Regulator dependence in quantum gravity
and
non perturvative renormalizability:
possible new perspectives

BACKGROUND INDEPENDENT STRING-LIKE MATTER COUPLED TO FOUR DIMENSIONAL BF THEORY

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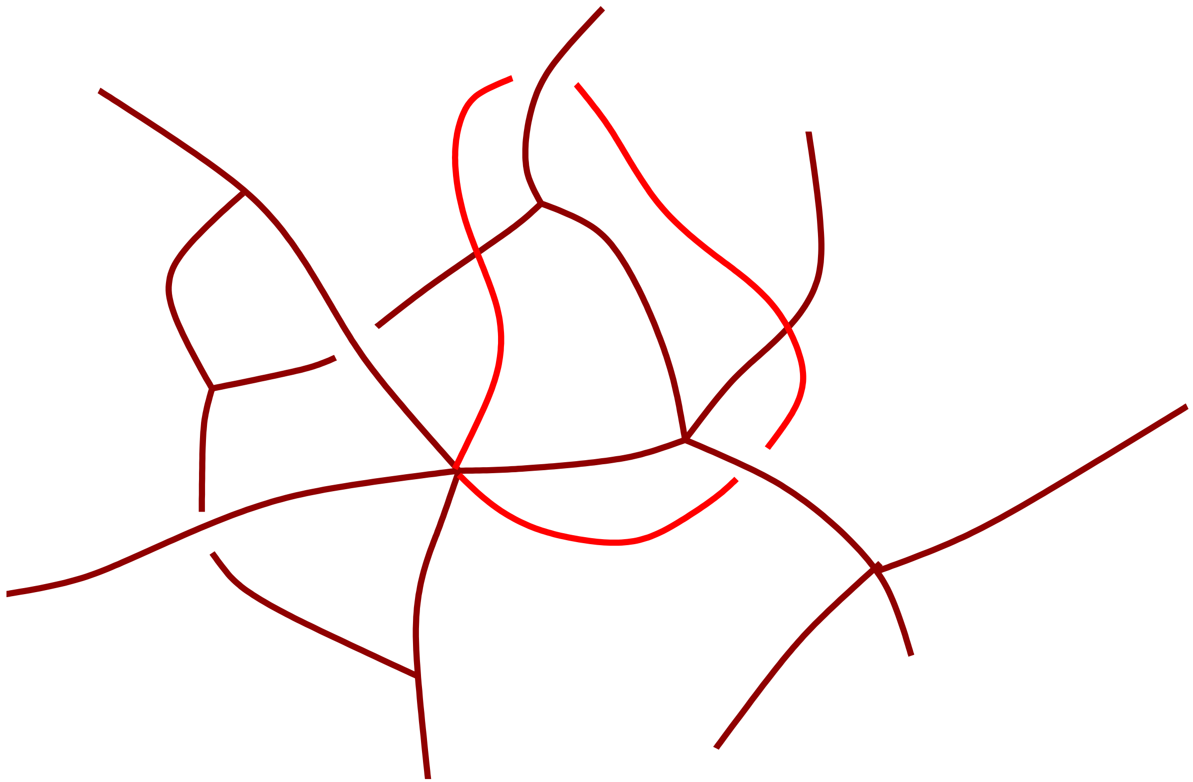
LOOPS07, June 2007, Morelia, Mexico.

One of the most important results of LQG is Thiemann's rigorous quantization of the Hamiltonian constraint of gravity coupled to matter.

Background Independence (Diff. Inv.)$\Rightarrow$ UV finiteness.



