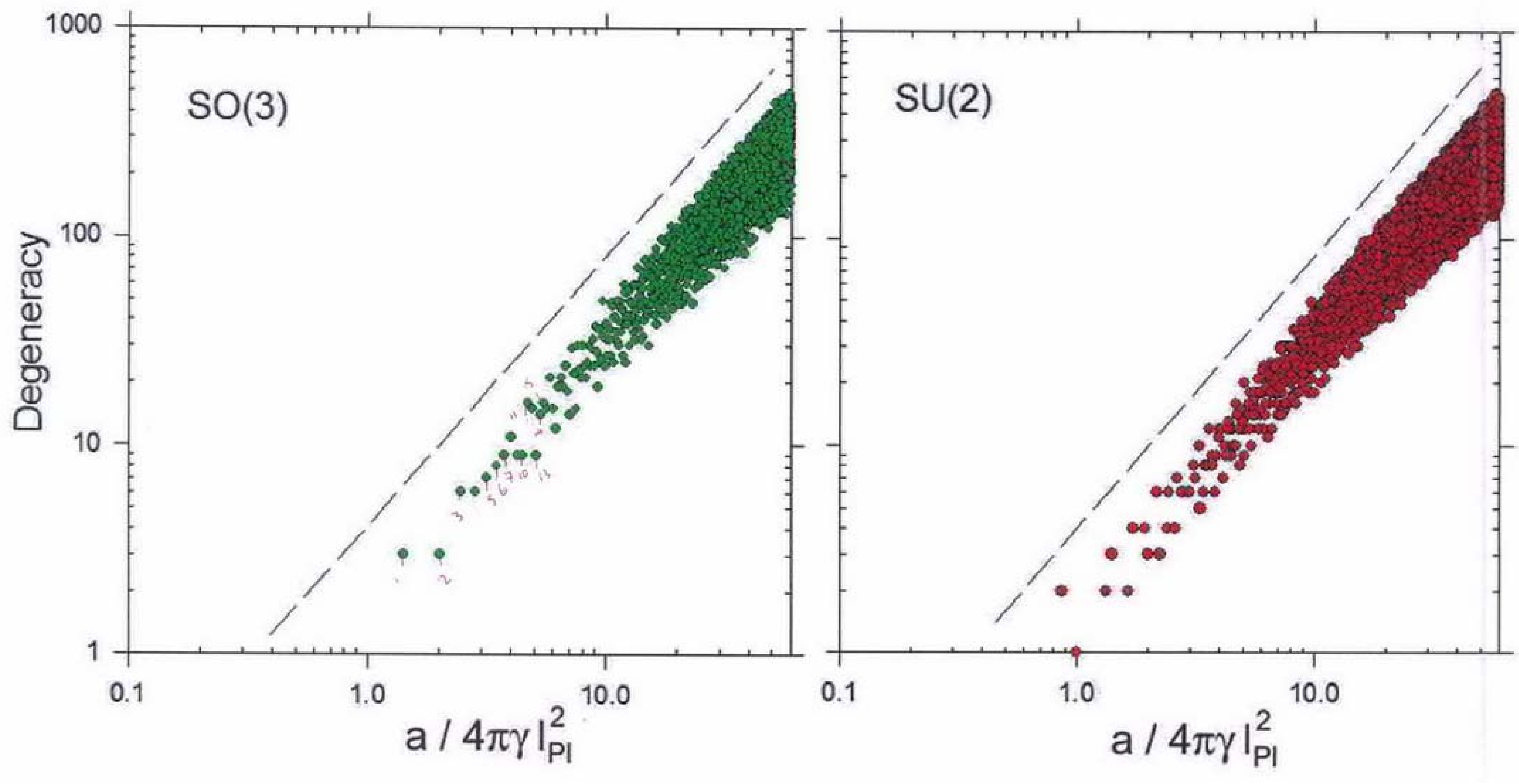


Another example :



all correspond to eigen value

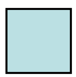
$$a_6 = \sqrt{3}\, a_o$$



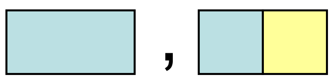
SO(3)
SU(2)
Degeneracy
1000
100
10
1
0.1
1.0
10.0
a / 4πγ l²_Pl

Total degeneracy

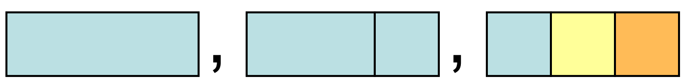
The degeneracy + The ladder symmetry ➔ the total degeneracy grows exponentially, not a power law.

