# The LQG vertex

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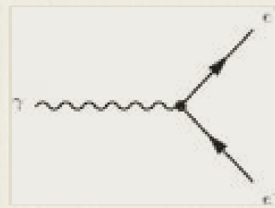
- In QED, the *kinematics* is given by the Fock space of the photons and the electrons.

  The *dynamics* can be covariantly coded by the single vertex amplitude :

$$e\,\gamma^\mu\,\delta(p_1-p_2-k)$$



- In LQG, the *kinematics* is well understood: it is given by the separable Hilbert space of the s-knots (abstract spin networks).

  What is the LQG vertex amplitude, which codes the *dynamics* covariantly ?

## Related issues

- Low energy limit of LQG, Newton's law
- Computing scattering amplitudes
- Relation hamiltonian LQG and spinfoam formalism: SO(4) versus SO(3) mismatch
- Which version of the dynamics is the good one?

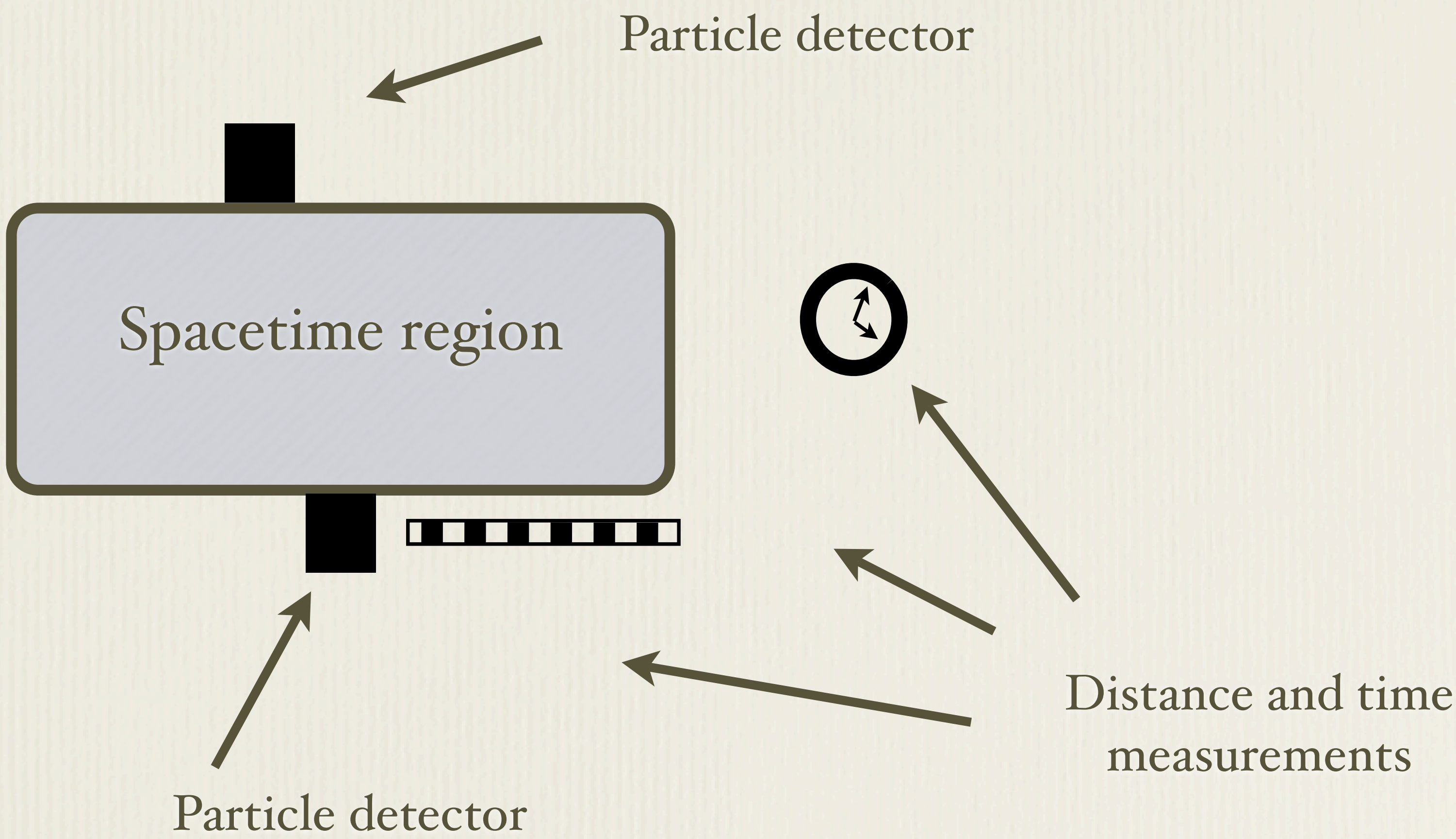
## Recent results (and plan of the talk)

1. Scattering amplitudes in a background independent theory
   (Oeckl 2003; Modesto, cr 2004)

2. Propagator (and Newton law) from LQG
   (cr 2005; Bianchi, Modesto, Speziale cr 2006; Speziale, Livine, Willis, Christensen 2006)

3. Nondiagonal terms: problems of the Barrett Crane vertex.
   (Alesci, cr 2007)

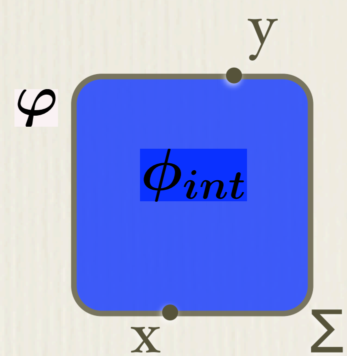
4. A new vertex amplitude
   (Engles, Pereira, cr 2007; Speziale, Livine 2007).




