The full Graviton propagator from LQG

Emanuele Alesci

University of Rome 3 and

Université de la Mediterranée

Loop 2007 Morelia

Tensorial Structure

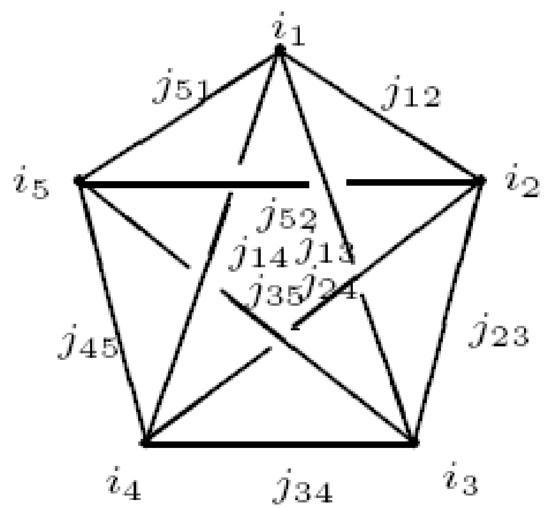
Try to extend the results of Bianchi, Modesto, Rovelli, Speziale

To calculate the complete tensorial structure

$$\mathbf{G}_{\mathbf{q}}^{abcd}(x,y) = \sum_{s,s'} \langle W|s'\rangle\langle s'|h^{ab}(x)\; h^{cd}(y)|s\rangle\langle s|\Psi_{\mathbf{q}}\rangle$$

To first order in λ $\langle W|s\rangle = W[s] = W[\Gamma, \mathbf{j}, \mathbf{i}]$ is non vanishing only if Γ is

$s =$



We have 5 intertwiners and 10 spins as variables

Inserting resolutions of the identity, using the base $|s\rangle = |\Gamma, \mathbf{j}, \mathbf{i}\rangle$

$$\mathbf{G}_{\mathbf{q}}^{abcd}(x,y) = \sum_{\mathbf{j},\mathbf{j}',\mathbf{i},\mathbf{i}'} W(\mathbf{j}',\mathbf{i}') \; < \mathbf{j}',\mathbf{i}'|h^{ab}(x)\; h^{cd}(y)|\mathbf{j},\mathbf{i} > \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

$W(\mathbf{j},\mathbf{i}) = W[\Gamma_5, \mathbf{j}, \mathbf{i}]$ PROPAGATION KERNEL

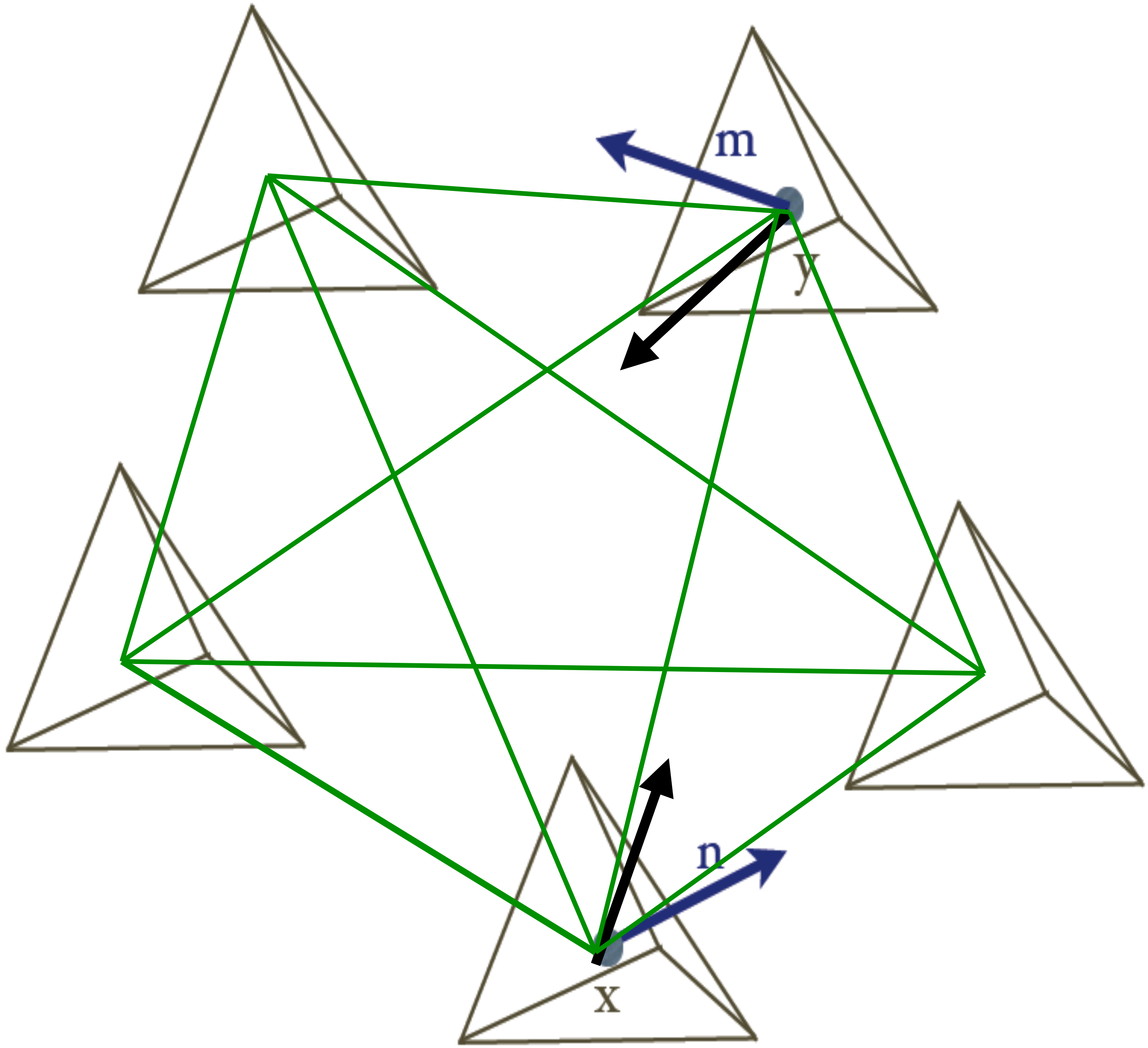
$\Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i}) = \Psi_{\mathbf{q}}[\Gamma_5, \mathbf{j}, \mathbf{i}] = \langle \Gamma_5, \mathbf{j}, \mathbf{i}|\Psi_{\mathbf{q}}\rangle$ BOUNDARY STATE

