Effective Field Theory of General Relativity

1) **Intro:** Watch your language!
   What can EFT do for you?

2) **Effective field theory in action**
   Potential between two nucleons
   Predictive use of non-renormalizeable theories

3) **Application to General Relativity**
   Renormalization of GR
   Quantum predictions

4) **Limitations of the effective field theory**

5) **Matching to LQG etc**
   John Donoghue
   Morelia, 2007
What is the problem with quantum gravity?

“Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct.”

“The existence of gravity clashes with our description of the rest of physics by quantum fields”

“Attempting to combine general relativity and quantum mechanics leads to a meaningless quantum field theory with unmanageable divergences.”

“The application of conventional field quantization to GR fails because it yields a nonrenormalizable theory”

“Quantum mechanics and general relativity are incompatible”
However these are inaccurate
- reflect an outdated view of field theory
- in particular, neglects understanding of effective field theory

Physics is an experimental science:
- constructed through discovery at accessible energies/distances
- description changes as we move to new domains
- new DOF and new interactions uncovered

- our present theories can only claim to hold at accessible scales
Crucial rephrasing:

Are QM and GR compatible over accessible scales, where we expect both to be correct?

Answer is **YES**
- this is what effective field theory can do for you

Effective field theory handles separation into known DOF at ordinary scales and unknown physics at extreme scales
- known = GR and matter
- the known part of the theory is very well behaved

“There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.”

F. Wilczek
Lessons of effective field theory for GR:

At ordinary energies/scales:

The quantum theory of general relativity exists and is of the form of an effective field theory.

The effective field theory treatment completes the covariant approach started by Feynman and DeWitt.

The non-renormalizability of QGR is not a problem - GR can be renormalized perturbatively.

Quantum general relativity is an excellent perturbative theory - we can make reliable predictions.

There IS a quantum theory of gravity at ordinary energies.
However, there are still problems in gravity theory
- issue is not QM vs GR per se
- problems are at “extremes”

EFT helps reformulate the issues:
“Quantum general relativity points to the limits of its perturbative validity”

“Quantum general relativity is most likely not the final theory”

“The extremes of quantum GR have raise deep and important problems.”

But, this IS progress!
- we have a solid foundation for the start of a more extensive theory
Effective Field Theory in action: I – The need

Example: Nucleon scattering
(here J=0, I=0 part only, and m_π=0 for clarity)

General principles
- causality
- unitarity
- crossing
imply a dispersive representation of scattering amplitude

\[ V(s, q^2) = \frac{1}{\pi} \int_{0}^{\infty} \frac{dt}{t - q^2} \rho(s, t) + (\text{L.H. cut} = \text{short range}) \]

\( \rho \) is Imag part of amplitude

Nuclear potential is F.T. of \( V(s, q^2) \)
What goes into the spectral function?
- all physical on-shell intermediate states
- lightest DOF are pions – need 2 pions
- no other state exists below $\sim 0.6 – 0.9$ GeV
Build interaction - product of on-shell tree level amplitudes

\[
\rho(s, t) = \frac{-1}{32\pi} \int \frac{d\Omega_k}{4\pi} \mathcal{M}_1(p_1, -k, p_1 - q, q - k) \mathcal{M}_2(p_3, -k, p_3 + q, k - q)
\]

\[
\rho(s, t) = c_3(s)t^{3/2} + \ldots
\]
The energy expansion in pictures:

**Result:** Longest distance piece

\[ V_C(r) = -\frac{9g_A^2}{4\pi F^2} \left( \frac{3g_A^2}{16M} + c_3 \right) \frac{1}{r^6} + \ldots \]
Reconstruct full low energy amplitude:
- on shell imaginary part generates real parts through dispersion relation
- Quantum effects without loops

How to find this result in field theory?
- loops generate exactly these real and imaginary parts

Loop diagrams capture physics required by general principles
But pion loops have wrong high energy behavior