In GR distance and time measurements are field measurements like the other ones: they determine the **boundary data** of the problem.

Scattering amplitudes in a background independent theory :
consider the boundary functional

$$W[\varphi] = \int_{\phi_{int}(\Sigma) = \varphi} D\phi_{int} \;\; e^{iS[\phi_{int}]}$$




and write the conventional two point function

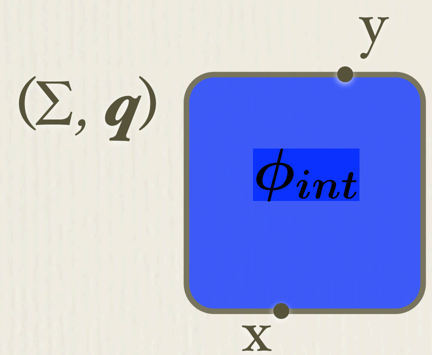
$$W(x, y) = \int D\phi \;\; \phi(x)\, \phi(y) \;\; e^{iS[\phi]}$$

in the form

$$W(x, y; q) = \int D\varphi \;\; \varphi(x)\, \varphi(y)\; W[\varphi] \;\; \Psi_q[\varphi]$$

where $\Psi_q[\varphi]$ is a state picked on a given *geometry q* of $\Sigma$.

Distance and time separation between x and y are now well defined *with respect to the mean boundary geometry q*.




# Give meaning to the expression

$$\boldsymbol{W(x, y; q) = \int D\varphi \; \varphi(x) \; \varphi(y) \; W[\varphi] \; \Psi_q[\varphi]}$$

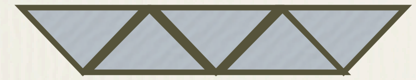
- $\int \mathrm{D}\varphi \;\; \boldsymbol{\rightarrow} \;\; \sum_{\text{s-knots}}$
- $\boldsymbol{W}[\varphi] \;\; \boldsymbol{\rightarrow} \;\; \boldsymbol{W}[\mathrm{s}]$ defined by GFT spinfoam model
- $\boldsymbol{\Psi_q} \;\; \boldsymbol{\rightarrow}$ a suitable coherent state on the geometry $\boldsymbol{q}$
- $\varphi(\mathrm{x}) \;\; \boldsymbol{\rightarrow}$ graviton field operator from LQG.

$$\boldsymbol{W^{abcd}(x, y; q) = N \sum_{ss'} \; W[s'] \; \langle s' | h^{ab}(x) h^{cd}(y) | s \rangle \; \Psi_q[s]}$$

# What is the meaning of the expansion in $\lambda$ ?

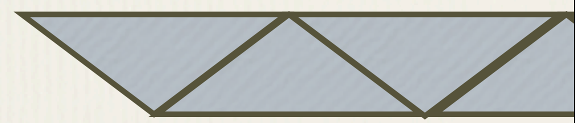
- It is the same cut off as GR on a finite Regge lattice

N=5

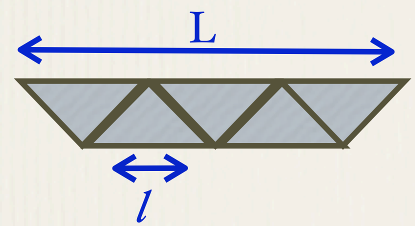


- It is *not* a long distance expansion (a Regge simplex can be arbitrarilly small)

- It is *not* a short distance expansion (a Regge simplex can be arbitrarilly large)



- It is a cut off of the degrees of freedom that are high frequency with respect to the maximal dimension



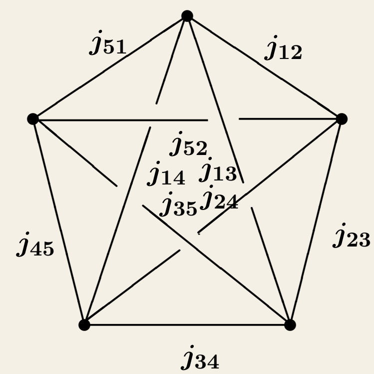

- That is it is an expansion in $l/L$ where $l$ is the mimimal wavelength in the process and $L$ is the size of the process