$< \mathbf{j}',\mathbf{i}'|h^{ab}(x)\; h^{cd}(y)|\mathbf{j},\mathbf{i} >$ QUANTUM OPERATORS

Now explicit dependance on the interwiners i

Consider the propagator projection on the normals $n_a^{(ni)}$ to the triangle t_{ni} that bounds the tetrahedra n and i and so on

$$\mathbf{G}_{\mathbf{q}\,n,m}^{ij,kl} := \mathbf{G}_{\mathbf{q}}^{abcd}(x_n, x_m)\; n_a^{(ni)} n_b^{(nj)}\; n_c^{(mk)} n_d^{(ml)}$$


m
y
n
x

Since $h^{ab} = g^{ab} - \delta^{ab} = E^{ai} E^b_i - \delta^{ab}$ defining $E_n^{(ml)} = E^a(\vec{x}) n_a^{(ml)}$

We have to calculate

$$\begin{aligned} \mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} &= \langle W | \left(E_n^{(ni)} \cdot E_n^{(nj)} - n^{(ni)} \cdot n^{(nj)} \right) \left(E_m^{(mk)} \cdot E_m^{(ml)} - n^{(mk)} \cdot n^{(ml)} \right) | \Psi_{\mathbf{q}} \rangle \\ &= \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i}) \left(E_n^{(ni)} \cdot E_n^{(nj)} - n^{(ni)} \cdot n^{(nj)} \right) \left(E_m^{(mk)} \cdot E_m^{(ml)} - n^{(mk)} \cdot n^{(ml)} \right) \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i}) \end{aligned}$$

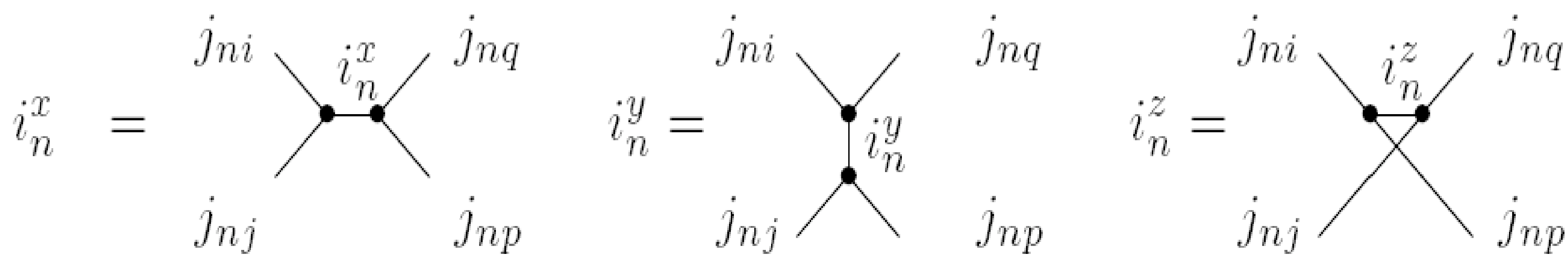
Understand the action of the non diagonal operator E.E on the spin networks states

$$E_n^{(ni)} \cdot E_n^{(nj)} \; |\Gamma, \mathbf{j}, \mathbf{i}\rangle$$

Use of Recoupling Theory paying attention to

Spinnetworks orientations and pairing of the virtual links

Three independent bases determined by the three possible coupling of the external links