- don’t know about true high energy theory

This is not a problem

- high energy = local
- can’t be confused with long distance effect
- EFT can separate the low E from high E parts
EFT in action – II – Nonrenormalizable Lagrangians

But this is not enough – chiral symmetry of QCD

\[
\begin{align*}
\psi_L & \to L\psi_L \\
\psi_R & \to R\psi_R \\
\end{align*}
\]

with L,R in SU(2) exact when

\[
\begin{align*}
m_q &= 0 \\
m_\pi &= 0
\end{align*}
\]

Implies the need for a **non-linear** Lagrangian

\[
U = e^{i\sigma \cdot \pi / F_\pi}
\]

\[
U \to LUR^\dagger
\]
The energy expansion:
- derivatives = energies

\[ \mathcal{L} = F_\pi^2 Tr(D_\mu U D^\mu U^\dagger) + L_1 [Tr(D_\mu U D^\mu U^\dagger)]^2 + L_2 Tr(D_\mu U D_\nu U^\dagger) Tr(D^\mu U D^\nu U^\dagger) + \ldots. \]

At low energy only the leading term dominates

This shares some features with gravity:
nonlinear action
nonrenormalizable theory
the energy expansion
non-trivial backgrounds (Skyrmions)
amplitudes that grow rapidly with energy
naïve unitarity violation
EFT in action – III – Renormalizing the Nonrenormalizable

Use background field method and dim. reg.

\[ U = \bar{U} e^{i\Delta/F} \]

Quantum Lagrangian:

\[ S = \int d^4x \left[ \mathcal{L}(\bar{U}) + \frac{1}{2} \Delta(D^2 + \sigma)\Delta \right] \]

Isolate one loop divergences:

\[ \Delta \mathcal{L} = \frac{1}{192\pi^2(d-4)} \left[ Tr(D_\mu U D^\mu U^\dagger) \right]^2 + 2 Tr(D_\mu U D_\nu U^\dagger) Tr(D^\mu U D^\nu U^\dagger) \]

Renormalize parameters:

\[ L_1^{\text{ren}} = L_1 + \frac{1}{192\pi^2(d-4)} \]

\[ L_2^{\text{ren}} = L_2 + \frac{2}{192\pi^2(d-4)} \]
EFT in action – IV – Predictions

Our example: Nuclear potential \( (m_\pi = 0) \)

High energy parts go into local operator:
- spurious pion loop behavior (and divergences)
- real QCD effects

\[
H_{contact} = G_S \bar{N} NN \bar{N} N .
\]

Result:

\[
V_C(r) = G_S^{ren} \delta^3(r) - \frac{9g_A^2}{4\pi F_\pi^2} \left( \frac{3g_A^2}{16M} + c_3 \right) \frac{1}{r^6}
\]

\[
\uparrow \quad \text{not predictive} \quad \downarrow \quad \text{predictive}
\]
Summary:

Full field theory treatment (loops) required by very general principles despite non-renormalizability of effective theory

**EFT pathway**

- identify low energy D.O.F and their low energy interactions
- allow general Lagrangian consistent with symmetries
- effects can be ordered by an energy expansion
- High energy parts of loops equivalent to local terms in Lagrangian and non-predictive
- Low energy parts of loops well defined and predictive
Aside: E&M also satisfies dispersive constraints

- best to use covariant gauge – includes Coulomb potential
- cut involves the Compton amplitude

\[
\rho(s, t) = -\frac{1}{32\pi} \int \frac{d\Omega_k}{4\pi} \mathcal{M}_{\lambda\mu}(p_1, -k, p_1 - q, q - k) \mathcal{M}^\lambda_{\mu
u}(p_3, -k, p_3 + q, k - q)
\]

\[
V_{EM}(r) = \frac{e^2}{4\pi r} \left[ 1 + \frac{e^2}{8\pi r} \frac{(m_A + m_B)}{m_A m_B c^2} - \frac{7e^2}{24\pi^2} \frac{\hbar}{m_A m_B c^3 r^2} \right]
\]