$\lambda \sim l / L$

- Good for $L \sim L_{Pl}$ or for large $L$ in the low energy approximation

To first order in λ, the only nonvanishing connected term in W[s] is for

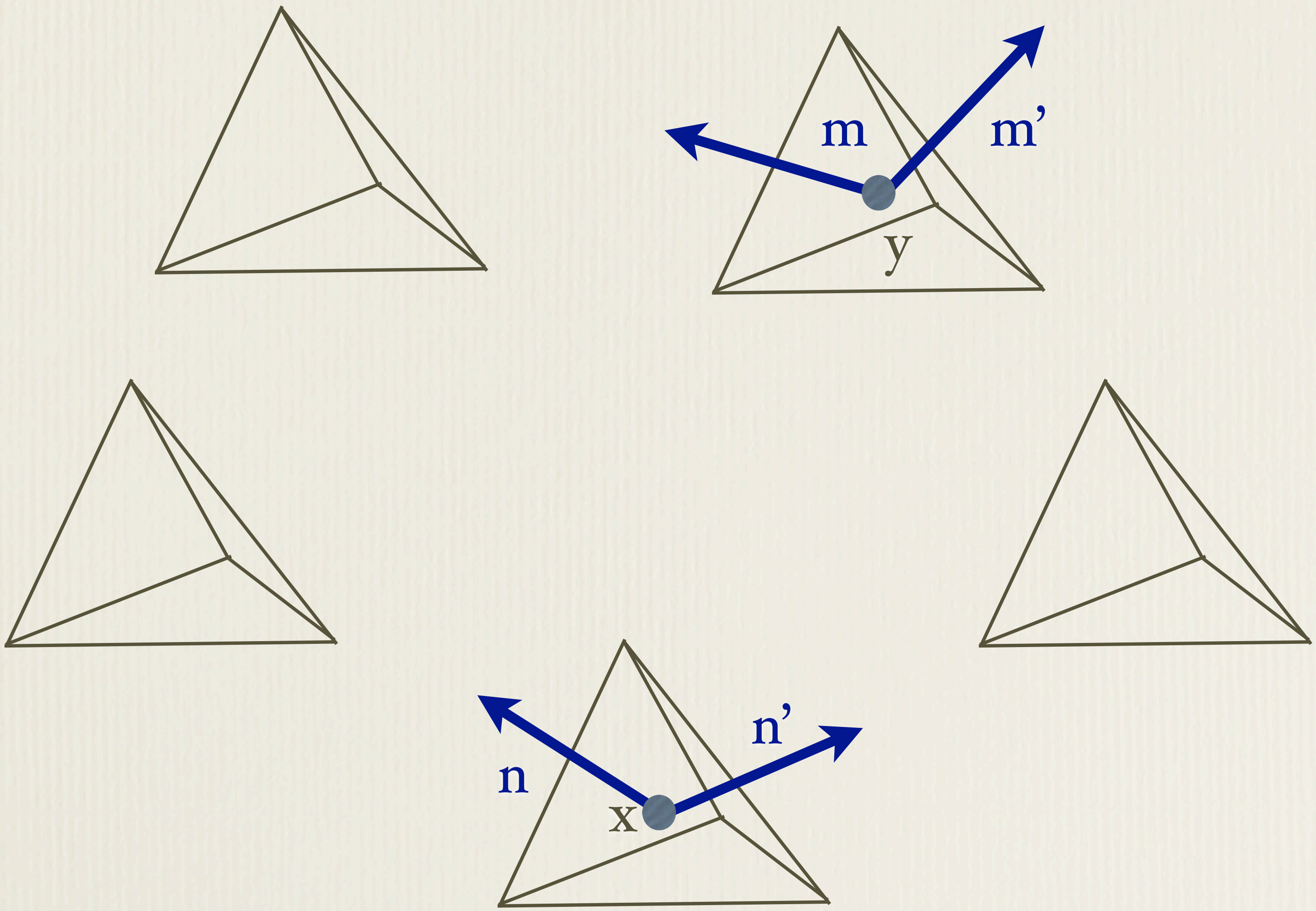
$$s =$$



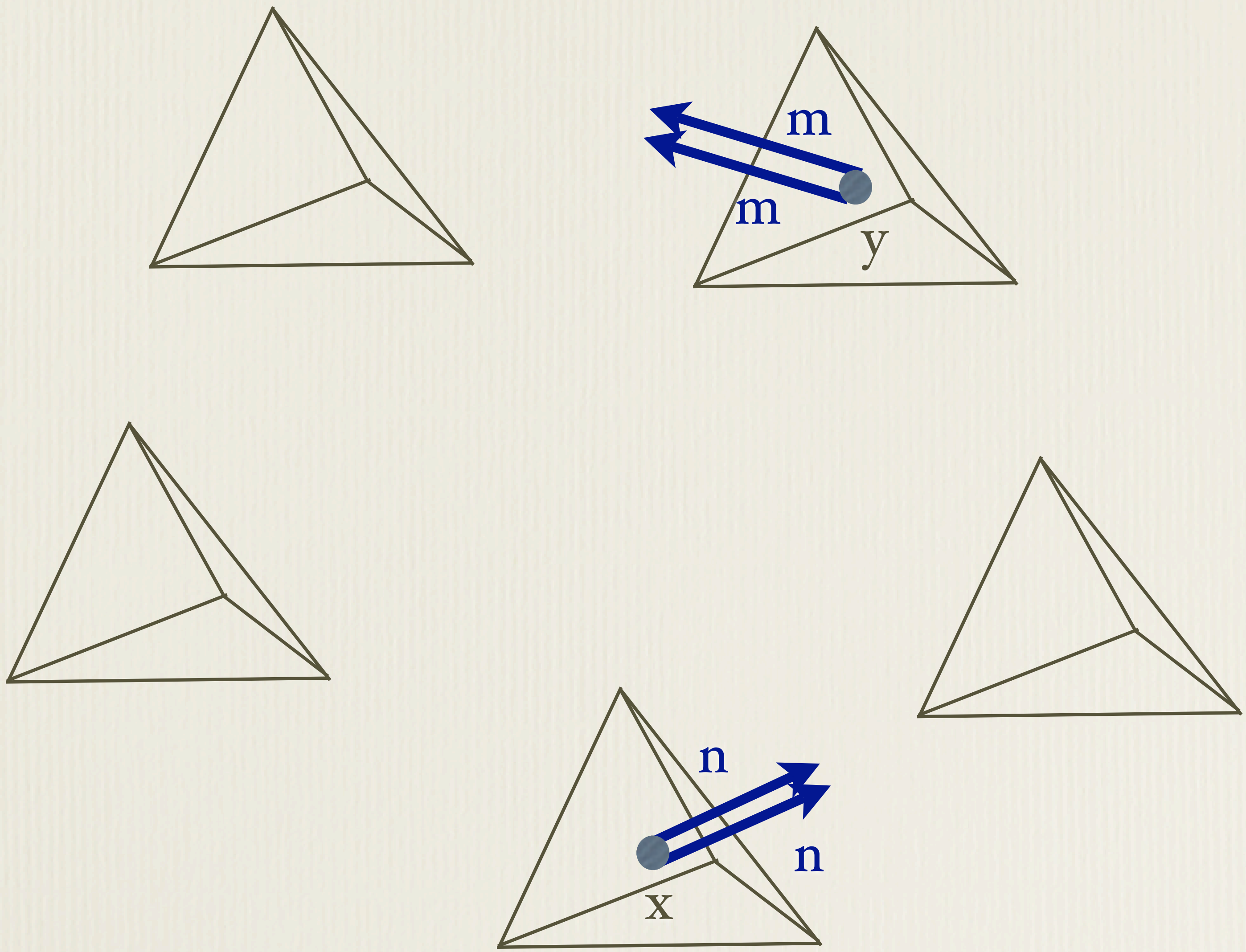

