Dynamics of Loop Quantum Schwarzschild Interior

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Loop Quantum Dynamics of Schwarzschild Interior
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Introduction

Recent advances in loop quantum cosmology (LQC) indicate replacement of big-bang with big-bounce for FRW cosmologies [Ashtekar, Pawlowski, Singh, KV, 2006-7]

Phenomenological effective theory incorporating holonomy features of Hamiltonian constraint operator provides explanation for bounce: Friedmann equation modified $H^2 = \frac{8\pi G}{3}\rho\,(1 - \frac{\rho}{\rho_c})$, gravity repulsive at high energies, bounce at $\rho = \rho_c$

Improved Hamiltonian constraint operator constructed ("$\overline{\mu}$" quantization vs "μ_0") - more physical results, good semi-classical limit, $\rho_c \approx \rho_{PL}$

What about black hole singularities?

Loop quantization of Schwarzschild interior Kantowski-Sachs model ("μ_0" quant) [Ashtekar, Bojowald, 2006]

Analysis of phenomenological effective dynamics performed [Modesto, 2006]

This talk: analyze consequences of the improved quantization successful in LQC applied to Schwarzschild interior

Schwarzschild Interior

Inside Schwarzschild horizon, switching temporal and radial coordinates, metric become spatially homogeneous Kantowski-Sachs type

$$ds^2 = -N^2(t)\,dt^2 + g_{xx}(t)\,dx^2 + g_{\Omega\Omega}(t)\,d\Omega^2$$

Two triad components p_b, p_c two connection components b, c

$$ds^2 = -N^2(t)\,dt^2 + \frac{p_b^2(t)}{|p_c(t)|}\,dx^2 + |p_c(t)|\,d\Omega^2$$

Dynamics determined from Hamiltonian

$$H = \frac{-N}{2G\gamma^2}\left[2\,b\,c\,\sqrt{p_c} + (b^2+\gamma^2)\frac{p_b}{\sqrt{p_c}}\right]$$

Schwarzschild solution

$$\begin{aligned}
N^2(t) &= \left(\frac{2m}{t} - 1\right)^{-1} \\
p_b(t) &= p_b^{(0)}\,t\sqrt{\frac{2m}{t} - 1} \qquad p_c(t) = t^2 \\
ds^2 &= -\left(\frac{2m}{t} - 1\right)^{-1} dt^2 + \left(\frac{2m}{t} - 1\right) dx^2 + t^2 d\Omega^2
\end{aligned}$$

Schwarzschild Interior

$$p_b(t) = p_b^{(0)} t \sqrt{\frac{2m}{t} - 1} \qquad p_c(t) = t^2$$

$$ds^2 = -\Big(\frac{2m}{t} - 1\Big)^{-1} dt^2 + \Big(\frac{2m}{t} - 1\Big) dx^2 + t^2 d\Omega^2$$

Singularity at $t = 0$: $p_c = 0$ and $p_b = 0$

Horizon at $t = 2m$: $p_c = 4m^2$ and $p_b = 0$

Interpretation:

p_c component directly determines two-sphere radius

Radial geodesics

$$\left(\frac{dt}{d\tau}\right)^2 = \Big(\frac{p_c}{p_b^2}\mathcal{E}^2 + 2\mathcal{L}\Big)\frac{1}{N^2}$$

$\mathcal{E}$ corresponds classically to energy at infinity, $\mathcal{L} = 0, 1$ for massless/massive test particle, τ is proper affine parameter/proper time for massless/massive particle

To interpret effective dynamics, calculate $p_c(\tau)$

Quantum Dynamics

Want to incorporate holonomy effects of Hamiltonian constraint operator into effective semi-classical description

Holonomies roughly exponentials of connection components $b, c \rightarrow e^{ib\delta_b}$ etc

Holonomy length parameters δ_b, δ_c measure the magnitude of quantum corrections - classical limit for $\delta_b, \delta_c \rightarrow 0$

Original quantization of Schwarzschild interior assumed δ_b, δ_c were constants analogous to μ_0 parameter of LQC ($\delta_b = \delta_c = \rho$ from talk of Pullin)

More recent work of LQC has length parameters dependant on triad components - better semi-classical limit, more physical results