Here, resultant field theory is renormalizable: QED
- but basic logic is the same

Field theory approach confirmed.
Quantum theory of General Relativity has the form of an effective field theory

Method for extracting the low energy quantum predictions
- uses low energy degrees of freedom and couplings
- independent of high energy completion of theory

Basic distinction:
- high energy effects (including divergences) are local
  - look like a term in a general Lagrangian
- low energy quantum effects are non-local
  - distinct from the general Lagrangian
EFT pathway:

- identify low energy D.O.F and their low energy interactions

= gravitational waves and General relativity

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The general Action

Need the most general Action consistent with general covariance.

Key: \( R \) depends on two derivatives of the metric

Order by the derivative expansion

\[
\begin{align*}
R_{\mu\nu} &= \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\mu\nu} \Gamma^\lambda_{\lambda\sigma} \\
\Gamma^\lambda_{\alpha\beta} &= \frac{g^{\lambda\sigma}}{2} (\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta})
\end{align*}
\]

Result:

\[
S_{\text{grav}} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}
\]
Parameters

\[ S_{\text{grav}} = \int d^4 x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\} \]

1) \( \Lambda = \) cosmological constant
\[ \Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4 \]
   - this is observable only on cosmological scales
   - neglect for rest of talk
   - interesting aspects

2) Newton’s constant
\[ \kappa^2 = 32\pi G \]

3) Curvature –squared terms \( c_1, c_2 \)
   - studied by Stelle
   - modify gravity at very small scales
   - essentially unconstrained by experiment
   \( c_1, c_2 \leq 10^{74} \)

\( M_P = 1.22 \times 10^{19} \text{ GeV} \)
Quantization of gravity is now well known:

- Covariant quantization: Feynman deWitt
  - gauge fixing, ghosts
- Background field method: ‘t Hooft Veltman
  - retains symmetries of GR

Background field:

\[ g_{\mu\nu} = \tilde{g}_{\mu\nu} + \kappa h_{\mu\nu} \]
\[ g^{\mu\nu} = \tilde{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_\lambda h^{\lambda\nu} + \ldots \]

Expand around this background:

\[ S_{grav} = \int d^4x \sqrt{-g} \left[ \frac{2\bar{R}}{\kappa^2} + \mathcal{L}^{(1)}_g + \mathcal{L}^{(2)}_g + \ldots \right] \]

\[ \mathcal{L}^{(1)}_g = \frac{\lambda}{\kappa} \left[ \tilde{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \]
\[ \mathcal{L}^{(2)}_g = \frac{1}{2} h_{\mu\nu\alpha} h^{\mu\nu\alpha} - \frac{1}{2} h_{\alpha} h^{\alpha\beta} \]
\[ + \bar{R} \left( \frac{1}{4} \kappa^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + (2h_{\mu} h_{\nu} - hh_{\mu\nu}) \bar{R}^{\mu\nu} \]

Linear term vanishes by Einstein Eq.
Gauge fixing:
- harmonic gauge

\[ \mathcal{L}_{gf} = \sqrt{-\bar{g}} \left\{ \left( h_{\mu;\nu} - \frac{1}{2} h_{;\mu} \right) \left( h^{\mu;\lambda} - h^{\mu} \right) \right\} \]

\[ h \equiv h^\lambda_\lambda \]

Ghost fields:

\[ \mathcal{L}_{\text{ghost}} = \sqrt{-\bar{g}} \eta^* \left\{ \eta_{\mu;\lambda} - \bar{R}_{\mu\nu} \eta^\nu \right\} \]
Feynman rules:

A.1 Scalar propagator
The massive scalar propagator is

\[ \frac{i}{q^2 - m^2 + i\epsilon} \]