And the dominant contribution for large j is given by the spinfoam σ dual to a *single* four-simplex. This is

$$W[s] = \frac{\lambda}{5!}\left(\prod_{n<m} dim(j_{nm})\right) A_{vertex}(j_{nm})$$

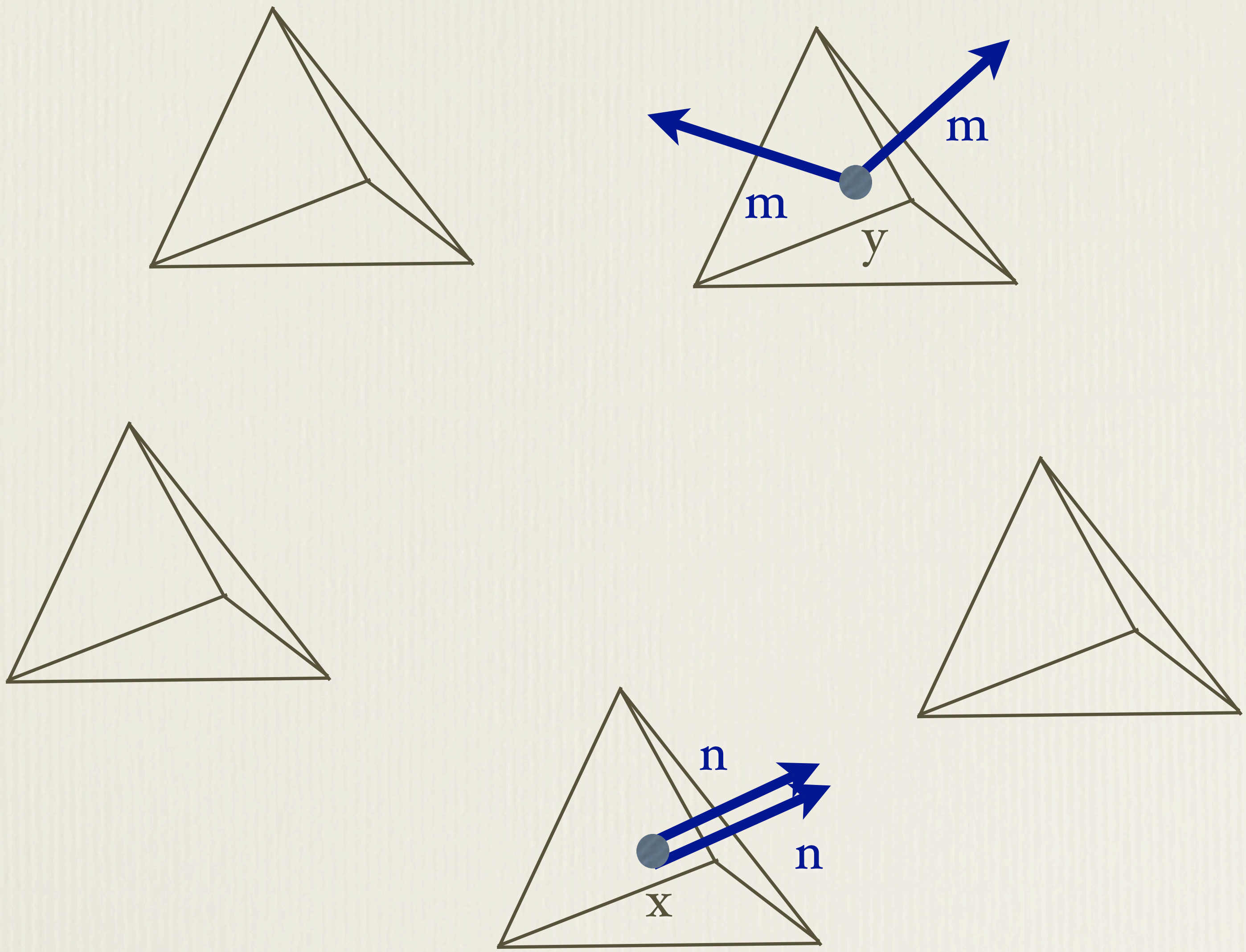
$$< 0 \, | h^{ab}(x) h^{cd}(y) | \, 0 > \quad n_a \, n'_b \, m_c \, m'_d \quad := \quad W$$