a_1 

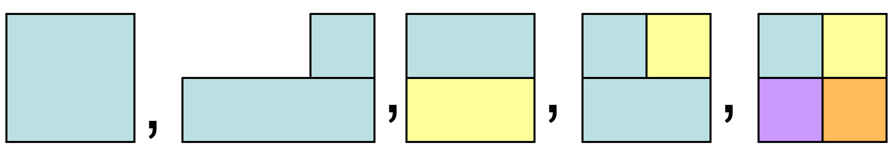
$\Omega_1 = g_1$

a_2 

$\Omega_2 = g_2 + 2g_1$

a_3 

$\Omega_3 = g_3 + g_2 g_1 + g_1^3$

a_4 

$\Omega_4 = g_4 + g_3 g_1 + g_2^2 + g_3 g_1 + g_1^4$

a_N

$\Omega_N \approx g_1^N$

Quantum fluctuations of horizon area:

Quantum fluctuations of the horizon may change Hawking radiation since the Hawking quanta will not be able to hover at a nearly fixed distance from the fluctuating horizon.

On a black hole energy is defined by

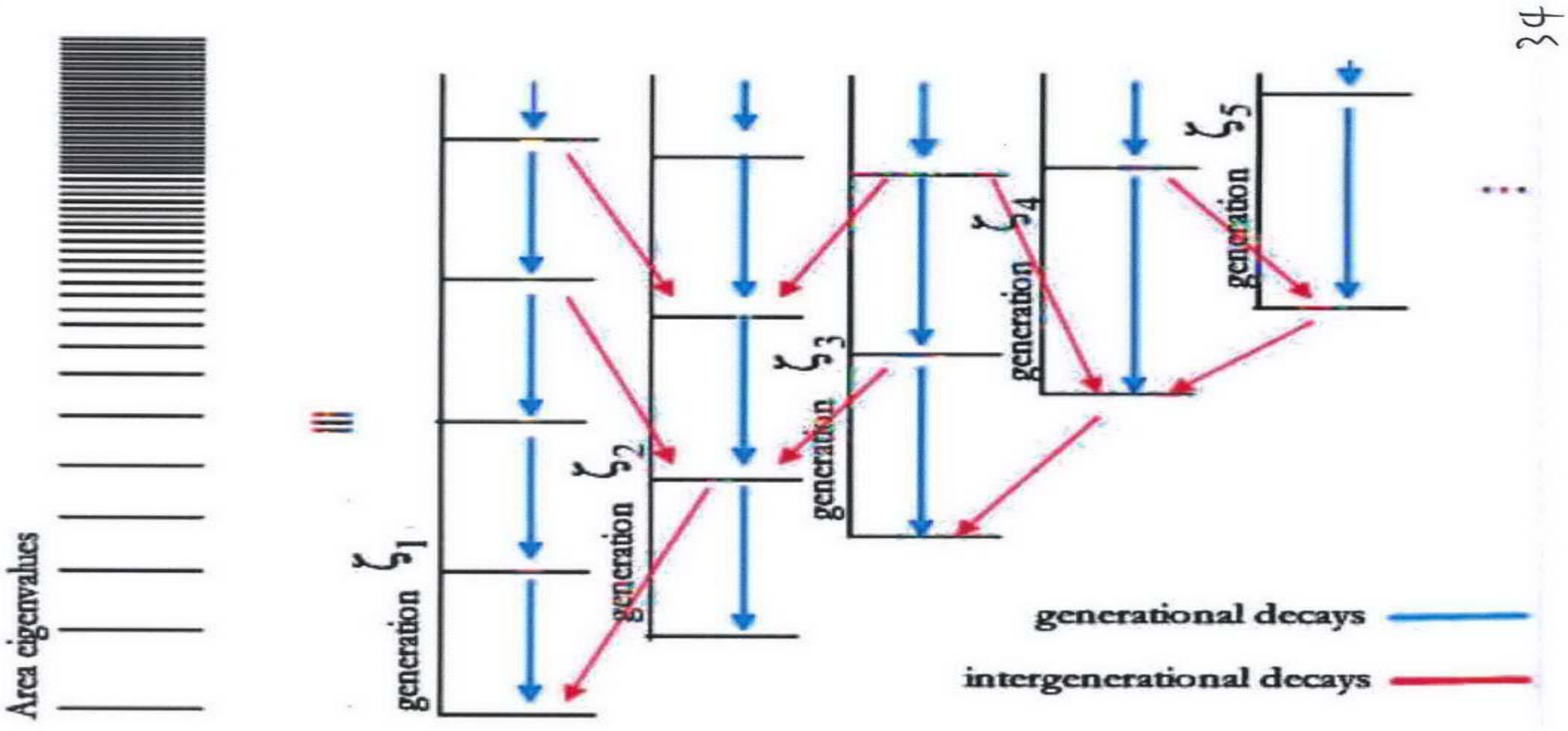
$$A = \frac{16\pi G^2}{c^4} M^2$$

Quanta of energy are proportional to the quanta of area for large black holes.

$$\delta A \propto \delta M$$

There are two types of decays:

- **generational**
- **inter-generational**



- Harmonic frequencies in **generational** decays: $\omega_n(\varsigma) = n\ \varpi(\varsigma),$

where fundamental frequencies of each generation $\varpi(\varsigma) = \omega_o\sqrt{\varsigma}$

- Inharmonic frequencies in **inter-generational** decays in all ranges of energy.