A.2 Graviton propagator
The graviton propagator in harmonic gauge can be written in the form

\[ \frac{i}{q^2 + i\epsilon} \]

where

\[ q^2 = \frac{1}{2} \left[ q^\mu q_\mu + q^\rho q_\rho - q^\sigma q_\sigma \right] \]

A.3 2-scalar-1-graviton vertex
The 2-scalar-1-graviton vertex is discussed in the literature. We write it as

\[ \tau^{\mu
u}(p, p', m) \]

where

\[ \tau^{\mu
u}(p, p', m) = -\frac{i\kappa}{2} \left( p^\mu p'^\nu + p'^\mu p^\nu - q^\sigma(p \cdot p' - m^2) \right) \]

A.4 2-scalar-2-graviton vertex
The 2-scalar-2-graviton vertex is also discussed in the literature. We write it here with the full symmetry of the two gravitons:

\[ \tau^{\mu\nu\lambda\rho}(p, p', m) = \]
**EFT pathway:**

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Performing quantum calculations

Next step: Renormalization
- divergences arise at high energies
- not of the form of the basic lagraingian

Solution: Effective field theory and renormalization
- renormalize divergences into parameters of the most general lagrangian ($c_1, c_2 \ldots$)

Power counting theorem: (pure gravity, $\Lambda = 0$)
- each graviton loop = 2 more powers in energy expansion
- 1 loop = Order $(\partial g)^4$
- 2 loop = Order $(\partial g)^6$
Renormalization

One loop calculation:    \( \text{‘t} \text{ Hooft and Veltman} \)

\[
Z[\phi, J] = Tr \ln D
\]

Divergences are local:

\[
\Delta \mathcal{L}_0^{(1)} = \frac{1}{8\pi^2 \epsilon} \left\{ \frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right\} \quad \epsilon = 4 - d
\]

**Renormalize** parameters in general action:

\[
c_1^{(r)} = c_1 + \frac{1}{960\pi^2 \epsilon}
\]

\[
c_2^{(r)} = c_2 + \frac{7}{160\pi^2 \epsilon}
\]

**Note:** Two loop calculation known in pure gravity    \( \text{Goroff and Sagnotti} \)

\[
\Delta \mathcal{L}^{(2)} = \frac{209 \kappa}{2880(16\pi^2)^2} \frac{1}{\epsilon} \sqrt{-g} R_{\alpha\beta}^{\gamma\delta} R_{\rho\sigma}^{\gamma\delta} R_{\rho\sigma}^{\alpha\beta}
\]

Order of six derivatives

**dim. reg. preserves symmetry**

**Pure gravity**

“one loop finite”

since \( R_{\mu\nu} = 0 \)
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What are the quantum predictions?

Not the divergences
- they come from the Planck scale
- unreliable part of theory

Not the parameters
- local terms in L
- we would have to measure them

Low energy propagation
- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can’t be Taylor expanded – can’t be part of a local Lagrangian
- long distance in coordinate space
Corrections to Newtonian Potential

Example of isolating low energy quantum corrections
- long distance corrections to potential
- divergence free, parameter free
- high energy effects only influence very short distance potential

Scattering potential of two heavy masses.

\[ \langle f|T|i \rangle \equiv (2\pi)^4 \delta^{(4)}(p - p')(\mathcal{M}(q)) \]
\[ = -(2\pi) \delta(E - E') \langle f|\hat{V}(q)|i \rangle \]

Potential found using from

\[ V(x) = \frac{1}{2m_1} \frac{1}{2m_2} \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q} \cdot \mathbf{x}} \mathcal{M}(\mathbf{q}) \]

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura
Lowest order:
one graviton exchange

\[ iM_{1(a)}(\vec{q}) = \tau_1^{\mu\nu}(k_1, k_2, m_1) \left[ \frac{iP_{\mu\nu}\alpha\beta}{q^2} \right] \tau_1^{\alpha\beta}(k_3, k_4, m_2) \]