“diagonal” terms :



a “non diagonal” term :



# The expression for the propagator is then well defined:

$$W(L) = W^{abcd}(x, y; q) n_a n_b m_c m_d =$$

$$N \frac{\lambda(\hbar G)^4}{5!} \sum_{j_{nm}} (j_{12}(j_{12}+1) - j_L^2)\,(j_{34}(j_{34}+1) - j_L^2)\ A_{vertex}(j_{nm})\ \ e^{-\frac{\alpha}{2}\sum_{n,m}(j_{nm}-j_L)^2 - i\Phi\sum_{n,m} j_{nm}}$$

Computing, and adjusting the numerical factors in the state, this gives

$$W(L) = i\frac{8\pi\hbar G}{4\pi^2}\frac{1}{L^2} = i\frac{8\pi}{4\pi^2}\frac{1}{|x-y|_q^2}$$

which is (one component) of the the correct graviton-propagator.

➞ This is equivalent to the Newton law

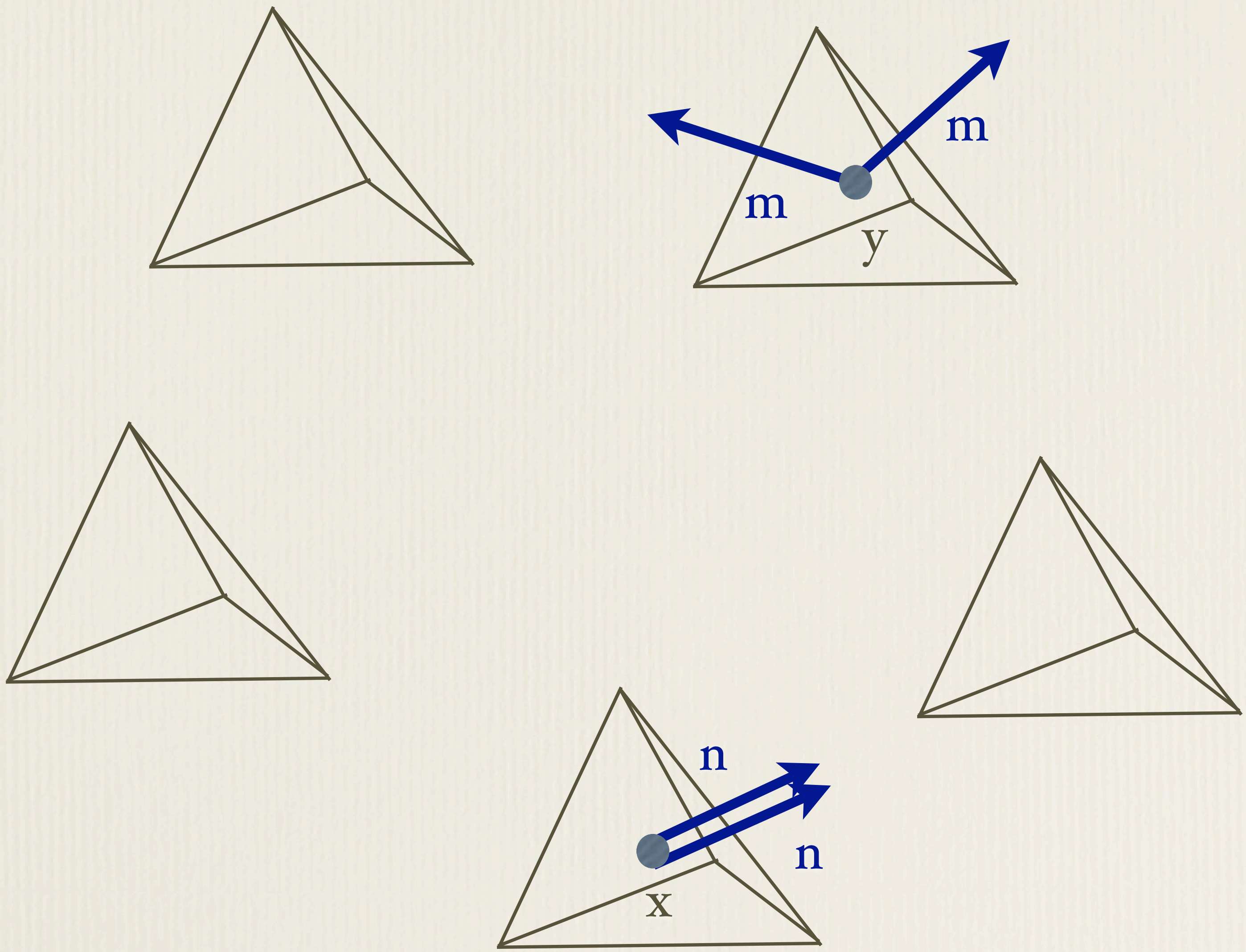
➞ This is only valid for $L^2 >> \hbar G$.