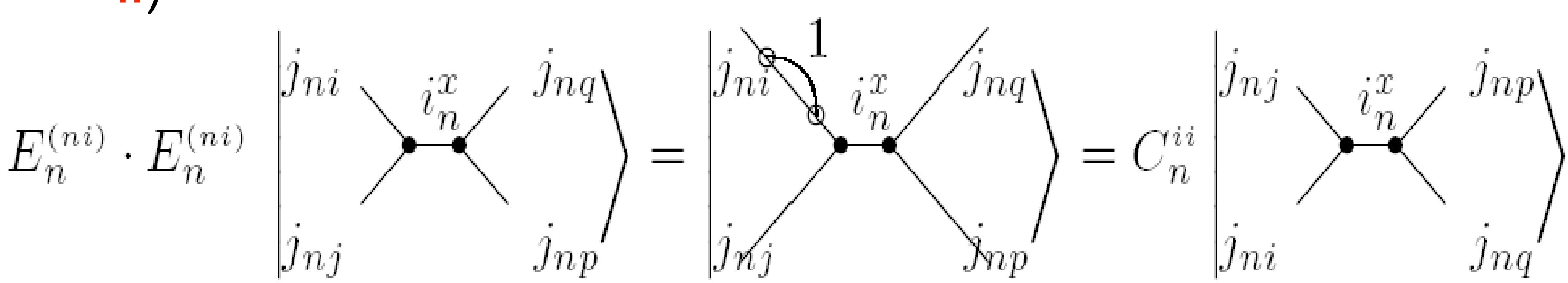


The three bases diagonalize the three non commuting operators

$$E_n^{(ni)} \cdot E_n^{(nj)} \qquad E_n^{(ni)} \cdot E_n^{(nq)} \qquad E_n^{(ni)} \cdot E_n^{(np)}$$

Action of the quantum operators

ii)



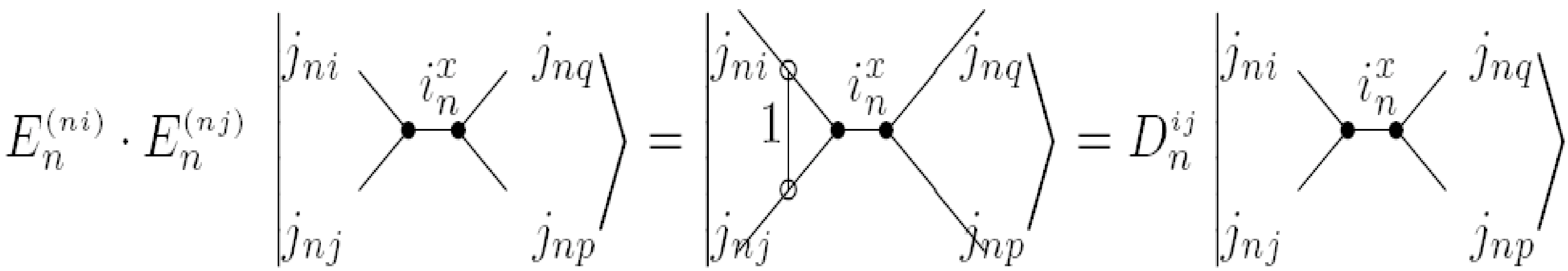
Where $C_n^{ii} = C^2(j_{ni})$

The action of the operators EE is diagonal if i=j

$E_n^{(ni)} \cdot E_n^{(ni)}$ Is the Area operator,it reads the Casimir

$C^2(j_{ni})$ of the link j_{ni} In our picture the area of the triangle t_{ni}

ij)



The actions of the operators EE with i≠j is diagonal only if the intertwiner has been resolved in the coupling (i,j)

Recoupling Theory gives

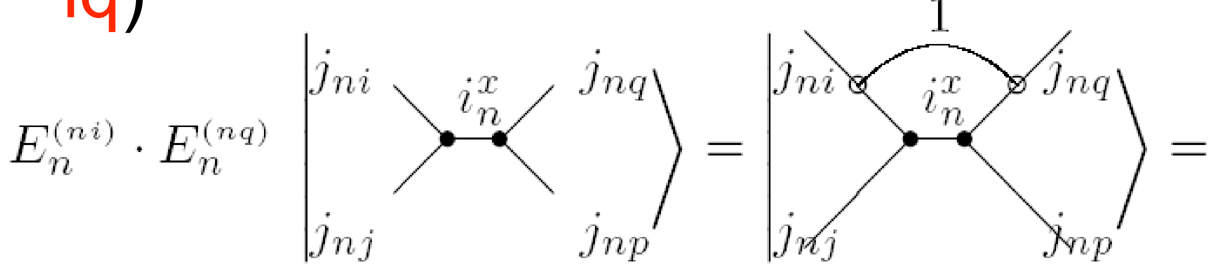
$$D_n^{ij} = \frac{C^2(i_n^x) - C^2(j_{ni}) - C^2(j_{nj})}{2}$$

