Effective Field Theory of General Relativity

1) Intro: Watch your language!
What can EFT do for you?

2) **Effective field theory in action**
  Potential between two nucleons
  Predictive use of non-renormalizeable theories

3) **Application to General Relativity**
  Renormalization of GR
  Quantum predictions

4) **Limitations of the effective field theory**

5) **Matching to LQG etc**

John Donoghue
Morelia, 2007

What is the problem with quantum gravity?

"*Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct.*"

"*The existence of gravity clashes with our description of the rest of physics by quantum fields*"

"*Attempting to combine general relativity and quantum mechanics leads to a meaningless quantum field theory with unmanageable divergences.*"

"The application of conventional field quantization to GR fails because it yields a nonrenormalizable theory"

"*Quantum mechanics and general relativity are incompatible*"

<u>However these are inaccurate</u>

- reflect an outdated view of field theory
- in particular, neglects understanding of **effective field theory**

<u>Physics is an experimental science</u>:

- constructed through discovery at accessible energies/distances
- description changes as we move to new domains
- new DOF and new interactions uncovered

- our present theories can only claim to hold at accessible scales

Crucial rephrasing:

Are QM and GR compatible over accessible scales, where we expect both to be correct?

Answer is **YES**

- this is what effective field theory can do for you

Effective field theory handles separation into known DOF at ordinary scales and unknown physics at extreme scales

- known = GR and matter
- the known part of the theory is very well behaved

"There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data."

F. Wilczek

Lessons of effective field theory for GR:

At ordinary energies/scales:

The quantum theory of general relativity exists and
is of the form of an effective field theory

The effective field theory treatment completes the covariant approach started by Feynman and DeWitt.

The non-renomalizability of QGR is not a problem
-GR can be renormalized perturbatively.

Quantum general relativity is an excellent perturbative theory
- we can make reliable predictions

There IS a quantum theory of gravity at ordinary energies

However, there there are still problems in gravity theory

- issue is not QM vs GR per se
- problems are at "extremes"

EFT helps reformulate the issues:

"*Quantum general relativity points to the limits of its perturbative validity*"

"*Quantum general relativity is most likely not the final theory*"

"*The extremes of quantum GR have raise deep and important problems.* "

But, this IS progress!

- we have a solid foundation for the start of a more extensive theory

Effective Field Theory in action: I – The need

Example: Nucleon scattering

(here J=0, I=0 part only, and m_π=0 for clarity)

General principles

- **causality**
- **unitarity**
- **crossing**

imply a dispersive representation of scattering amplitude

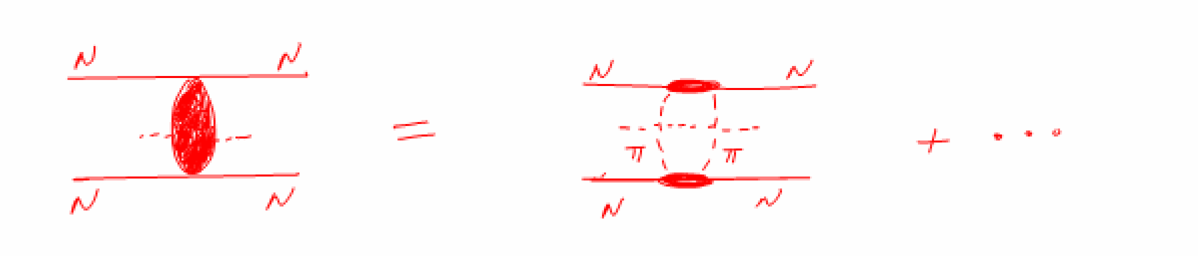
$$V(s,q^2) = \frac{1}{\pi}\int_0^\infty \frac{dt}{t-q^2}\rho(s,t) \ + \ (\mathrm{L.H.cut} = \mathrm{short\ range})$$

ρ is Imag part of amplitude

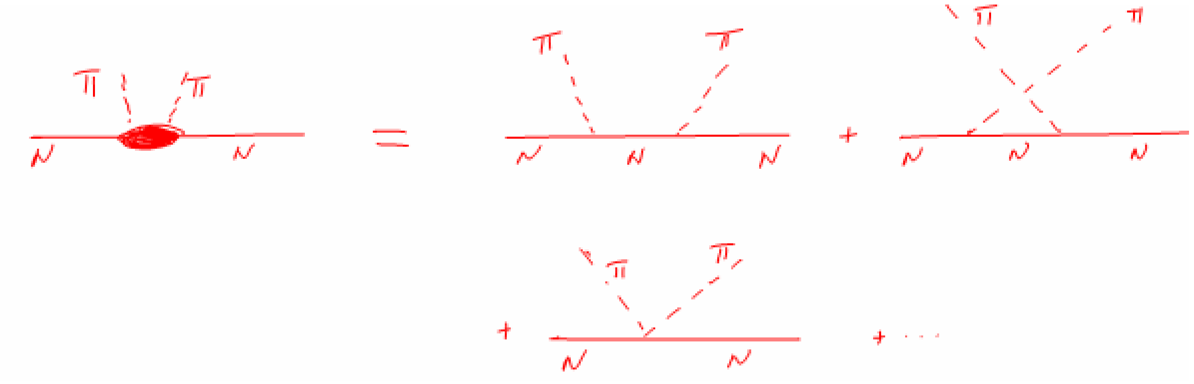
Nuclear potential is F.T. of $V(s,q^2)$

What goes into the spectral function?