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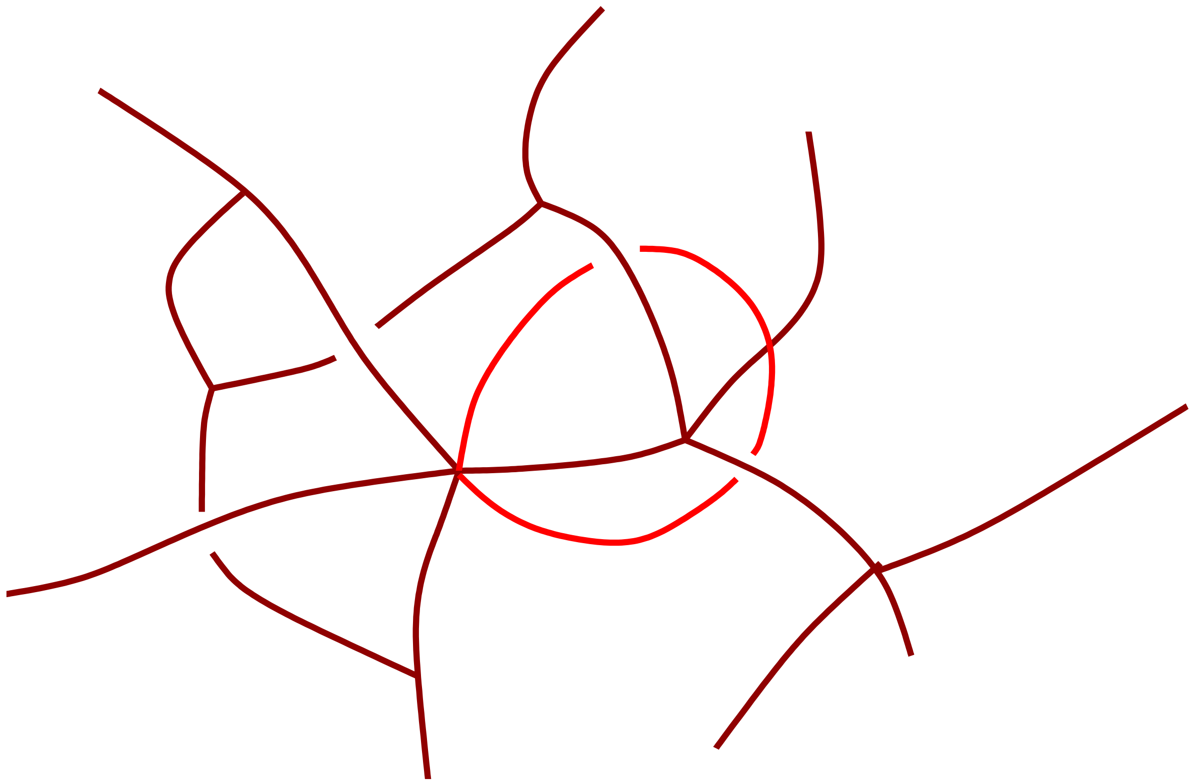

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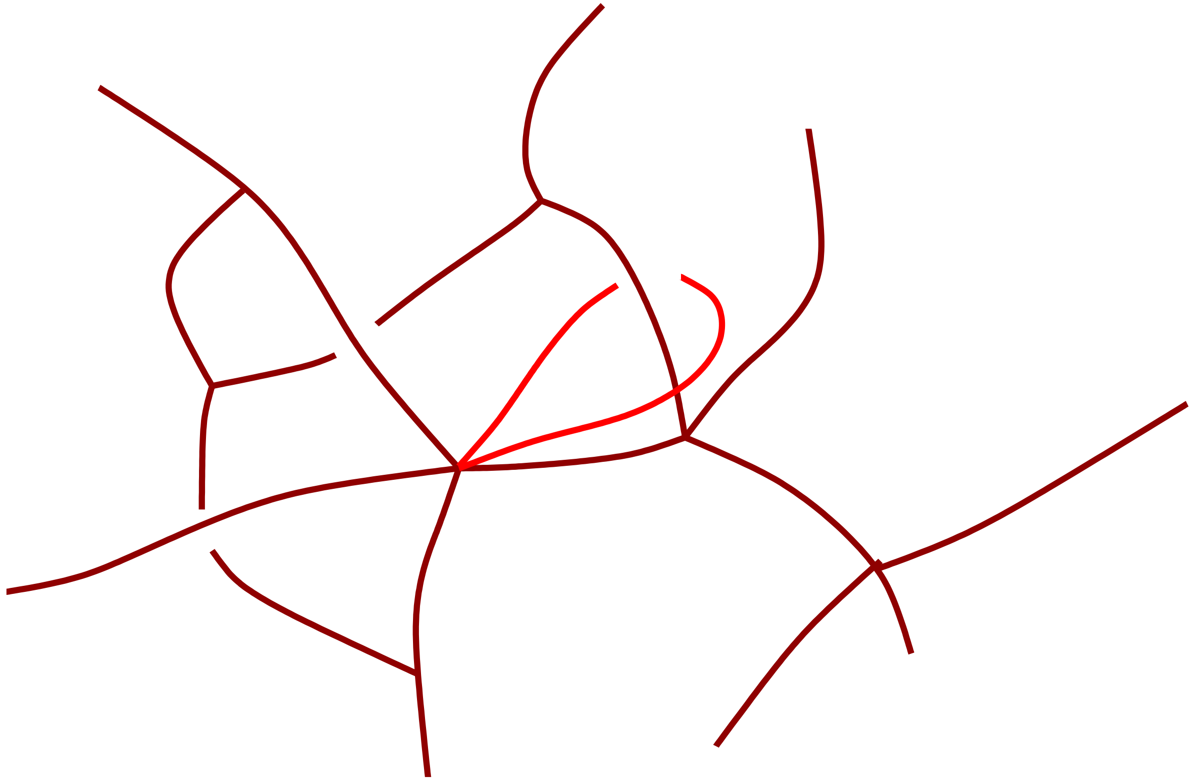