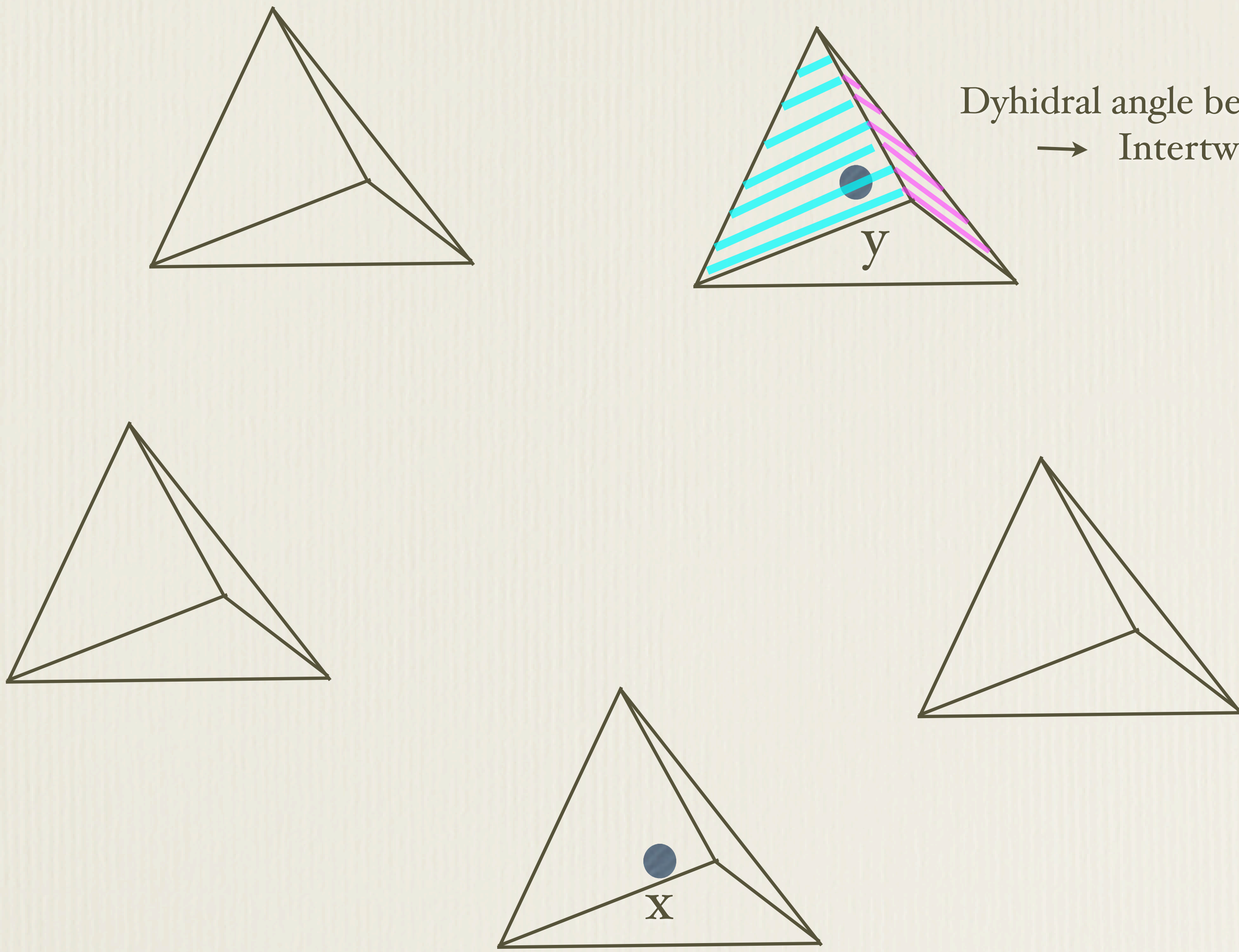
For small L, the propagator is affected by quantum gravity effects, and is given by the 10j symbol combinatorics.