Non-relativistic reduction:

\[ M_{1(a)}(\vec{q}) = -\frac{4\pi G m_1 m_2}{\vec{q}^2} \]

Potential:

\[ V_{1(a)}(r) = -\frac{G m_1 m_2}{r} \]
What to expect:

General expansion:

\[ V(r) = -\frac{GMm}{r} \left[ 1 + \frac{G(M + m)}{rc^2} + \frac{G\hbar}{r^2c^3} \right] + cG^2Mm\delta^3(r) \]

Classical expansion parameter

Quantum expansion parameter

Relation to momentum space:

\[ \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r} \]

\[ \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2r^2} \]

\[ \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \ln(|\mathbf{q}|^2) = -\frac{1}{2\pi^3} \]

Momentum space amplitudes:

\[ V(q^2) = \frac{G^2Mm}{q^2} \left[ 1 + a'G(M + m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2 \right] \]

Classical quantum short range

Non-analytic analytic short range
**Parameter free and divergence free**

Recall: divergences like local Lagrangian $\sim R^2$

Also unknown parameters in local Lagrangian $\sim c_1, c_2$

But this generates only “short distance term”

Note: $R^2$ has 4 derivatives $R^2 \sim q^4$

Then:
Treating $R^2$ as perturbation

$$V_{R^2} \sim \frac{G^2 M m}{q^2} \frac{1}{q^2} \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 M m \delta^3(x)$$

Local lagrangian gives only short range terms
The calculation:

Lowest order:

Vertex corrections:

Vacuum polarization: (Duff 1974)

Box and crossed box

Others:
Results:

Pull out non-analytic terms:
- for example the vertex corrections:

\[
M_{5(a)+5(b)}(q) = 2G^2m_1m_2\left(\frac{\pi^2(m_1 + m_2)}{|q|} + \frac{5}{3} \log q^2\right)
\]

\[
M_{5(c)+5(d)}(q) = -\frac{52}{3} G^2m_1m_2 \log q^2
\]

Sum diagrams:

\[
V(r) = -\frac{Gm_1m_2}{r} \left[ 1 + 3 \frac{G(m_1 + m_2)}{r} + \frac{41}{10\pi} \frac{G'h}{r^2} \right]
\]

Classical correction
(Iwasaki; Gupta + Radford)  
Quantum correction
Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Duff; JFD; Muzinich and Vokos; Hamber and Liu;
   Akhundov, Bellucci, and Sheikh )
   - other potentials or mistakes

5) Why not done 30 years ago?
   - power of effective field theory reasoning
Aside: Classical Physics from Quantum Loops:

Field theory folk lore:
Loop expansion is an expansion in “Proofs” in field theory books $\hbar$

This is not really true.
- numerous counter examples – such as the gravitational potential

- can remove a power of $\hbar$ via kinematic dependence

\[ \sqrt{\frac{m^2}{-q^2}} = \frac{m}{\hbar \sqrt{-k^2}} \]

- classical behavior seen when massless particles are involved
2 new results on quantum potential:

Universal quantum corrections:

- Calculations for a variety of spins
  - different diagrams, different factors

  Yet, both classical and quantum pieces are universal

Dispersive confirmation of the potential:

- demonstrates generality
- may be useful for understanding universality
Dispersive treatment of quantum potential

Interactions from sewing together tree amplitudes
- only involves on-shell gravitons
- classical and quantum corrections emerge from low $q^2$ limit

Tree amplitudes easier to calculate than loops

Universality of low energy limit:
- leading corrections identical for different spins
Basic dispersive framework:

diagrams satisfy analyticity requirements
leads to a dispersive representation

\[ V_2(s, q^2) = \frac{1}{\pi} \int_0^\infty \frac{dt}{t - q^2} \rho(s, t) + (\text{L.H. cut} = \text{short range}) \]