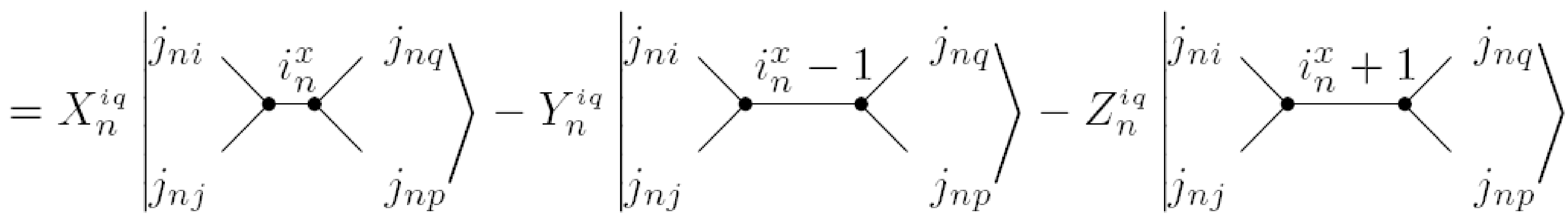
$E_n^{(ni)} \cdot E_n^{(nj)}$ involves directly the intertwiners dependance

This is the operator associated with the dihedral angle

between the triangles t_{ni} and t_{nj}

iq)





The calculated operators satisfy

$$E_n^{(ni)} \cdot E_n^{(ni)} + E_n^{(ni)} \cdot E_n^{(nj)} + E_n^{(ni)} \cdot E_n^{(np)} + E_n^{(ni)} \cdot E_n^{(nq)} = 0$$

The four normals of a tetrahedron sum up to 0 $\sum_{i \neq n} n_a^{(ni)} = 0$

New boundary state

To compute the DIAGONAL terms it was sufficient to consider a state of the kind

$$\Psi_{\mathbf{q}}[\mathbf{j},\mathbf{i}] = C \exp\left\{ -\frac{1}{2j^0} \sum_{(ij)(mr)} \alpha_{(ij)(mr)} \left(j^{(ij)} - j^0\right)\left(j^{(mr)} - j^0\right) + i\Phi \sum_{(ij)} j^{(ij)} \right\}$$

q is the geometry of the 3d boundary (Σ,*q*) of a spherical 4d ball, with linear size L >> √ħG

Ψ*q*(*s*) is a Gaussian state with correlation matrix α peacked on the "background" spins j^0.
Three free parameters in α to respect the symmetry of the sphere

The Φ are the background dihedral angles between tetrahedra (Variables coniugate to spins). They code the *extrinsic* 3-geometry *q*

The graviton operators call into play the intertwiners, we have to consider the kinematics of intertwiners and introduce an intertwiner dependance in the boundary state

New state

$$\Psi_{\rm q}[{\rm j},{\rm i}] = C\exp\left\{-\frac{1}{2j^0}\sum_{(ij)(mr)}\alpha_{(ij)(mr)}\,(j^{(ij)}-j^0)(j^{(mr)}-j^0)+i\Phi\sum_{(i,j)}j^{(ij)}\right\}\cdot$$
$$\cdot\exp\left\{-\sum_n\left(\frac{(i_n-i^0)^2}{4\sigma_{i_n}}+\sum_{a\neq n}\phi_{j_{na}\,i_n}(j^{(na)}-j^0)(i_n-i^0)+i\chi_{i_n}(i_n-i^0)\right)\right\}$$

Also gaussian in the intertwiners around the background value i^0 (background dihedral angles) with variance S , phase factor □, correlation spin-intertwiner f.

THE FUNCTIONAL HAS TO BE PEACKED ON THE VALUES OF ALL DIHEDRAL ANGLES but the three bases in differing pairing don’t commute,
The three bases are related by 6j symbols

With an appropriate choise of s,c,f we can create a state with mean value i^0 in every pairing, in each node and also with vanishing relative uncertainties

$$\frac{\langle\Psi_{\rm q}|\,i_n\,|\Psi_{\rm q}\rangle}{\langle\Psi_{\rm q}|\Psi_{\rm q}\rangle}=i_0\quad\text{and}\quad\frac{\sqrt{\frac{\langle\Psi_{\rm q}|(i_n)^2|\Psi_{\rm q}\rangle}{\langle\Psi_{\rm q}|\Psi_{\rm q}\rangle}-\left(\frac{\langle\Psi_{\rm q}|i_n|\Psi_{\rm q}\rangle}{\langle\Psi_{\rm q}|\Psi_{\rm q}\rangle}\right)^2}}{\frac{\langle\Psi_{\rm q}|i_n|\Psi_{\rm q}\rangle}{\langle\Psi_{\rm q}|\Psi_{\rm q}\rangle}}\to 0\quad\text{when } j^0\to\infty$$

A similar state has good semiclassical properities but it is not symmetric in the fluctuations. The boundary functional has to be symmetric in the fluctuations, otherwise it would threat different pairings with different weights, and in the quantum theory different pairings mean different directions: priviledging a chosen pairing we would break the symmetry of the space