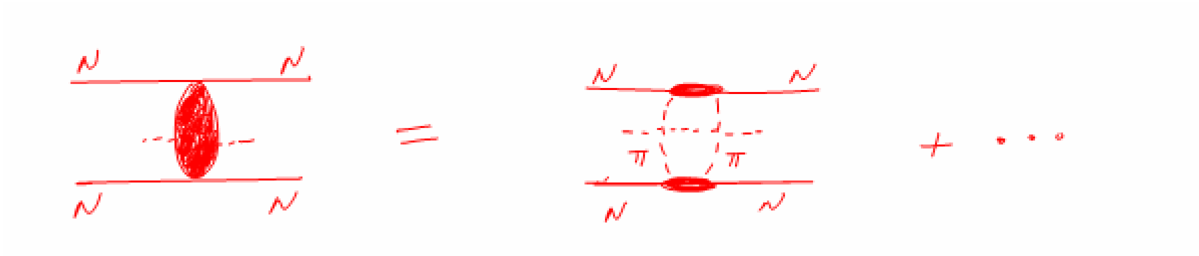
- all physical **on-shell** intermediate states
  - lightest DOF are pions – need 2 pions
  - no other state exists below ~ 0.6 – 0.9 GeV



Build interaction - product of on-shell tree level amplitudes



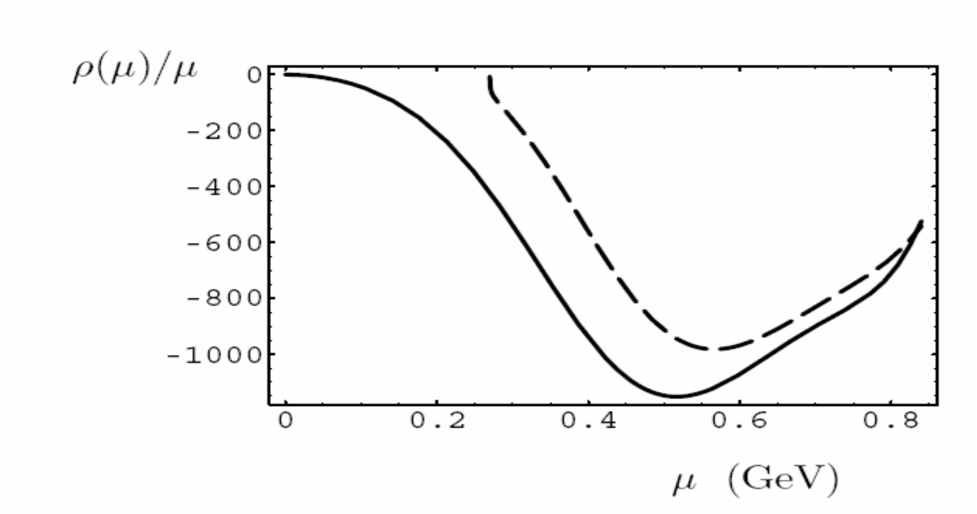
$$\rho(s,t) = \frac{-1}{32\pi}\int \frac{d\Omega_k}{4\pi}\,\mathcal{M}_1(p_1,-k,p_1-q,q-k)\mathcal{M}_2(p_3,-k,p_3+q,k-q)$$



$$\rho(s,t) = c_3(s)t^{3/2} + ...$$

Plugging in known and measured interactions:

The energy expansion in pictures:



Result: Longest distance piece

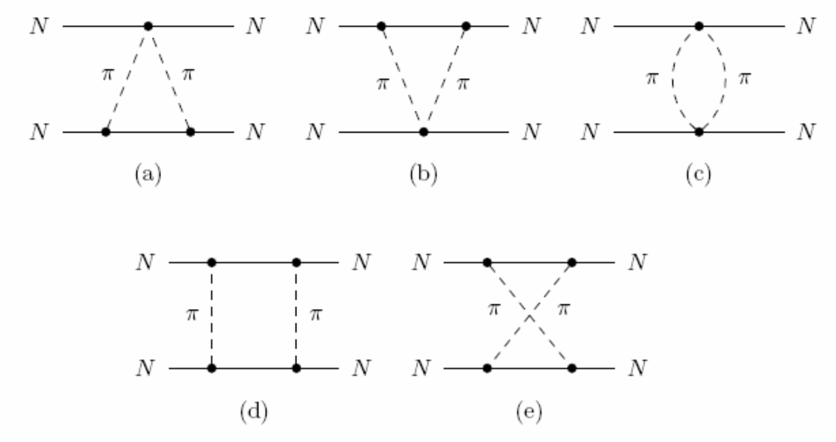
$$V_C(r) = -\frac{9g_A^2}{4\pi F_\pi^2}(\frac{3g_A^2}{16M} + c_3)\frac{1}{r^6} + ...$$

Reconstruct full low energy amplitude:

- on shell imaginary part generates real parts through dispersion relatio
  - Quantum effects without loops

How to find this result in field theory?