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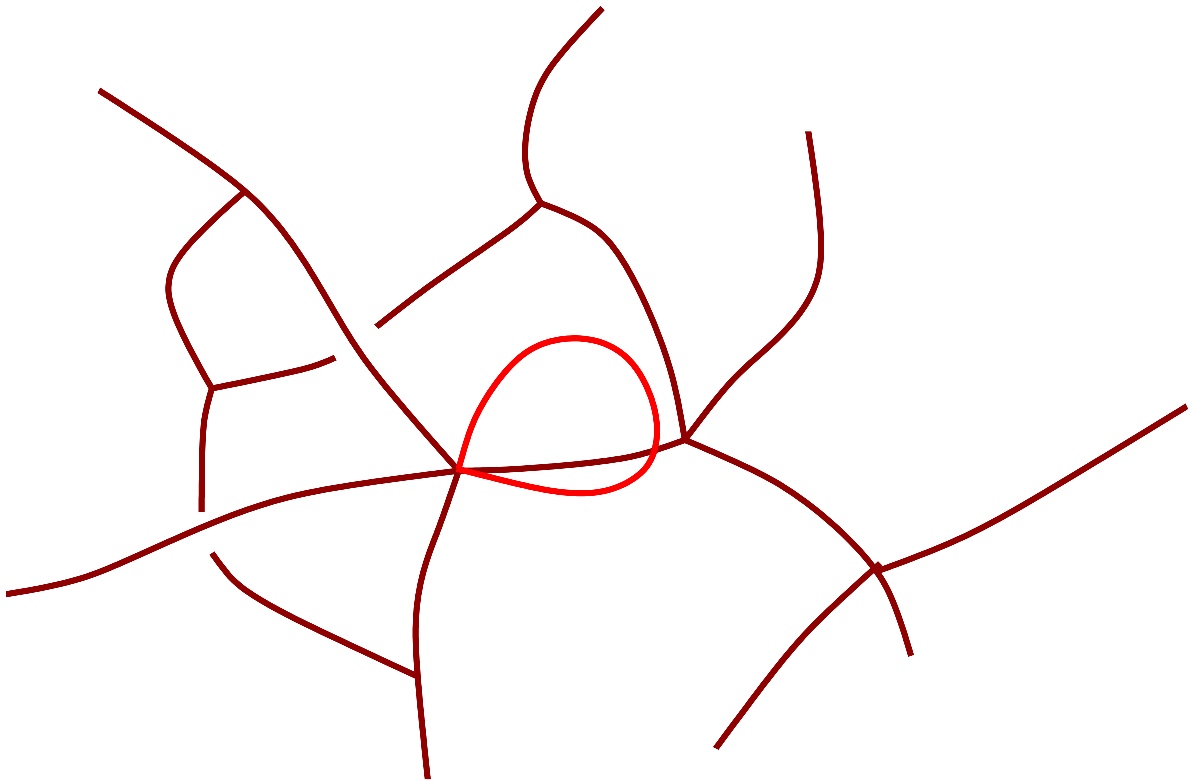