SOLUTION

We symmetryze the state on each node summing over the three possible pairings

$$|\Psi_{\mathbf{q}}\rangle = \sum_{m_n} \sum_{\mathbf{j}} \sum_{\mathbf{i}_n^{m_n}} C_{\mathbf{j}\,\mathbf{i}_n^{m_n}} \left|\mathbf{j}, \mathbf{i}_n^{m_n}\right\rangle$$

m_n=x,y,z

n=1,2..,5

$$\mathbf{j} = \{j^{12}, j^{13}, .., j^{21}, j^{23} .., j^{53}, j^{54}\}$$

$$C_{\mathbf{j}\,\mathbf{i}_n^{m_n}} = e^{-\frac{1}{2j^0}\sum \alpha_{(ij)(mr)}\delta j^{ij}\delta j^{mr} + i\sum \Phi \delta j^{ij}} e^{-\sum_n \left(\frac{3(\delta i_n^{m_n})^2}{4j^0} - i\left(\sum_a \frac{3}{4j^0}\delta j^{an} + \frac{\pi}{2}\right)\delta i_n^{m_n} \right)}$$

Calculation with BC vertex

In the calculation of the diagonal terms, was used a BC vertex with a projection map

$$W(\mathbf{j},\mathbf{i}) = W(\mathbf{j}) \prod_n \langle i_{BC} | i_n \rangle = W(\mathbf{j}) \prod_n (2 i_n + 1)$$

Where W(j) is the 10j symbol

Map Simple SO(4)->SU(2) supported by the physical interpretation

We have to calculate terms of the kind,

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i}) \left(D^{ij}_n - n^{(ni)} \cdot n^{(nj)}\right) \left(D^{kl}_m - n^{(mk)} \cdot n^{(ml)}\right) \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

Keeping the dominant terms (we are interested in the large j^0 limit)

$$D^{(ij)(ik)}_i - n^{(ij)} \cdot n^{(ik)} = \delta i_i \; i_0 - \delta j_{ij} j_0 - \delta j_{ik} j_0$$

Dominant term of operators: intertwiners and spins as variables

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = j_0^2 \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i}) \left(\frac{2}{\sqrt{3}}\,\delta i_n - \delta j_{ni} - \delta j_{nk}\right) \left(\frac{2}{\sqrt{3}}\,\delta i_m - \delta j_{mk} - \delta j_{ml}\right) \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = j_0^2 \sum_{\mathbf{j},\mathbf{i}} W(\mathbf{j},\mathbf{i}) \left(\frac{2}{\sqrt{3}}\,\delta i_n - \delta j_{ni} - \delta j_{nk} \right) \left(\frac{2}{\sqrt{3}}\,\delta i_m - \delta j_{mk} - \delta j_{ml} \right) \Psi_{\mathbf{q}}(\mathbf{j},\mathbf{i})$$

$$W(\mathbf{j}) \approx e^{iS_{Regge}} + e^{-iS_{Regge}} + D$$

Barrett, Williams, Baez, Christensen, Egan, Freidel, Louapre

$$\Psi_{\mathbf{q}}[\mathbf{j},\mathbf{i}] \approx e^{-\frac{1}{2j^0}\sum \alpha_{(ij)(mr)}\delta j^{ij}\delta j^{mr} + i\sum \Phi \delta j^{ij}}\, e^{-\sum_n \frac{3(\delta i_n)^2}{4j^0} - i\sum_a \frac{3}{4j^0}\delta j^{an}\delta i_n + i\frac{\pi}{2}\delta i_n}$$

$$S_{\text{Regge}}(j_{nm}) = \Phi \sum_{nm} j_{nm} + \frac{1}{2} G_{(mn)(pq)} \delta j_{mn} \delta j_{pq}$$

The rapidly oscillating phase in the state (green) cancel or double the phase in the dynamics (green).

Only the term without phase survives (This was the key feature of the Diagonal terms)

BUT now there is also a phase term (pink) in the state UNCOMPENSED by the dynamics

PROBLEM OF THE MODEL:

THE DYNAMICS DOESN'T SPEAK WITH THE INTERTWINERS

$i\frac{\pi}{2}\sum_p i_p$ The Phase Factor is not compensated by the dynamics

SUPPRESS THE SUM

If we proceed with the calculation, we can recast the problem introducing the 15 components vectors and the 15 x 15

$$\delta I^\alpha = (\delta j^{ab}, \delta i_n) \qquad \delta\Theta^\alpha = (0, \chi i_n)$$

Correlation Matrix M that contains the 3 free parameters of the gaussian plus dynamics

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} = \mathcal{N}' j_0^2 \int d\delta I^\alpha \left(\frac{2}{\sqrt{3}}\,\delta i_n - \delta j_{ni} - \delta j_{nj}\right)\left(\frac{2}{\sqrt{3}}\,\delta i_m - \delta j_{mk} - \delta j_{ml}\right) e^{-\frac{M_{\alpha\beta}}{j^0}\delta I^\alpha \delta I^\beta}\; e^{i\Theta_\alpha \delta I^\alpha}$$

We get a sum of terms of the kind

$$\left(\frac{M^{-1}_{\alpha\beta}}{j_0} - M^{-1}_{\alpha\gamma}\Theta^\gamma M^{-1}_{\beta\delta}\Theta^\delta\right) j^0 \to \infty$$

→ Dominant term CONSTANT

Wrong large distance propagator

Proposal

We simply assume that a vertex can be defined such that in the large distance expansion it has the same asymptotic behavior as the Barrett-Crane vertex on the spins j, and it has also a dependence on the intertwiners i.
Guided by the compensation present in the diagonal case
we assume a vertex which asymptotic expansion up to second order is

$$W_{Asymp}(\mathbf{j},\mathbf{i}) = e^{i\frac{G}{2}\delta j \delta j} e^{i\Phi \delta j} e^{i\chi_{i_n}\delta i_n} e^{i\phi_{j i_n}\delta j \delta i_n} + e^{-i(\text{same expression})}$$

Same as BC but with the crucial phase (pink) in the intertwiner variable able to compensate the one in the boundary state.
Correlation spin-intertwiner usefull but not crucial.