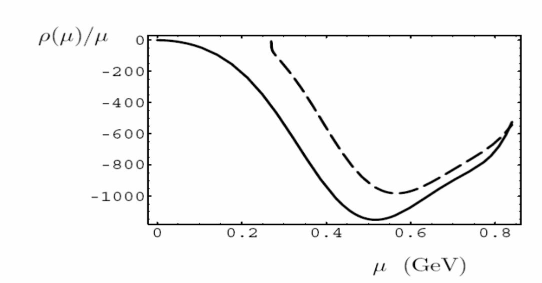
- loops generate exactly these real and imaginary parts



Loop diagrams capture physics required by general principles

But pion loops have wrong high energy behavior

- don’t know about true high energy theory



This is not a problem

-high energy = local
-can’t be confused with long distance effect
-EFT can separate the low E from high E parts

EFT in action – II – Nonrenormalizable Lagrangians

But this is not enough – chiral symmetry of QCD

$$\psi_L \to L\psi_L$$

$$\psi_R \to R\psi_R$$

with L,R in SU(2)

exact when
m_q=0
m_π=0

Implies the need for a **non-linear** Lagrangian

$$U = e^{i\sigma\cdot\pi/F_\pi}$$

$$U \to LUR^\dagger$$

The energy expansion:

- derivatives = energies

$$\mathcal{L} = F_\pi^2 Tr(D_\mu U D^\mu U^\dagger) + L_1 [Tr(D_\mu U D^\mu U^\dagger)]^2$$

$$+ L_2 Tr(D_\mu U D_\nu U^\dagger)\, Tr(D^\mu U D^\nu U^\dagger) + ...$$

At low energy only the leading term dominates

This shares some features with gravity:
nonlinear action
nonrenormalizeable theory
the energy expansion
non-trivial backgrounds (Skyrmions)
amplitudes that grow rapidly with energy
naïve unitarity violation

EFT in action – III – Renormalizing the Nonrenormalizable

Use background field method and dim. reg.

$$U = \bar{U} e^{i\Delta/F}$$

quantum field

background

Quantum Lagrangian:

$$S = \int d^4x \left[\mathcal{L}(\bar{U}) + \frac{1}{2}\Delta(D^2 + \sigma)\Delta \right]$$

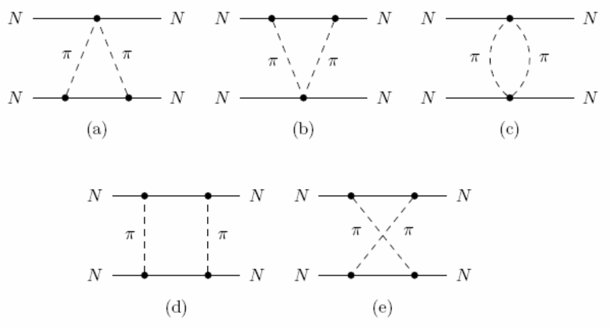
Isolate one loop divergences:

$$\Delta\mathcal{L} = \frac{1}{192\pi^2(d-4)} \left[[Tr(D_\mu U D^\mu U^\dagger)]^2 + 2Tr(D_\mu U D_\nu U^\dagger)\ Tr(D^\mu U D^\nu U^\dagger) \right]$$

Renormalize parameters:

$$L_1^{ren} = L_1 + \frac{1}{192\pi^2(d-4)} \qquad L_2^{ren} = L_2 + \frac{2}{192\pi^2(d-4)}$$

EFT in action – IV – Predictions



Our example: Nuclear potential (m_π=0)

High energy parts go into local operator

- spurious pion loop behavior (and divergences)
- real QCD effects

$$H_{contact} = G_S \bar{N} N \bar{N} N \, .$$

Result:

$$V_C(r) = G_S^{ren} \delta^3(r) - \frac{9g_A^2}{4\pi F_\pi^2}(\frac{3g_A^2}{16M} + c_3)\frac{1}{r^6}$$

not predictive

predictive

Summary:

Full field theory treatment (loops) required by very general principles despite non-renormalizability of effective theory

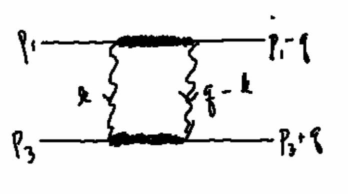
EFT pathway

- identify low energy D.O.F and their low energy interactions
- allow general Lagrangian consistent with symmetries
- effects can be ordered by an energy expansion
- High energy parts of loops equivalent to local terms in Lagrangian and non-predictive
- Low energy parts of loops well defined and predictive