Holonomy effects incorporated in form of effective Hamiltonian with connection components replaced by holonomy equivalents

$$\begin{aligned} H_{cl} &= \frac{-N}{2G\gamma^2}\Big[2\,b\,c\,\sqrt{p_c} + (b^2+\gamma^2)\frac{p_b}{\sqrt{p_c}}\Big] \\ H_{eff} &= -\frac{N}{2G\gamma^2}\Big[2\frac{\sin b\delta_b}{\delta_b}\frac{\sin \delta_c c}{\delta_c}\sqrt{p_c} + \big(\frac{\sin^2 b\delta_b}{\delta_b^2} + \gamma^2\big)\frac{p_b}{\sqrt{p_c}}\Big] \end{aligned}$$

Effective Dynamics

Effective Hamiltonian

First step in analyzing quantum corrections - not rigorous derivation of effects, possible additional corrections

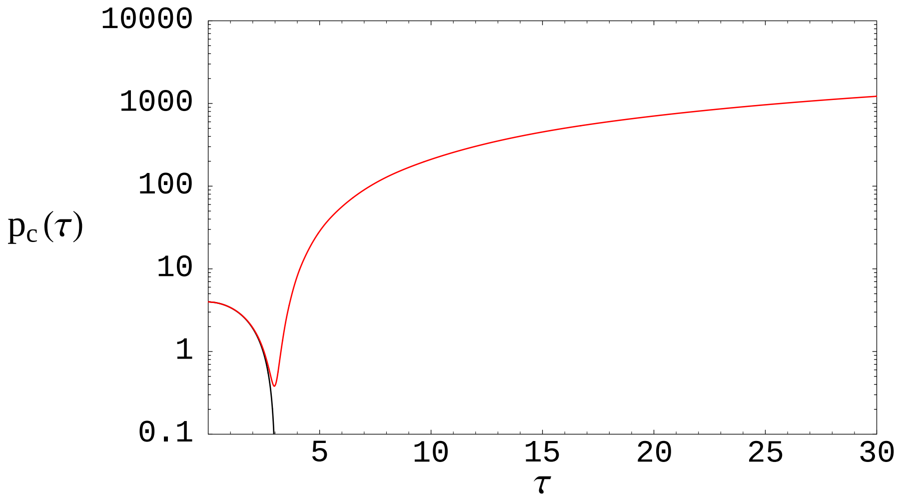
Has provided excellent accounting of big-bounce results of LQC for massless scalar field with Λ

Interested in phenomenological effects of these corrections, not necessarily final word

Effective Hamiltonian for $\delta_b, \delta_c = \text{const}$ results: talks by Pullin and Modesto

Singularity avoided, bounce in two-sphere radius: $p_c \geq \gamma\delta m$

Solution matches classical before bounce, connects to
another classical solution with different mass in general (can be made symmetric)



Improved Effective Dynamics

In LQC, more physical results when $\overline{\mu} = \frac{\Delta}{p^{1/2}}$

Constrain by shrinking loop of Hamiltonian constraint to have minimum LQG area - $\Delta = A_{min}$

Schwarzschild interior anisotropic, so more possible ways to implement the δ_c, δ_b parameters

Two most interesting schemes:

A) More geometric approach - constrain classical area of holonomy loops to have minimum area

End result

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_c}} \qquad \delta_c = \sqrt{\Delta}\frac{\sqrt{p_c}}{p_b}$$

B) Alternative approach - loop area dependent on transversal holonomy

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_b}} \qquad \delta_c = \frac{\sqrt{\Delta}}{\sqrt{p_c}}$$

A favored by stability analysis of quantum difference equation [Bojowald, Cartin, Khanna, 2007]

B applied to Bianchi I model [Chiou, 2006]

B gives similar results to the constant δ case

Improved Effective Dynamics

Focus on scheme A:

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_c}} \qquad \delta_c = \sqrt{\Delta}\frac{\sqrt{p_c}}{p_b}$$

$$H_{eff} = -\frac{N}{2G\gamma^2}\Big[2\frac{\sin b\delta_b}{\delta_b}\frac{\sin \delta_c c}{\delta_c}\sqrt{p_c} + \big(\frac{\sin^2 b\delta_b}{\delta_b^2} + \gamma^2\big)\frac{p_b}{\sqrt{p_c}}\Big]$$