Quantum Amplification effect:

There exist many different copies of each harmonics made in different levels of a generation

BUT

There exist only one copy of inharmonics.

A discrimination:

The population of harmoonic frequencies exceeds the one of inharmonics.

For instance: $M = 10^{12}$ Kg

→A= 10^{-25} cm^2 ,

→T = 10^{11} K, ω_o = 10 Kev

Such a horizon is 40 order of magnitude larger than a quantum of area.

→

"Quantum Amplification Effect" makes a huge difference between harmonics and inharmonics.

Intensity

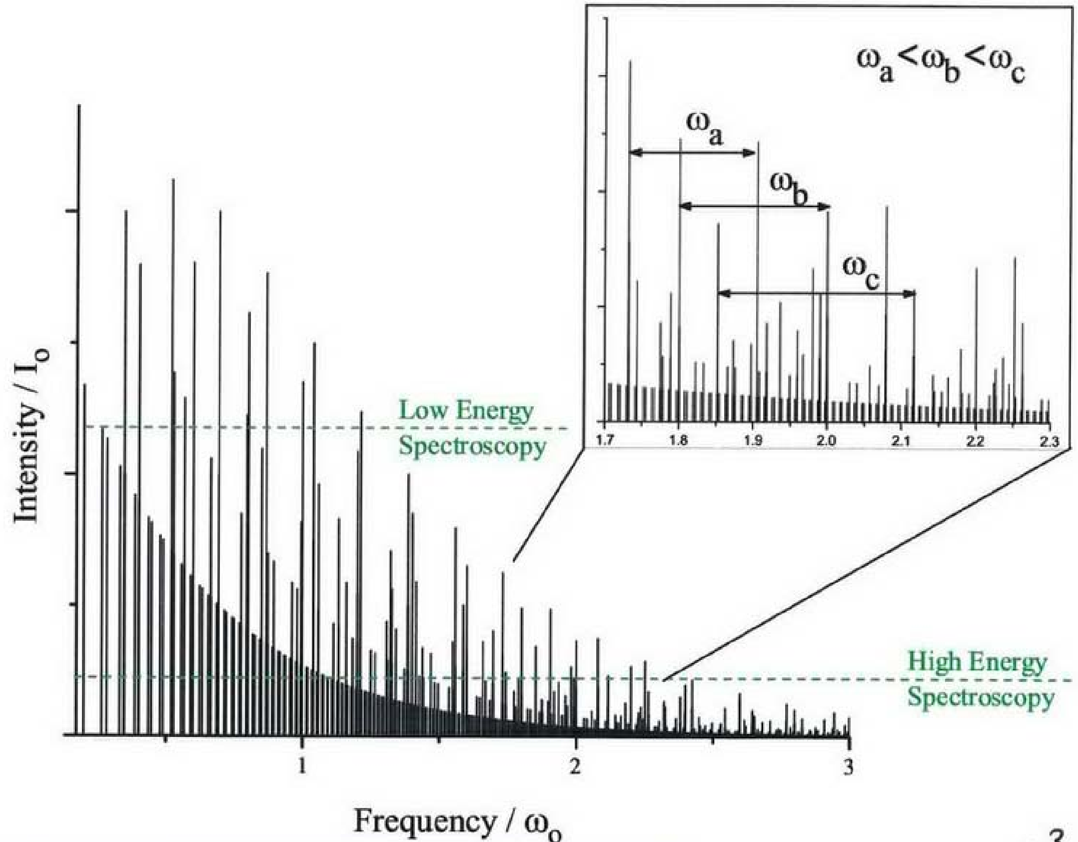
Width:

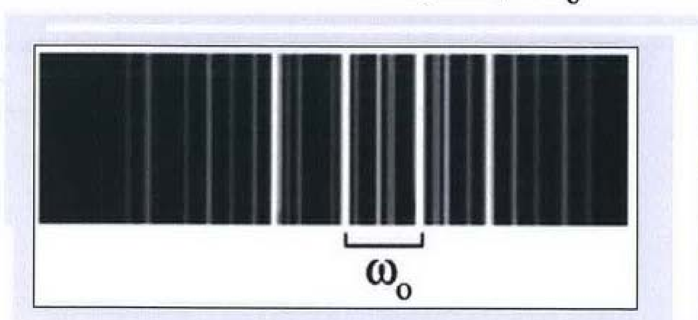
$\Delta\omega = 0.0001\ \omega_o$

For instance:

$M = 10^{12}$ Kg

→ $A = 10^{-25}\ cm^2$,
→ $T = 10^{11}$ K,
→ ωo = 10 Kev





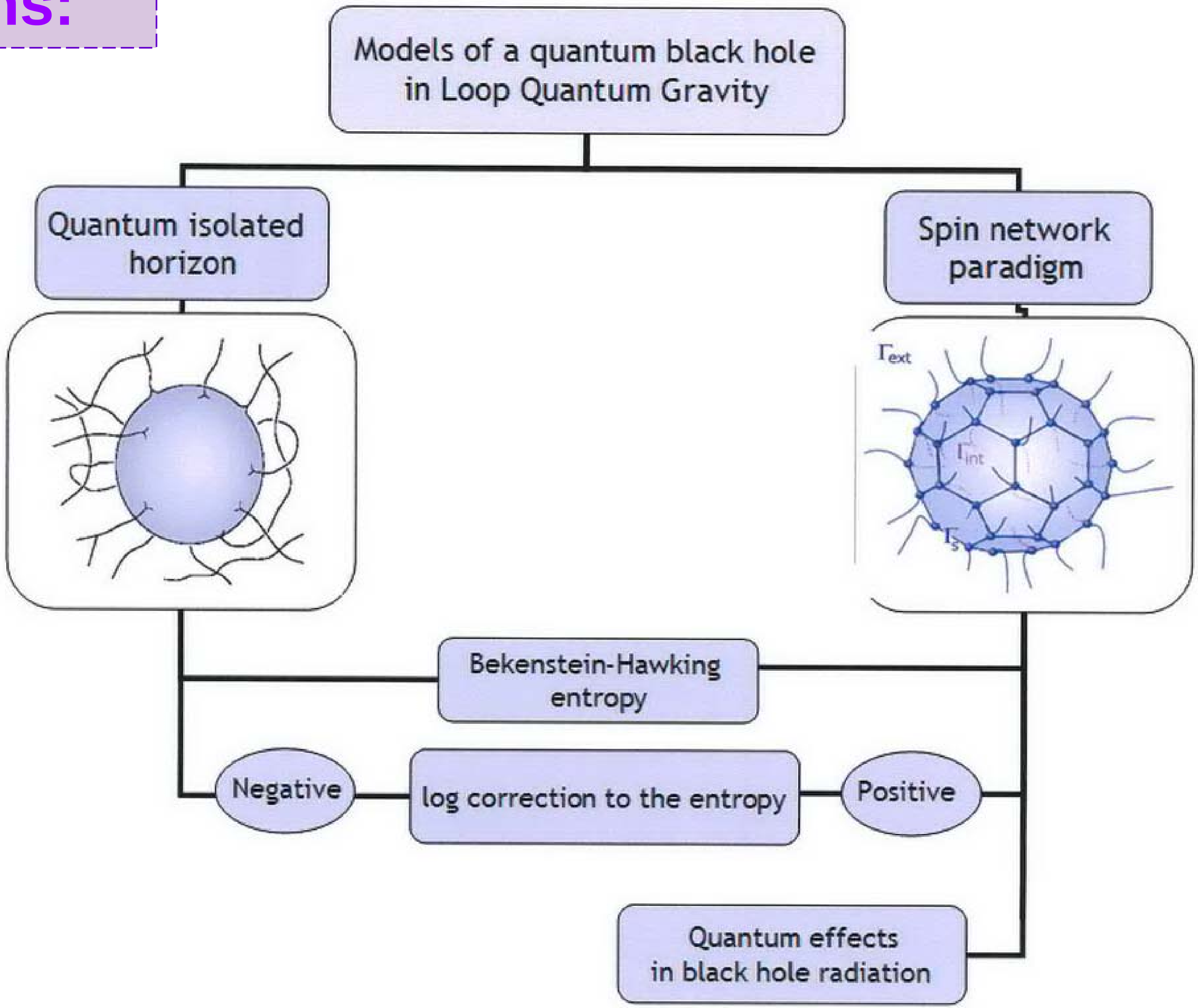
$$\omega_o = \frac{c^3}{8GM} \times \sqrt{3}$$

Conclusions:

1- A different paradigm for defining a quantum black hole,

2- Black hole entropy,

3- A macroscopic effect in black hole radiation.



Reference:

M. Ansari (2006)