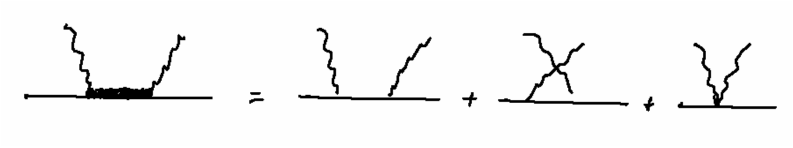
Aside: E&M also satisfies dispersive constraints

Feinberg
Sucher

- best to use **covariant gauge** – includes Coulomb potential
- cut involves the Compton amplitude



with



$$\rho(s,t)=\frac{-1}{32\pi}\int\frac{d\Omega_k}{4\pi}\,\mathcal{M}_{A\mu\nu}(p_1,-k,p_1-q,q-k)\mathcal{M}_B^{\mu\nu}(p_3,-k,p_3+q,k-q)$$

$$V_{EM}(r)=\frac{e^2}{4\pi r}\left[1+\frac{e^2}{8\pi r}\frac{(m_A+m_B)}{m_Am_Bc^2}-\frac{7e^2}{24\pi^2}\frac{\hbar}{m_Am_Bc^3r^2}\right]$$

Here, **resultant field theory is renormalizable**: QED

- but basic logic is the same

Field theory approach confirmed.

Holstein and Ross

Quantum theory of General Relativity has the form of an effective field theory

Weinberg
Donoghue

Method for extracting the low energy quantum predictions
- uses low energy degrees of freedom and couplings
- independent of high energy completion of theory

Basic distinction:
- high energy effects (including divergences) are local
  - look like a term in a general Lagrangian
- low energy quantum effects are non-local
  - distinct from the general Lagrangian

EFT pathway:

-identify low energy D.O.F and their low energy interactions

= **gravitational waves and General relativity**

- allow general Lagrangian consistent with symmetries

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EFT pathway:

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The general Action

Need the most general Action consistent with general covariance.

Key: R depends on two derivatives of the metric
Order by the derivative expansion

$$\begin{aligned} R_{\mu\nu} &= \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\mu\nu}\Gamma^\lambda_{\lambda\sigma} \\ \Gamma^\lambda_{\alpha\beta} &= \frac{g^{\lambda\sigma}}{2}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right) \end{aligned}$$

Result:

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

Parameters

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

1) Λ = cosmological constant

$$\Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4$$

$$M_P = 1.22 \times 10^{19} \text{ GeV}$$

-this is observable only on cosmological scales
-neglect for rest of talk
-interesting aspects

2) Newton's constant

$$\kappa^2 = 32\pi G$$

3) Curvature –squared terms c_1, c_2

- studied by Stelle
- modify gravity at very small scales
-essentially unconstrained by experiment

$$c_1, c_2 \leq 10^{74}$$

Quantization

Quantization of gravity is now well known:

-Covariant quantization Feynman deWitt
 -gauge fixing, ghosts
- -Background field method ‘t Hooft Veltman
 -retains symmetries of GR

Background field:

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \\ g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \ldots \end{aligned}$$

Expand around this background:

$$S_{grav} = \int d^4x \sqrt{-\bar{g}} \left[\frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \ldots \right]$$

$$\begin{aligned} \mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa} \left[\bar{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \\ \mathcal{L}_g^{(2)} &= \frac{1}{2} h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} - \frac{1}{2} h_{;\alpha} h^{;\alpha} + h_{;\alpha} h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha} h^{\mu\alpha;\beta} \\ &\quad + \bar{R} \left(\frac{1}{4} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + \left(2h^{\lambda}_{\mu} h_{\nu\lambda} - h h_{\mu\nu} \right) \bar{R}^{\mu\nu} \end{aligned}$$

Linear term vanishes by Einstein Eq.

Gauge fixing:

-harmonic gauge

$$\mathcal{L}_{gf} = \sqrt{-\bar{g}}\left\{\left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2}h_{;\mu}\right)\left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu}\right)\right\}$$

$$h \equiv h^{\lambda}_{\lambda}$$

Ghost fields:

$$\mathcal{L}_{ghost} = \sqrt{-\bar{g}}\eta^{*\mu}\left\{\eta_{\mu;\lambda}{}^{;\lambda} - \bar{R}_{\mu\nu}\eta^{\nu}\right\}$$

Feynman rules:

A.1 Scalar propagator

The massive scalar propagator is:

$$= \frac{i}{q^2 - m^2 + i\epsilon}$$

A.2 Graviton propagator

The graviton propagator in harmonic gauge can be written in the form:

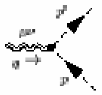
$$= \frac{i\mathcal{P}^{\alpha\beta\gamma\delta}}{q^2 + i\epsilon}$$

where

$$\mathcal{P}^{\alpha\beta\gamma\delta} = \frac{1}{2}\left[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\beta\gamma}\eta^{\alpha\delta} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right]$$

A.3 2-scalar-1-graviton vertex

The 2-scalar-1-graviton vertex is discussed in the literature. We write it as:



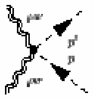
$$= \tau_1^{\mu\nu}(p, p', m)$$

where

$$\tau_1^{\mu\nu}(p, p', m) = -\frac{i\kappa}{2}\left[p^\mu p'^\nu + p^\nu p'^\mu - \eta^{\mu\nu}\left((p \cdot p') - m^2\right)\right]$$

A.4 2-scalar-2-graviton vertex

The 2-scalar-2-graviton vertex is also discussed in the literature. We write it here with the full symmetry of the two gravitons:



$$= \tau_2^{\eta\lambda\rho\sigma}(p, p', m)$$

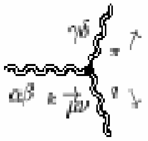
$$\tau_2^{\eta\lambda\rho\sigma}(p, p') = i\kappa^2\Bigg[\left\{I^{\eta\lambda\alpha\delta}I^{\rho\sigma\beta}{}_{\delta} - \frac{1}{4}\left\{\eta^{\eta\lambda}I^{\rho\sigma\alpha\beta} + \eta^{\rho\sigma}I^{\eta\lambda\alpha\beta}\right\}\right\}(p_\alpha p'_\beta + p'_\alpha p_\beta)$$
$$-\frac{1}{2}\left\{I^{\eta\lambda\rho\sigma} - \frac{1}{2}\eta^{\eta\lambda}\eta^{\rho\sigma}\right\}\left[(p \cdot p') - m^2\right]\Bigg] \qquad (61)$$

with

$$I_{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}).$$

A.5 3-graviton vertex

The 3-graviton vertex can be derived via the background field method and has the form[9],[10]



$$= \tau_3{}^{\mu\nu}_{\alpha\beta\gamma\delta}(k, q)$$

where

$$\tau_3{}^{\mu\nu}_{\alpha\beta\gamma\delta}(k,q) = -\frac{i\kappa}{2} \times \Bigg(\mathcal{P}_{\alpha\beta\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\right]$$
$$+ 2q_\lambda q_\sigma\left[I_{\alpha\beta}{}^{\sigma\lambda}I_{\gamma\delta}{}^{\mu\nu} + I_{\gamma\delta}{}^{\sigma\lambda}I_{\alpha\beta}{}^{\mu\nu} - I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta}{}^{\nu\lambda} - I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta}{}^{\nu\lambda}\right]$$
$$+ \Bigg[q_\lambda q^\mu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\nu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\nu\lambda}\right) + q_\lambda q^\nu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\lambda}\right)$$
$$- q^2\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\nu} - \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\nu}\right) - \eta^{\mu\nu}q_\sigma q_\lambda\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\sigma\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\sigma\lambda}\right)\Bigg]$$
$$+ \Big[2q_\lambda\big(I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\nu}(k-q)^\mu + I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\mu}(k-q)^\nu - I_{\gamma\delta}{}^{\lambda\sigma}I_{\alpha\beta\sigma}{}^{\nu}k^\mu - I_{\gamma\delta}{}^{\lambda\sigma}$$
$$+ q^2\left(I_{\alpha\beta\sigma}{}^{\mu}I_{\gamma\delta}{}^{\nu\sigma} + I_{\alpha\beta}{}^{\nu\sigma}I_{\gamma\delta\sigma}{}^{\mu}\right) + \eta^{\mu\nu}q_\sigma q_\lambda\left(I_{\alpha\beta}{}^{\lambda\rho}I_{\gamma\delta\rho}{}^{\sigma} + I_{\gamma\delta}{}^{\lambda\rho}I_{\alpha\beta\rho}{}^{\sigma}\right)\Big]$$
$$+ \Big\{(k^2 + (k-q)^2)\big[I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta\sigma}{}^{\nu} + I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta\sigma}{}^{\nu} - \frac{1}{2}\eta^{\mu\nu}\mathcal{P}_{\alpha\beta\gamma\delta}\big]$$
$$- \left(I_{\gamma\delta}{}^{\mu\nu}\eta_{\alpha\beta}k^2 + I_{\alpha\beta}{}^{\mu\nu}\eta_{\gamma\delta}(k-q)^2\right)\Big\}\Bigg) \qquad (62)$$

EFT pathway:

- identify low energy D.O.F and their low energy interactions

- allow general Lagrangian consistent with symmetries

- effects can be ordered by an energy expansion

- **High energy parts of loops equivalent to local terms in Lagrangian and non-predictive**

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Performing quantum calculations

Next step: Renormalization

-divergences arise at high energies

- not of the form of the basic lagraingian

Solution: Effective field theory and renormalization

- renormalize divergences into parameters of the most general lagrangian (c_1,c_2…)

Power counting theorem: (pure gravity, Λ=0)

-each graviton loop=2 more powers in energy expansion

-1 loop = Order $(\partial g)^4$

-2 loop = Order $(\partial g)^6$

Renormalization

One loop calculation: ‘t Hooft and Veltman

$$Z[\phi, J] = Tr \ln D$$

Divergences are local:

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \qquad \epsilon = 4 - d$$

dim. reg.
preserves
symmetry

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$

$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity
“one loop finite”
since $R_{\mu\nu} = 0$

Note: Two loop calculation known in pure gravity Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

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What are the quantum predictions?