Spectral functions calculated via Cutkosky rules
- on shell intermediate states

At low energy:

\[ \rho(s, t) = a_2(s) \frac{1}{\sqrt{t}} + a_3(s) + \ldots. \]

\[ V_2(s, q^2) = a_2(s) \frac{1}{\sqrt{-q^2}} - \frac{a_3(s)}{\pi} \ln(-q^2) + \ldots. \]

\[ V_2(r) = \frac{1}{8\pi^2 m_A m_B} \left[ \frac{a_2}{r^2} + \frac{a_3}{r^3} \right] \]

Note: leading non-analytic structures – independent of possible subtractions
Gravitational potential via dispersive techniques:
- cut involves gravitational Compton amplitude

\[ \rho_g(s, t) = \frac{-1}{32\pi} \int \frac{d\Omega_k}{4\pi} \mathcal{M}_A^{\mu\nu,\lambda\sigma}(p_1, -k, p_1 - q, q - k) \mathcal{M}_B^{\alpha\beta,\gamma\delta}(p_3, -k, p_3 + q, k - q) P_{\mu\nu,\alpha\beta} P_{\lambda\sigma,\gamma\delta} \]

Amplitudes are more complicated, but procedure is the same:

Reproduce usual result – diagram by diagram

\[ V(r) = -\frac{GMm}{r} \left[ 1 + 3\frac{G(M + m)}{rc^2} + \frac{41}{10\pi} \frac{G\hbar}{r^2 c^3} \right] \]

VERY strong check on loop calculation
Universality of the quantum corrections:

Can we prove universality?:
   Electromagnetic amplitude has multipole expansion
      - at low energy, E1 transitions dominate – fixed tensor structure
      - E1 transition has fixed $q^2 \rightarrow 0$ limit, normalized to charge

Reasons to expect that gravitational interaction is similar
   - universal form - factorization
   - low energy is square of E1 amplitude – fixed form

⇒ all gravity spectral functions have the same low energy structure
⇒ same classical and quantum corrections
**Example 2: Graviton –graviton scattering**

Fundamental quantum gravity process

**Lowest order amplitude:**

\[ A^{\text{tree}}(++;++) = \frac{i \kappa^2 s^3}{4tu} \]

**One loop:**

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

\[
A^{1-\text{loop}}(++;--) = -i \frac{\kappa^4}{30720\pi^2} \left( s^2 + t^2 + u^2 \right) \\
A^{1-\text{loop}}(++;+-) = -i \frac{\kappa^2}{3} A^{1-\text{loop}}(++;--) \\
A^{1-\text{loop}}(++;++) = \frac{\kappa^2}{4(4\pi)^2} \left( \Gamma(1 - \epsilon) \Gamma(1 + \epsilon) \right) \frac{A^{\text{tree}}(++;++) \times (stu)}{\Gamma(1 - 2\epsilon)} \\
\times \left[ \frac{2}{\epsilon} \left( \frac{\ln(-u)}{st} + \frac{\ln(-t)}{su} + \frac{\ln(-s)}{tu} \right) + \frac{1}{s^2} f \left( \frac{-t}{s}, \frac{-u}{s} \right) \right] \\
+ 2 \left( \frac{\ln(-u) \ln(-s)}{su} + \frac{\ln(-t) \ln(-s)}{tu} + \frac{\ln(-t) \ln(-s)}{ts} \right)
\]

where

\[
f \left( \frac{-t}{s}, \frac{-u}{s} \right) = \frac{(t + 2u)(2t + u)(2t^4 + 2t^3u - t^2u^2 + 2tu^3 + 2u^4)}{s^6} \left( \ln^2 \frac{t}{u} + \pi^2 \right) \\
+ \frac{(t - u)(341t^4 + 1609t^3u + 2566t^2u^2 + 1609tu^3 + 341u^4)}{30s^5} \ln \frac{t}{u} \\
+ \frac{1922t^4 + 9143t^3u + 14622t^2u^2 + 9143tu^3 + 1922u^4}{180s^4},
\]
**Infrared safe:**