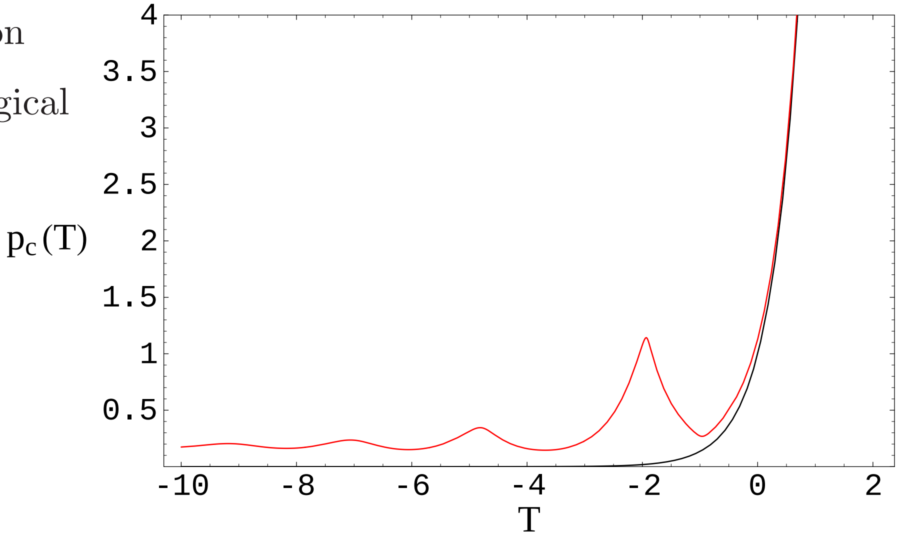
Equations of motion e.g. $\dot{p_c} \propto \partial H_{eff}/\partial c$ etc.

Too complicated for analytical solution, numerical integration of $p_c(T)$:

Again, no singularity, asymptotes to a Nariai type solution

Nariai solution of classical GR: constant p_c with cosmological constant Λ

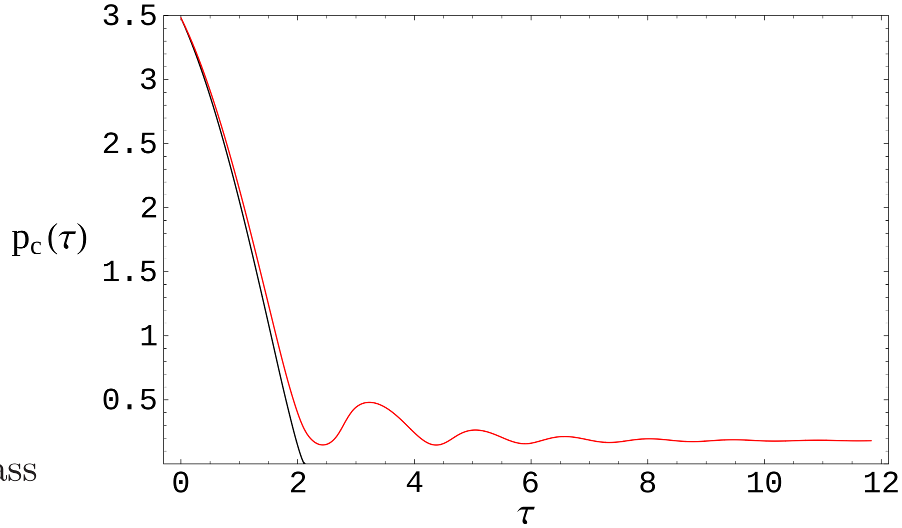
$ds^2 = -dt^2 + A\cosh^2(\sqrt{\Lambda}t)dx^2 + 1/\Lambda d\Omega^2$



Improved Effective Dynamics

Radial geodesic:

- Decrease in two-sphere radius as classical singularity approached
- Damped oscillations in radius
- particle settles in at finite radius dependant only on Δ
- For $\Delta = A_{min}$, $p_c \approx .2 l_p^2$
- Final radius Planckian, independent of black-hole mass
- Interpret as repulsive gravity, similar to bouncing results of LQC



Caveat - effective Hamiltonian also predicts deviations from classical behavior near classical horizon.

Not clear if problem with quantization scheme, or Kantowski-Sachs approximation not to be trusted there, or boundary matching more complicated

Conclusion/Outlook

Each phenomenological study indicates singularity resolution of Schwarzschild black hole analogously to LQC results

Detailed consequences dependant on quantization scheme

Two-interesting outcomes - wormhole like solution matching connecting two black holes, asymptote into Nariai type space-time - in-falling particle trapped at finite Planckian radius

Results are indicative and arise from simple effective theory - requires more justification by analyzing semi-classical states in quantum theory

Future - apply results to inhomogeneous spherically symmetric models for instance work of Campiglia, Gambini, Pullin. Can investigate true collapse models.