Not the divergences

-they come from the Planck scale

-unreliable part of theory

Not the parameters

-local terms in L

-we would have to measure them

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Low energy propagation

-not the same as terms in the Lagrangian

- most always non-analytic dependence in momentum space

-can’t be Taylor expanded – can’t be part of a local Lagrangian

-long distance in coordinate space

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein,
Bjerrum-Bohr 2002
Kriplovich Kirilin

Example of isolating low energy quantum corrections

- long distance corrections to potential
- divergence free, parameter free
- high energy effects only influence very short distance potential

Scattering potential of two heavy masses.

$$\begin{aligned}\langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q))\\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle\end{aligned}$$

Potential found using from

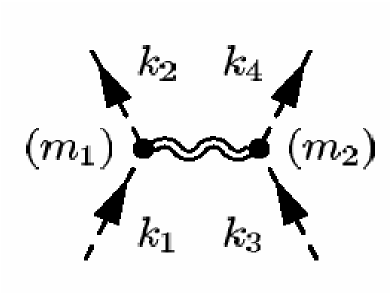
$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3}e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

Lowest order:

one graviton exchange



$$iM_{1(a)}(\vec{q}) = \tau_1^{\mu\nu}(k_1,k_2,m_1)\left[\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{q^2}\right]\tau_1^{\alpha\beta}(k_3,k_4,m_2)$$

Non-relativistic reduction:

$$M_{1(a)}(\vec{q}) = -\frac{4\pi G m_1 m_2}{\vec{q}^2}$$

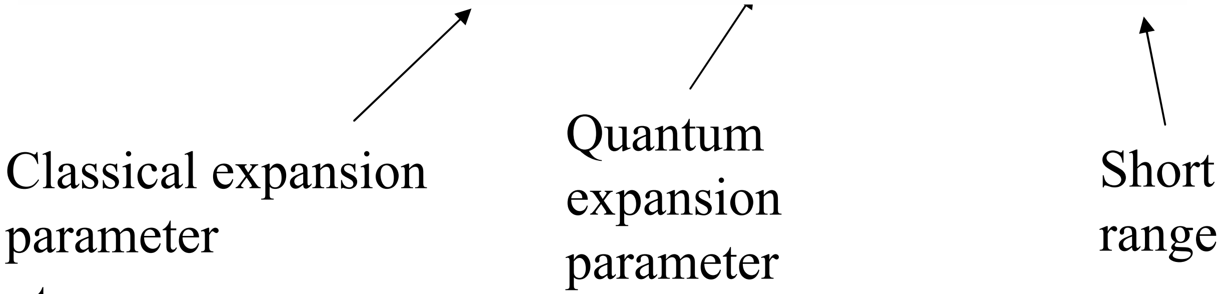
Potential:

$$V_{1(a)}(r) = -\frac{G m_1 m_2}{r}$$

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$



Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

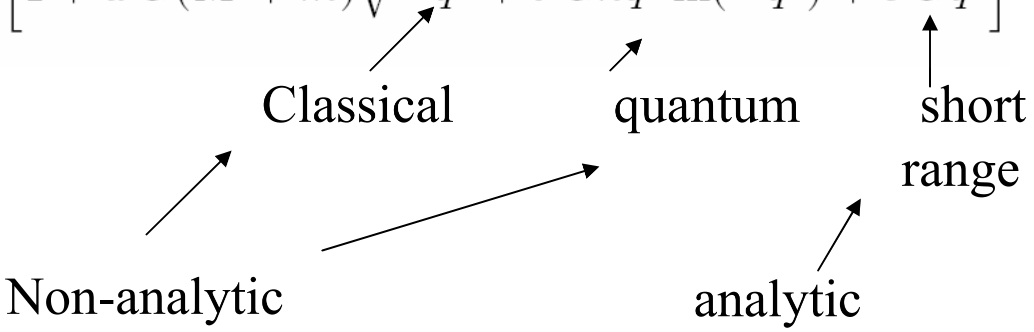
$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}}\ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$



Classical

quantum

short range

Non-analytic

analytic

Parameter free and divergence free

Recall: divergences like local Lagrangian ~R^2

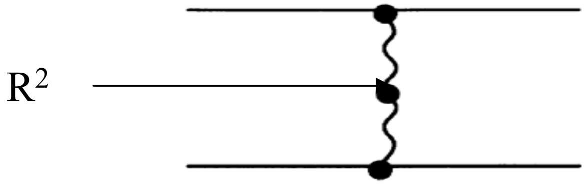
Also unknown parameters in local Lagrangian ~c_1,c_2

But this generates only "short distance term"

Note: R^2 has 4 derivatives $R^2 \sim q^4$

Then:

Treating R^2 as perturbation

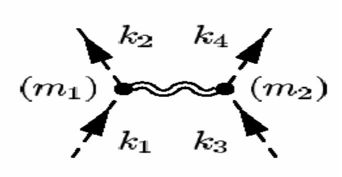


$$V_{R^2} \sim G^2 Mm \ \ \frac{1}{q^2}\, q^4\, \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 Mm\delta^3(x)$$

