Quantum field theory on curved spacetimes, and the problem of background dependence in perturbative quantum gravity

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Introduction

Main problem of quantum gravity:

In quantum physics, space and time are a priori structures which enter the definition of the theory as well as its interpretation in a crucial way.

This motivates rather radical new approaches (string theory, loop quantum gravity,. . .)

Difficulty: relation to actual physics.

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Conservative approach:

First step: spacetime is a given Lorentzian manifold, on which quantum fields live.

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This approach meets in the second step severe obstructions

- The arising theory is nonrenormalizable, in the sense that infinitely many counter terms arise in the process of renormalization.
- The background metric determines the causal structure of the theory.

But also the first step is by no means trivial. Namely , the standard formalism of quantum field theory is based on the symmetries of Minkowski space. Its generalization even to the most symmetric spacetimes (de Sitter, anti-de Sitter) poses problems.

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- No vacuum
- No particles
- No S-matrix
- No Feynman propagator
- No momentum space
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- No diffeomorphism covariant path integral

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Solution by concepts of algebraic quantum field theory and methods from microlocal analysis:

Haag-Kastler axioms, generalized to generic spacetimes:
Difficulty:

Causal structure well defined but absence of nontrivial symmetries

Question: What is the meaning of repeating an experiment?
(Crucial for the probability interpretation of quantum theory)

Candidate for symmetries: Diffeomorphisms
But causal structure is changed, in general

$\Rightarrow$ cannot induce algebraic homomorphisms
(conflict with locality and primitive causality)

Related problem: Generally covariant version of the spectrum condition

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Kay: Hadamard condition as a local characterization of admissible states (Verch)

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Locally Covariant Quantum Field Theory

Verch, Hollands-Wald, Brunetti-Fredenhagen-Verch (2001-2003)
Idea: Construct theory simultaneously on all spacetimes (of a given class) in a coherent way

$\mathcal{M}$ globally hyperbolic, oriented, time oriented Lorentzian 4d spacetime

Global hyperbolicity $\Rightarrow$ $\mathcal{M}$ is diffeomorphic to $\mathbb{R} \times \Sigma$, Σ Cauchy surface of $\mathcal{M}$

Comparison of spacetimes by admissible embeddings:

An embedding $\mathcal{N} \to \mathcal{M}$ is called admissible, if it is isometric, time orientation and orientation preserving, and causally convex in the following sense: If γ is a causal curve in $\mathcal{M}$ with endpoints $p, q \in \chi(\mathcal{N})$ then $\gamma = \chi \circ \gamma'$ with a causal curve γ' in $\mathcal{N}$.

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Axioms:

1. $\mathcal{M} \mapsto \mathfrak{A}(\mathcal{M})$ unital (C)*-algebra
2. $\chi : \mathcal{N} \to \mathcal{M}$ admissible embedding $\Rightarrow$ $\alpha_\chi : \mathfrak{A}(\mathcal{N}) \to \mathfrak{A}(\mathcal{M})$ homomorphism
3. Let $\chi : \mathcal{N} \to \mathcal{M}$, $\chi' : \mathcal{M} \to \mathcal{L}$ be admissible embeddings then
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Axioms 1 to 3:

$\mathfrak{A}$ is a functor
from the
category Loc of globally hyperbolic Lorentzian spacetimes
with admissible embeddings as morphisms
to the
category Obs of unital (C)*-algebras with
homomorphisms as morphisms,
such that

$$\mathfrak{A}\chi = \alpha_\chi$$

Question: Is there a categorical interpretation of axioms 4 and 5?