The $1/\varepsilon$ is from infrared
-soft graviton radiation
-made finite in usual way
$1/\varepsilon \to \ln(1/\text{resolution})$ (gives scale to loops)
cross section finite

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{tr}} + \left( \frac{d\sigma}{d\Omega} \right)_{\text{rad}} + \left( \frac{d\sigma}{d\Omega} \right)_{\text{non-rad.}} =
\]
\[
= \frac{\kappa^4 s^5}{2048 \pi^2 t^2 u^2} \left\{ 1 + \frac{\kappa^2 s}{16 \pi^2} \left[ \ln \frac{-t}{s} \ln \frac{-u}{s} + \frac{tu}{2s^2} f \left( \frac{-t}{s}, \frac{-u}{s} \right) \right] \right.
\]
\[
- \left( \frac{t}{s} \ln \frac{-t}{s} + \frac{u}{s} \ln \frac{-u}{s} \right) \left( 3 \ln(2\pi^2) + \frac{s}{\Lambda^2} + \frac{\sum_{ij} \eta_i \eta_j F^{(1)}(\gamma_{ij})}{\sum_{ij} \eta_i \eta_j F^{(0)}(\gamma_{ij})} \right) \right\}.
\]

* Beautiful result:
-low energy theorem of quantum gravity
Hawking Radiation

Exploratory calculation

- remove high energy contributions
- Pauli Villars regulators

-flux from local limit of Green’s function

$$\mathcal{F} \equiv -\langle T_{t^*} \rangle = -\langle T_{t^*} \rangle$$

$$= -\frac{1}{2} \lim_{x' \to x} \left( \frac{\partial}{\partial t'} \frac{\partial}{\partial r^*} + \frac{\partial}{\partial r^{*'}} \frac{\partial}{\partial t} \right) G(x, x').$$

-dependence on regulator vanishes exponentially

-radiation appears to be property of the low energy theory
Limitations of the effective field theory

Corrections grow like \[ Amp \sim A_0\left[1 + Gq^2 + Gq^2 \ln q^2\right] \]

Overwhelm lowest order at \( q^2 \sim M_p^2 \)

Also sicknesses of \( R+R^2 \) theories beyond \( M_p \)

(J. Simon)

Effective theory predicts its own breakdown at \( M_p \)
- could in principle be earlier

Needs to be replaced by more complete theory at that scale
The extreme IR limit

Singularity theorems:
- most space times have singularities
- EFT breaks down near singularity

Can we take extreme IR limit?
- wavelength greater than distance to nearest singularity?
- past black holes?

Possible treat singular region as source
- boundary conditions needed

deSitter horizon in IR
Matching to LQG/ spin foams etc

EFT should be low curvature limit of a more fundamental theory

Find low curvature backgrounds (4D)
Identify spin 2 gravitons (without extra massless DOF)
Verify leading coupling to stress energy

There exists Deser theorem => GR

Verify causality, unitarity, crossing
Why gravity may be the best case to study

- EFT reasoning

**Applequist Carrazzone theorem:**

*Effects from high energy either appear as shifts in coupling constants (eg. $c_1, c_2$) or are suppressed by powers of the heavy scale (eg. $M_{Pl}$).*

**But all gravity interactions are suppressed by $M_{Pl}^2$**

- relative effect could then be much bigger ~$O(1)$?
- small violations of unitarity, causality….
- approximate general covariance?

**Analogy:** $\pi^0 \rightarrow \gamma\gamma$ reveals # of colors of QCD
Summary

We have a quantum theory of general relativity
- quantization and renormalization
- perturbative expansion

It is an effective field theory
- valid well below the Planck scale
- corrections are very well behaved

Effective field theory techniques allow predictions
- finite, parameter free
- due to low energy (massless) propagation

Need full theory at or before Planck scale
- many interesting questions need full theory
- not conflict between QM and GR, but lack of knowledge about fundamental high energy theory