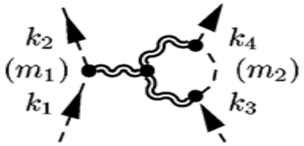
Local lagrangian gives only short range terms

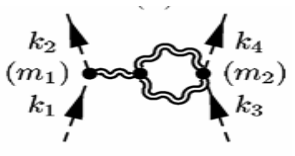
The calculation:

Lowest order:

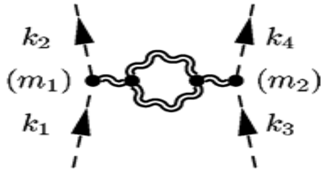


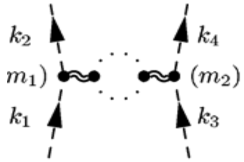
Vertex corrections:



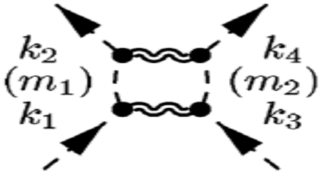


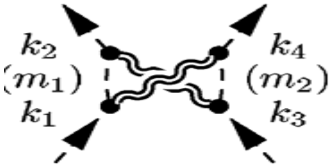
Vacuum polarization:
(Duff 1974)



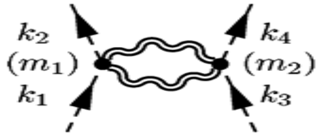


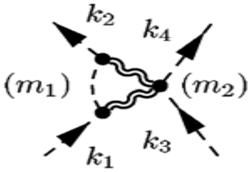
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$\begin{aligned} M_{5(a)+5(b)}(\vec{q}) &= 2G^2 m_1 m_2 \left(\frac{\pi^2 (m_1 + m_2)}{|\vec{q}|} + \frac{5}{3} \log \vec{q}^2 \right) \\ M_{5(c)+5(d)}(\vec{q}) &= -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2 \end{aligned}$$

Sum diagrams:

$$\boxed{V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]}$$

Classical
correction
(Iwasaki ;
Gupta + Radford)

Quantum
correction

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Duff; JFD; Muzinich and Vokos; Hamber and Liu;
   Akhundov, Bellucci, and Sheikh)
   -other potentials or mistakes

5) Why not done 30 years ago?
   - power of effective field theory reasoning

Aside: Classical Physics from Quantum Loops:

JFD, Holstein
2004 PRL

Field theory folk lore:

Loop expansion is an expansion in
“Proofs” in field theory books $\hbar$

This is not really true.

- numerous counter examples – such as the gravitational potential

- can remove a power of $\hbar$ via kinematic dependence

$$\sqrt{\frac{m^2}{-q^2}} = \frac{m}{\hbar\sqrt{-k^2}}$$

- classical behavior seen when massless particles are involved

2 new results on quantum potential:

Universal quantum corrections:

Holstein
Ross

Calculations for a variety of spins
- different diagrams, different factors

Yet, both classical and quantum pieces are universal

Dispersive confirmation of the potential:
- demonstrates generality
- may be useful for understanding universality

Donoghue
Holstein

Dispersive treatment of quantum potential

JFD
Holstein

Interactions from sewing together tree amplitudes

- only involves on-shell gravitons
- classical and quantum corrections emerge from low q^2 limit

Tree amplitudes easier to calculate than loops

Universality of low energy limit:

- leading corrections identical for different spins

Basic dispersive framework:

diagrams satisfy analyticity requirements
leads to a dispersive representation

$$V_2(s,q^2) = \frac{1}{\pi}\int_0^\infty \frac{dt}{t-q^2}\rho(s,t) \;+\; (\text{L.H.cut} = \text{short range})$$

Spectral functions calculated via Cutkosky rules
- on shell intermediate states

quantum
without loops

At low energy:

$$\rho(s,t) = a_2(s)\frac{1}{\sqrt{t}} + a_3(s) + \ldots.$$

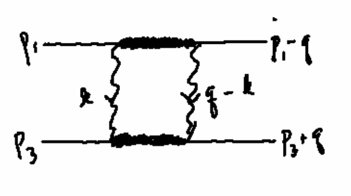
$$V_2(s,q^2) = a_2(s)\frac{1}{\sqrt{-q^2}} - \frac{a_3(s)}{\pi}\ln(-q^2) + \ldots.$$

$$V_2(r) = \frac{1}{8\pi^2 m_A m_B}\left[\frac{a_2}{r^2} + \frac{a_3}{r^3}\right]$$

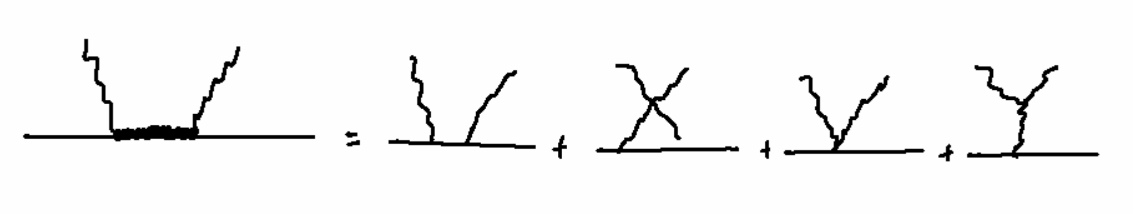
Note: leading non-analytic structures – independent of possible subtractions