The same kind of terms as before becomes

$$\mathbf{G}_{\mathbf{q}\,n,m}^{ij,kl} = \mathcal{N}' j_0^2 \int d\delta I^\alpha \left(\frac{2}{\sqrt{3}}\, \delta i_n - \delta j_{ni} - \delta j_{nj} \right) \left(\frac{2}{\sqrt{3}}\, \delta i_m - \delta j_{mk} - \delta j_{ml} \right)\, e^{-\frac{M_{\alpha\beta}}{j^0} \delta I^\alpha \delta I^\beta}\, e^{i\Theta_\alpha \delta I^\alpha}$$

The propagator is then a sum of terms of the kind

$$\frac{M'^{-1}_{\alpha\beta}}{j_0}$$

Right large distance behavior:

Remember $A = 8\pi\hbar G\, \sqrt{j^0(j^0+1)}$

$$\frac{k\hbar G M'^{-1}_{\alpha\beta}}{L^2}$$

$M'^{-1}_{\alpha\beta}$ Contains a linear combination of the derivatives of Regge Action and of the correlation matrix in the gaussian

The complete tensorial structure

In the Euclidean theory the linearized expression for the Graviton Propagator in the harmonic gauge is

$$G^{\mu\nu\rho\sigma}_{linearized} = \frac{1}{2L^2}\left(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\rho\sigma}\right)$$

In the non-perturbative theory we calculate the propagator projecting on the normals to the tetrahedra faces

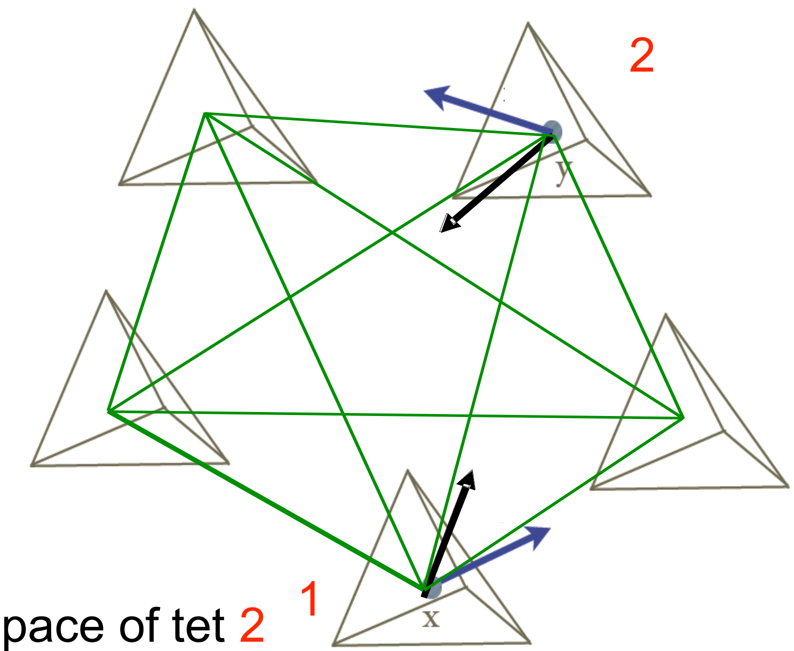
$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,n,m} := \mathbf{G}^{abcd}_{\mathbf{q}}(x_n, x_m)\; n^{(ni)}_a n^{(nj)}_b \; n^{(mk)}_c n^{(ml)}_d$$

Fixing n,m equal to 1,2 we have

$$\mathbf{G}^{ij,kl}_{\mathbf{q}\,1,2} := \mathbf{G}^{abcd}_{\mathbf{q}}(x_1, x_2)\; n^{(1i)}_a n^{(1j)}_b \; n^{(2k)}_c n^{(2l)}_d$$

We can compare it with

$$G^{(1i)(1j)(2k)(2l)}_{linearized} \equiv G^{abcd}_{linearized}(x, y)\; n^{(1i)}_a n^{(1j)}_b \; n^{(2k)}_c n^{(2l)}_d$$