# Developements

- 3d calculation (Speziale)
- Numerical check of the (drastic) approximation taken (Christensen, Speziale)
- Planck length corrections (Livine, Speziale, Willis)
- Improved boundary state (Livine, Speziale)
- Three point function (Bianchi, Modesto)
- Higher order terms in $\lambda$ (Bianchi, Modesto, Speziale, cr; Mamone...)
- Nondiagonal terms ($\langle 0|g_{ab}(x)g_{cd}(y)|0\rangle$ $a\neq b$ $c\neq d$) (Alesci, cr) ← **WRONG !**

“non diagonal” term :






Dyhidral angle between faces

→ Intertwiners

The problem is that the Barrett Crane vertex depends only on boundary spins, and not on boundary intertwiners.

The BC spinfoams are bounded by spin networks that have fixed intertwiners. The boundary state of the BC model is not the same as the one of hamiltonian LQG: SO(4) - SO(3) mismatch

Is there a spinfoam model, different from Barrett Crane, that does not have this difficulty?

# Origin of the problem: strong versus weak simplicty constraints

The BC model can be obtained adding the simplicity constraints to quantum BF theory:

BF action: $S_{BF} = \int B \wedge F,$ GR action , $S_{GR} = \int * (e \wedge e) \wedge F$

Simplicity constraints: $C_{ab} = * B_a \bullet B_b = 0$ imply: $B = * (e \wedge e)$

In the BC model, the simplicity constraints are are imposed *strongly* on the states $C_{ab}\, |\Psi\rangle = 0.$ This yields the BC intertwiner and kills the dependence of the boundary states from the intertwiners. But they are *second class* $\{C_{ab}, C_{ab}\} \neq 0$, and second class constraint must *not* be imposed strongly !

$\Rightarrow$ Can we, instead, impose the simpicity constraints weakly ? (cfr Gupta-Beuler, strings...)

$$\langle \Phi \,| C_{ab} |\Psi\rangle = 0$$

Does this solve the SO(4) - SO(3) mismatch and frees the intertwiners degrees of freedom?

# Yes !

Strategy:

1. Discretize GR on a triangulation with boundaries (Regge calculus).

2. Choose appropriate variables (two-form B and SO(4) group elements).

3. Write the proper boundary phase space (same as SO(4) lattice YM).

4. Quantize as in lattice QCD.

5. Find a proper subspace of the boundary state space where the simplicity constraints vanish *weakly*.

6. Compute the amplitude associated at one 4-simplex by a (Feynman-like) integration on the bulk variables on the simplex.

This can be done. See the seminar by Roberto Pereira for details.
The result is the following modification of BC (Engle, Pereira, CR, 2007).

## The resulting model

$$Z_{GR} = \sum_{j_f, i_e} \prod_f (\dim \tfrac{j_f}{2})^2 \prod_v A(j_f, i_e)$$

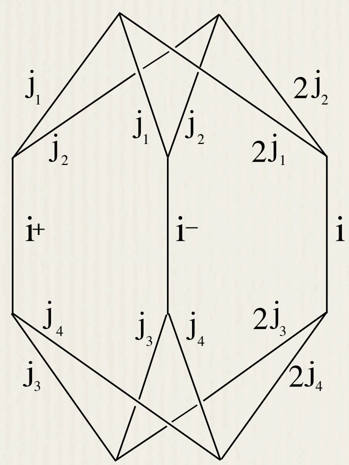
where the vertex amplitude is given by

$$A(j_f, i_e) = 15j_{SO(4)}\big((\tfrac{j_f}{2}, \tfrac{j_f}{2}), f(i_e)\big)$$

where

$$f(i) = \sum_{i^+,\, i^-} f^i_{i^+ i^-} \; (i^+, i^-)$$

$$f^i_{i^+ i^-} =$$



## Properties

- Derives from a proper quantization of a discretization of GR
- Boundary states match SO(3) LQG states precisely
- Intertwiners degrees of freedom remain free
- Simplicity constraints hold weakly
- Similar to SO(4) Barrett-Crane model, but with larger intertwiner space
- SO(4) covariant vertex amplitude for LQG.