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Locality and tensor structure

Claim: Axiom 4 (Locality) $\Leftrightarrow$ $\mathfrak{A}$ is a tensor functor

Tensor structure in Loc: disjoint union

Tensor structure in Obs: algebraic tensor product

embeddings $i : \mathcal{M} \to \mathcal{M} \otimes \mathcal{N}$, $j : \mathcal{N} \to \mathcal{M} \otimes \mathcal{N} \Rightarrow$

$$\alpha_i(A) = A \otimes 1_{\mathfrak{A}(\mathcal{N})} \ , \ \alpha_j(B) = 1_{\mathfrak{A}(\mathcal{M})} \otimes B$$

$\chi : \mathcal{M} \otimes \mathcal{N} \to \mathcal{L}$ admissible

$\Updownarrow$

$\chi \circ i$, $\chi \circ j$ are admissible

and

$\chi(\mathcal{M})$ and $\chi(\mathcal{N})$ cannot be connected by a causal curve in $\mathcal{L}$

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Time slice axiom and cobordisms

Σ Cauchy surface of $\mathcal{M}$.

$$\mathfrak{A}(\Sigma) := \lim_{\overleftarrow{\mathcal{N}\supset\Sigma}} \mathfrak{A}(\mathcal{N})$$

$\mathfrak{A}(\Sigma) \ni A =$ (germ of)$(A_{\mathcal{N}})_{\mathcal{N}\supset\Sigma}$ with $\alpha_{\mathcal{N}_1\mathcal{N}_2}(A_{\mathcal{N}_2}) = A_{\mathcal{N}_1}$ where $\mathcal{N}_1\mathcal{N}_2$ denotes the natural embedding of $\mathcal{N}_2$ into $\mathcal{N}_1$.

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Time slice axiom $\Longrightarrow$ $\alpha_{\mathcal{M}\Sigma}$ is an isomorphism $\Longrightarrow$ $\exists$ propagator between Cauchy surfaces

$$\alpha^{\mathcal{M}}_{\Sigma_1\Sigma_2} := \alpha^{-1}_{\mathcal{M}\Sigma_1} \circ \alpha_{\mathcal{M}\Sigma_2}$$

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Cobordisms:

$$\Sigma_- \xrightarrow{\mathcal{M}} \Sigma_+$$

$\mathcal{M}$ Lorentzian spacetime with future/past boundary $\Sigma_\pm$

$$\Sigma \mapsto \mathfrak{A}(\Sigma)$$

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Warning: In general $\mathfrak{A}(\Sigma)$ depends on the germ of Σ as a submanifold of $\mathcal{M}$.
Free scalar field: $\mathfrak{A}(\Sigma)$ algebra of canonical commutation relations in terms of Cauchy data $(f, p) \in \mathcal{C}^\infty_c(\Sigma) \times \Omega^{d-1}_c(\Sigma)$.
Enlarged algebra containing Wick products (Brunetti-Fredenhagen-Köhler,Dütsch-Fredenhagen):
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Perturbative quantum gravity

Proposal:

Split of the metric

$$g_{ab} = g_{ab}^{(0)} + h_{ab}$$

g^0 background metric, h quantum field.

Renormalize h by the Epstein-Glaser method (interaction restricted to a compact region between two Cauchy surfaces)

Compute $\alpha_{\Sigma_1 \Sigma_2}^{\mathcal{M}}$ for two background metrics which differ only in a compact region between the 2 Cauchy surfaces

Renormalization condition: Propagator is independent of the background metric

Infinitesimal version: The interacting metric satisfies Einstein's equation

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Obstructions:

- Nonrenormalizability: In every order new counter terms (hopefully small)
- Constraints have to be imposed. Best developped within perturbation theory: BRST
- Local BRST cohomology is presumably trivial, hence one has to use global objects

Candidates for global quantities: Fields (considered a natural transformations between the functor of test function spaces and the quantum field theory functor)

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Conclusions and outlook

- A construction of quantum field theory on generic Lorentzian spacetime is possible, in accordance with the principle of general covariance.
- A consistent incorporation of the quantized gravitational field seems to be possible.
- Relation to other field theoretical approaches to quantum gravity (Reuter, Bjerrum-Bohr,. . .) has to be investigated.
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