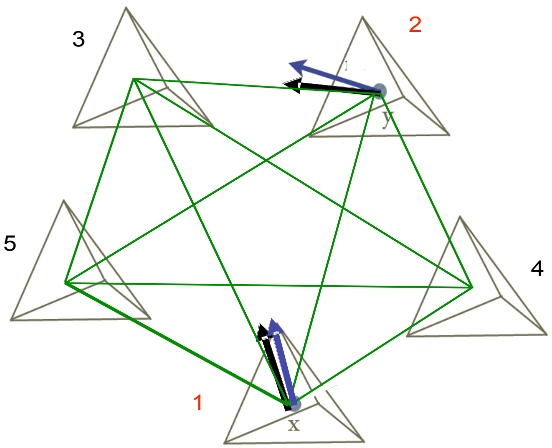


Indexes a,b in the 3d space of tet 1, c,d in the space of tet 2

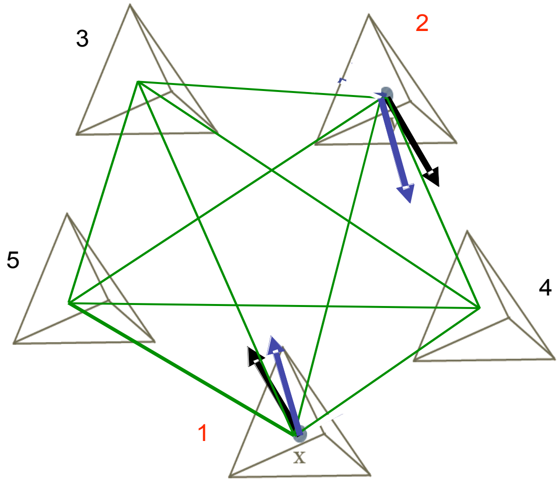
$$G^{(1i)(1j)(2k)(2l)}_{linearized} \equiv G^{abcd}_{linearized}(x,y)\; n_a^{(1i)} n_b^{(1j)}\;\; n_c^{(2k)} n_d^{(2l)}$$

4x4x4x4 tensor, but due to symmetry and Closure Relations there are

ONLY 5 INDEPENDENT COMPONENTS



G-lin13132323



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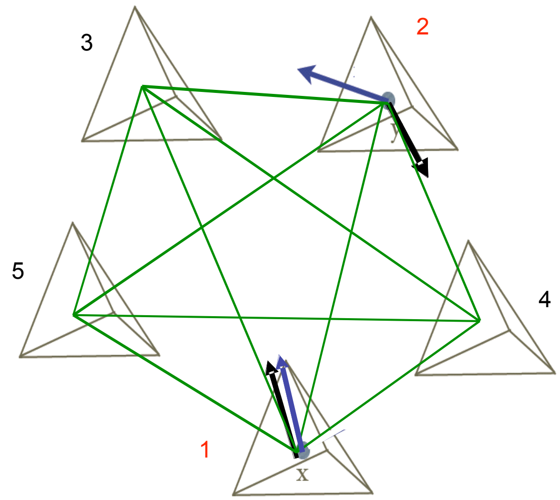

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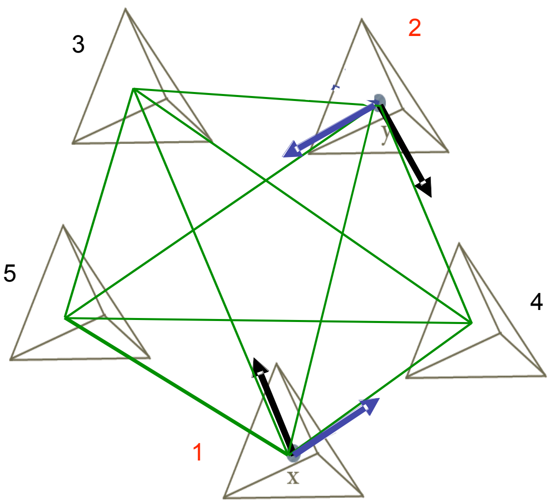

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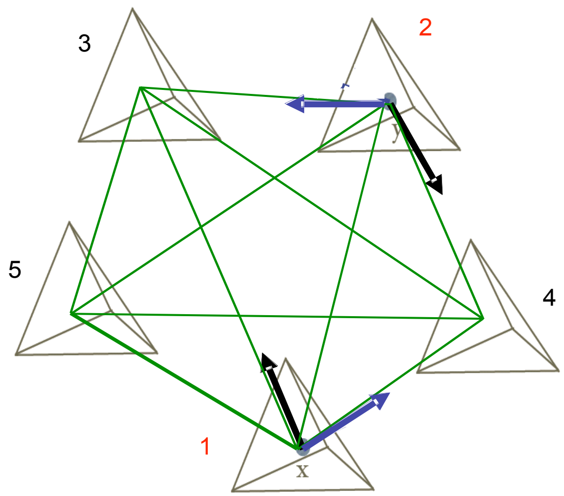