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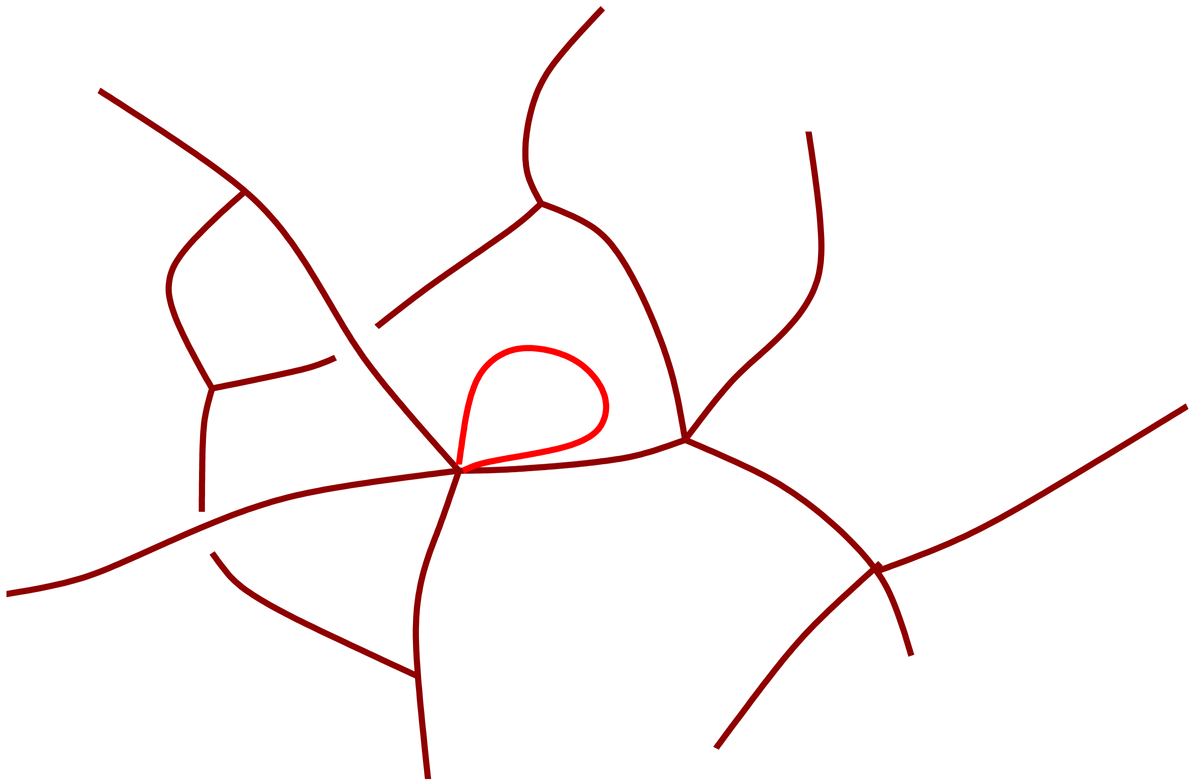

Nevertheless, renormalizability remains open issue in LQG

- (Canonical) Loop quantum gravity: UV finite; but regularization ambiguities.
- (Covariant LQG) Spin Foam Models: UV finite; but discretization dependent. Many interesting ideas[1] but so far it is not clear how to take the continuum limit.
- Group field theory formulation[2]: UV finite; There are ideas concerning non perturbative information[3].

$$S(\phi) = \int \phi^2 + \lambda_0 \phi^3 + \lambda_1 \phi^4 + \lambda_3 \phi^5 + \lambda_4 \phi^6 + \cdots$$

However, in contrast with matrix models, no evidence of universality.

[1] Reisenberger, Zapata, Oeckl, Markopoulou,...
[2] DePietri-Freidel-Krasnov-Rovelli, Reisenberger-Rovelli
[3] Freidel-Louapre, Livine, Oriti

Topological quantum field theories are 'renormalizable'

- Canonical Quantization: there are regularization ambiguities. However they dissapear when considering physical amplitudes[4]

- Covariant quantization (spin foam representation[5]) is regulator independent (discretization independent[6]).

- However, they have only finitely many degrees of feedom !

Want to define a background independent QFT with local degrees of freedom

- IDEA: make topology dynamical by suming over topologies in the path integral formuation. Hopeless in 4d, very difficult dont know how to start in 3d, but might be possible in 2d!!!

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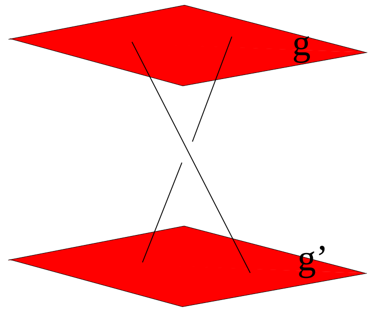
[4] AP 2005

[5] Noui-AP

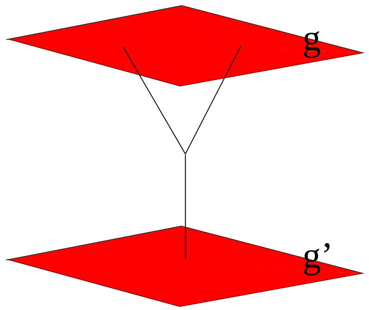
[6] Girelli-Oeckl-AP

Interesting example: 2+1 gravity plus point particle

$$\langle g', 2 | g, 2 \rangle =$$



For fixed number of topological defects (point particles) the quantum theory is well defined



$=??????$

Interactive 'Transition amplitudes' amplitudes are well defined but have not clear-cut interpretation in the quantum theory (int. as Feynman diag. of scalar field theory proposed[7].).