Gravitational potential via dispersive techniques:

- cut involves gravitational Compton amplitude



with



$$\rho_g(s,t)=\frac{-1}{32\pi}\int\frac{d\Omega_k}{4\pi}\,\mathcal{M}_A^{\mu\nu,\lambda\sigma}(p_1,-k,p_1-q,q-k)\mathcal{M}_B^{\alpha\beta,\gamma\delta}(p_3,-k,p_3+q,k-q)P_{\mu\nu,\alpha\beta}P_{\lambda\sigma,\gamma\delta}$$

Amplitudes are more complicated, but procedure is the same:

Reproduce usual result –diagram by diagram

Ghosts done
by hand for
now

$$V(r)=-\frac{GMm}{r}\left[1+3\frac{G(M+m)}{rc^2}+\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]$$

VERY strong check on loop calculation

Universality of the quantum corrections:

Can we prove universality?:

Electromagnetic amplitude has multipole expansion Weinberg

- at low energy, E1 transitions dominate – fixed tensor structure
- E1 transition has fixed q^2 -> 0 limit, normalized to charge

Reasons to expect that gravitational interaction is similar

- universal form - factorization
- low energy is square of E1 amplitude – fixed form

⇒all gravity spectral functions have the same low energy structure
⇒same classical and quantum corrections

Example 2: Graviton –graviton scattering

Fundamental quantum gravity process

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

Cooke;
Behrends Gastmans
Grisaru et al

One loop:

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right) \\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--) \\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3) \\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right. \\
&\quad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right) \\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u} \\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

Infrared safe:

JFD +
Torma

The 1/ε is from infrared
- -soft graviton radiation
- -made finite in usual way

1/ε -> ln(1/resolution) (gives scale to loops)

-cross section finite

$$
\begin{aligned}
&\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \\
&= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right)\right.\right. \\
&\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \underset{\uparrow *}{\ln\frac{s}{\Lambda^2}} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}. \qquad (29)
\end{aligned}
$$

finite

Beautiful result:
- -low energy theorem of quantum gravity

Hawking Radiation

Hambli,
Burgess

Exploratory calculation

-remove high energy contributios
 -Pauli Villars regulators

-flux from local limit of Green's function

$$\begin{aligned}\mathcal{F} &\equiv -\langle T_t{}^r\rangle = -\langle T_{tr^\star}\rangle \\ &= -\frac{1}{2}\lim_{x'\to x}\left(\frac{\partial}{\partial t'}\frac{\partial}{\partial r^\star}+\frac{\partial}{\partial r^{\star\prime}}\frac{\partial}{\partial t}\right)\,G(x,x').\end{aligned}$$

-dependence on regulator vanishes exponentially

-radiation appears to be property of the low energy theory

Limitations of the effective field theory

Corrections grow like $Amp \sim A_0\left[1 + Gq^2 + Gq^2 \ln q^2\right]$

Overwhelm lowest order at $q^2 \sim M_P{}^2$

Also sicknesses of $R+R^2$ theories beyond M_P
(J. Simon)

Effective theory predicts its own breakdown at M_P
- could in principle be earlier

Needs to be replaced by more complete theory
at that scale

The extreme IR limit

Singularity theorems:
- most space times have singularities
- EFT breaks down near singularity

Can we take extreme IR limit?
- wavelength greater than distance to nearest singularity?
- past black holes?

Possible treat singular region as source
- boundary conditions needed

deSitter horizon in IR

Matching to LQG/ spin foams etc

EFT should be low curvature limit of a more fundamental theory

Find low curvature backgrounds (4D)
Identify spin 2 gravitons (without extra massless DOF)
Verify leading coupling to stress energy

There exists Deser theorem => GR

Verify causality, unitarity, crossing

Why gravity may be the best case to study

- EFT reasoning

Applequist Carrazone theorem:

Effects from high energy either appear as shifts in coupling constants (eg. c_1,c_2) or are suppressed by powers of the heavy scale (eg. M_{Pl}).

But all gravity interactions are suppressed by $\mathbf{M_{Pl}^2}$

- relative effect could then be much bigger ~O(1)?
- small violations of unitarity, causality….
- approximate general covariance?

Analogy: $\pi^0 \rightarrow \gamma\gamma$ reveals # of colors of QCD

Summary

We have a quantum theory of general relativity
- quantization and renormalization
- perturbative expansion

It is an effective field theory
- valid well below the Planck scale
- corrections are very well behaved

Effective field theory techniques allow predictions
- finite, parameter free
- due to low energy (massless) propagation

Need full theory at or before Planck scale
- many interesting questions need full theory
- not conflict between QM and GR, but lack of knowledge about fundamental high energy theory