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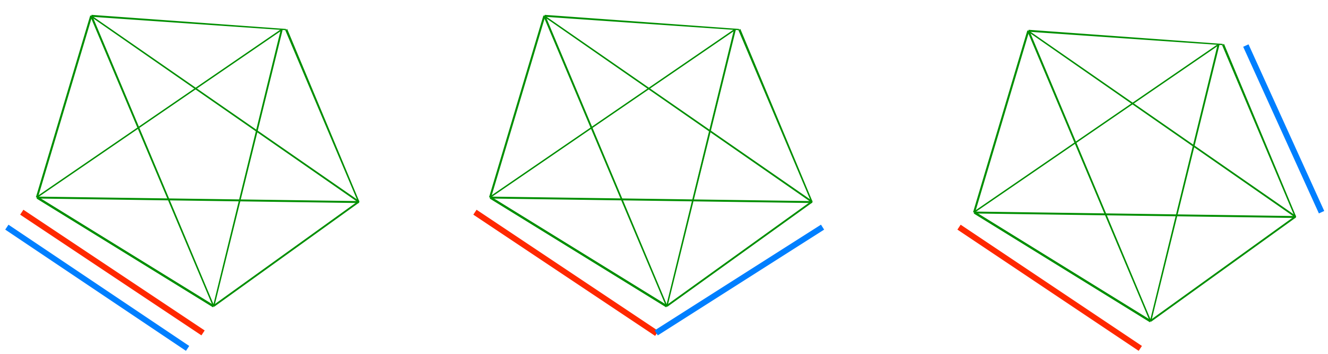
In the non perturbative theory, the so computed operators satisfy the closure relations:
The symmetrization procedure on the state allows to reproduce
the spacetime symmetry

We only have to fix 5 components to reproduce the entire tensorial structure
Do we have 5 independent parameters?

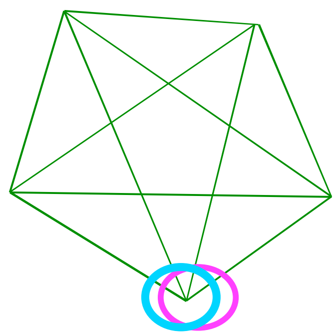
The only free parameters are in the boundary state:
to respect the symmetry of the sphere we have 7 possible correlations

3 link-link: free parameters

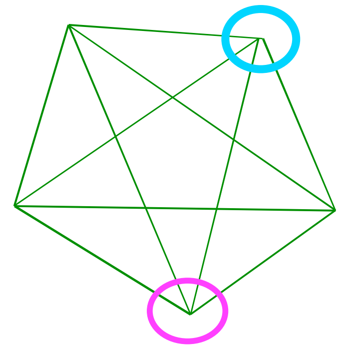


Free Free Free

2 intertwiner-intertwiner but as we have seen, the diagonal correlation int-int has to be fixed to reproduce the right classical behavior (mean values and dispersion relations) of the boundary state

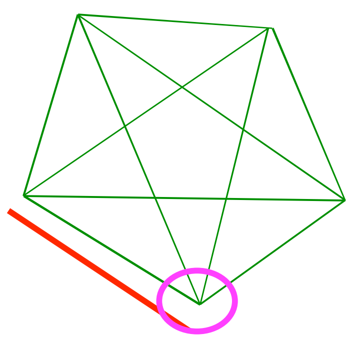


Fixed



Free

2 link-intertwiner but the correlation link-intertwiner of the same node, has to be fixed to reproduce the right classical behavior (mean values and dispersion relations) of the boundary state



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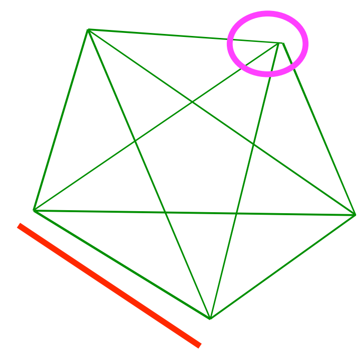

Free

Using a symmetrized gaussian state containing 5 free parameters as a Boundary state we can reproduce exactly the same tensorial structure and the same behavior of the linearized theory

WE HAVE FOUND THE FULL GRAVITON PROPAGATOR FROM LQG

CONCLUSIONS

CHANGE THE DYNAMICS: THE BARRET CRANE MODEL HAS NO INTERTWINER DEPENDANCE; USING A BC VERTEX WE ARE NOT ABLE TO REPRODUCE THE RIGHT LONG DISTANCE BEHAVIOR OF THE GRAVITON PROPAGATOR. In this sense the BC VERTEX DOESN'T WORK

Alesci,Rovelli to appear

Full tensorial structure and right long distance behavior: ASSUMING A VERTEX WITH NON TRIVIAL INTERTWINER DEPENDANCE, WITH GIVEN ASYMPTOTIC, IT IS POSSIBLE TO RECOVER THE FULL GRAVITON PROPAGATOR OF THE LINEARIZED THEORY FROM LQG USING ROVELLI'S TECHNIQUES TO COMPUTE SCATTERING AMPLITUDES IN A BACKGROUND INDEPENDENT FORMALISM

Alesci,Rovelli to appear

FUTURE DIRECTIONS

VERTEX ABLE TO REPRODUCE THE GIVEN ASYMPTOTICS Engle, Pereira,Rovelli

HIGHER ORDER TERMS IN λ

N POINT FUNCTIONS

MODIFICATION TO NEWTONIAN POTENTIAL

SCATTERING AMPLITUDES

LIMIT OF SMALL DISTANCES

CORRECTIONS TO THE GRAVITON PROPAGATOR WITH A THEORY NOT PLAGUED BY NOT RENORMALIZABILITY