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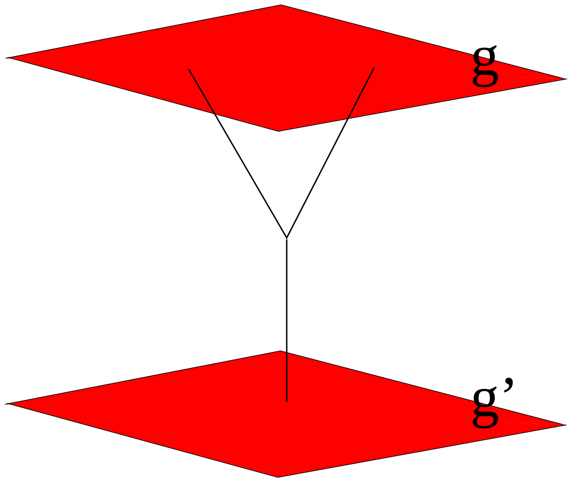
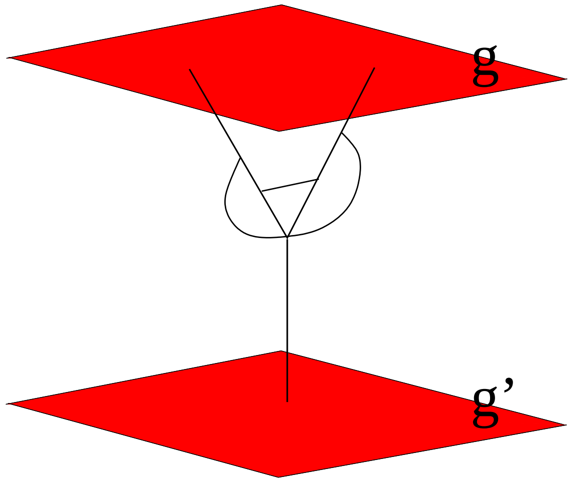
[7] Freidel et al.

IDEA: make topology dynamical!

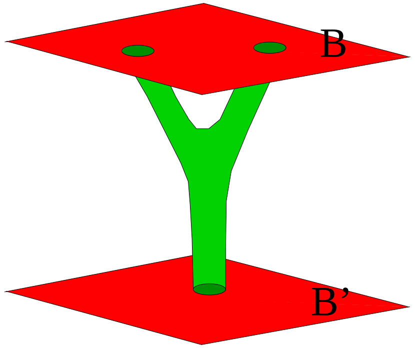
$\langle g', 1 | g, 2 \rangle =$ 

$+ \cdots +$ 

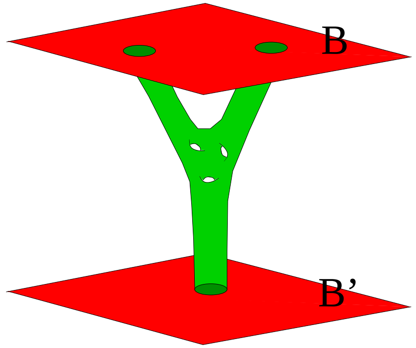
The sum does not converge. The number of different diagrams grows too fast with the number of interactions. Moreover, one can have interactions of any type (e.g. think ϕ^n, for arbitrary n).

With strings would be better ...

$$\langle B'1|B,2\rangle =$$



$$+\cdots+$$



The sum can converge (e.g. 2d BF theory[8]). Number of inequivalent diagrams grow with genus of worldsheet. Unique interaction vertex.

They are natural objects in four dimensions

Due to the fact that strings are extended, we might be able to include more interesting internal degrees of freedom preserving topological nature.

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[8] Livine-Rovelli-AP, Buzzi-AP in preparation.