# Intertwiner spaces

SO(4) intertwiner space : $\boldsymbol{K = Inv[H_{j_1^+ j_1^-} \otimes H_{j_2^+ j_3^-} \otimes H_{j_3^+ j_3^-} \otimes H_{j_4^+ j_4^-}]}$

Under SO(3) ⊂ SO(4) : $\boldsymbol{H_{jj} = H_j \otimes H_j = H_0 \oplus H_1 \oplus \ldots \oplus H_{2j-1} \oplus H_{2j}}$

BC intertwiner space: $\boldsymbol{H_{BC} = Inv[H_0 \otimes H_0 \otimes H_0 \otimes H_0]}$

New intertwiner space : $\boldsymbol{H_{new} = Inv[H_{2j_1} \otimes H_{2j_2} \otimes H_{2j_3} \otimes H_{2j_4}]}$

In $[\boldsymbol{H_0 \otimes H_0 \otimes H_0 \otimes H_0}]$ $J^{IJ}$ satisfies $J^{ij} = 0$ (I,J=1,...,0, i,j=1,3)

In $[\boldsymbol{H_{2j_1} \otimes H_{2j_2} \otimes H_{2j_3} \otimes H_{2j_4}}]$ $J^{IJ}$ satisfies $J^{0j} = 0$

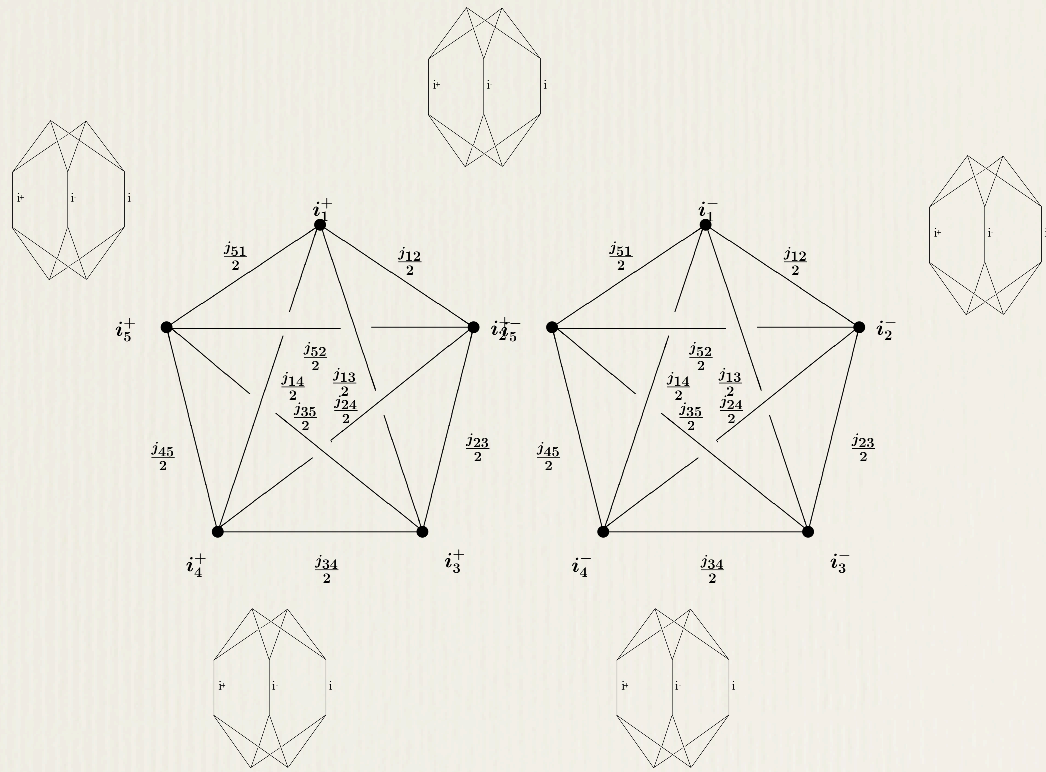
But the simplicty constraints are equivalent to $\Sigma^{0j} = e^0 \wedge e^j = 0$, that is, each tetrahedron is on a 3d hyperplane

Hence the new quantization is simply $\Sigma \Rightarrow J$ instead of $B = {}^*\Sigma \Rightarrow J$

(cfr. Ashtekar change of variables !)

# In conclusion, the vertex for quantum gravity is

$$A(j_{nm}, i_n) = \sum_{i^+_n \, i^-_n}$$




Livine and Speziale have recovered this same vertex with a different argument, based on SU(2) coherent states.

Pessimistic view:    one more nonsense model !

But recall that in 1912 Einstein had the correct math and physics of GR, but he published a series of papers with many (wrong) field equations before finding the good ones in 1915.

Optimistic view:  we are in a similar process !

# Summary

1. There is a way to compute diff-invariant scattering amplitudes. This gives us good control on the low-energy limit of LQG. The BC propagator has the correct spacetime dependence; its diagonal terms are correct, and give the Newton law in this limit.

2. However, the BC tensorial structure is incorrect, because of the mismatch between spinfoam states and LQG states.

3. This can be corrected by imposing the simplicity constraints weakly. This defines a new vertex amplitude, compatible with the LQG kimenatics.