Strings as conical singularities in BF theory

Baez-AP gr-qc/0605087, to appear in ATMP.,
and
work in progress in collaboration with W. Fairbairn

The point particle in 2+1 gravity[9]

Given $\mathfrak{su}(1,1)$-valued functions q and p, the Sousa Gerbert action is

$$S(A,e,q,p) = \int_{\mathscr{M}} \mathrm{tr}[e \wedge F(A)] + \int_{\gamma} \mathrm{tr}[(e + d_A q)\, p].$$

Gauge Symmetries	$e \mapsto geg^{-1}$ $A \mapsto gAg^{-1} + gdg^{-1}$	$q \mapsto gqg^{-1}$ $p \mapsto gpg^{-1}$	$e \mapsto e + d_A \eta$ $q \mapsto q - \eta$

with $g \in SO(2,1)$ and $\eta = \mathfrak{g}$-valued 1-form.

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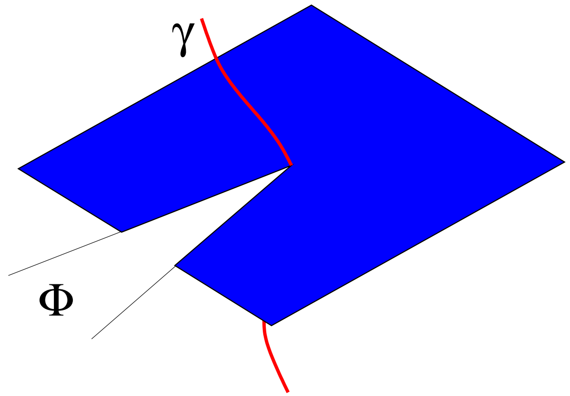
[9] Noui-AP, Freidel-Louapre, Barrett, Krasnov

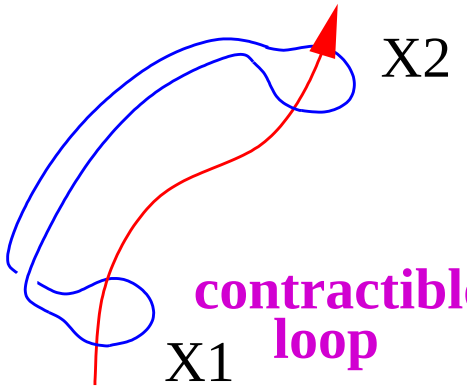
PARTICULES AS CONICAL SINGULARITIES

The equations of motion

$$F(A) = p\ \delta_\gamma$$
$$d_A e = [q, p]\ \delta_\gamma$$
$$d_A p|_\gamma = 0,\ (e + d_A q)|_\gamma = 0$$

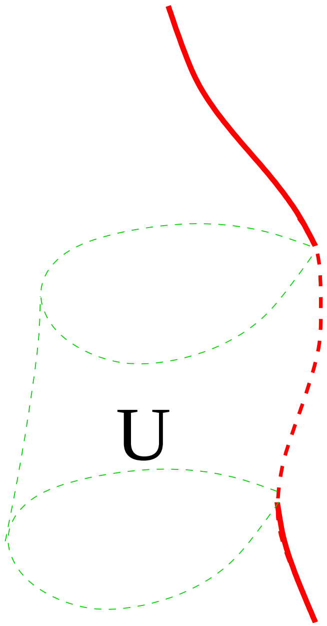


The momentum remains in the same conj−class



$\Longrightarrow p = mv$ with $v \cdot v = -1$

Geometric interpretation (flat-background gauge): Use coordinates $X : U \subset \mathscr{M} \to \mathbb{R}^3 \approx \mathfrak{su}(1,1)$ valid in U.



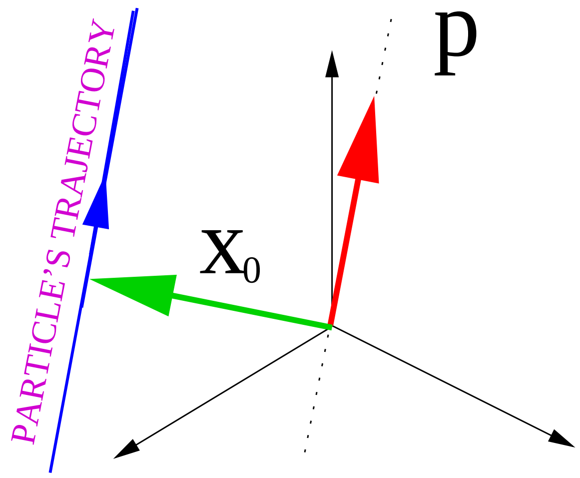
Locally all solutions are gauge equivalent.
$A = 0$, $e = 0$, $q = X_0$, $p = p_0$ a solution of the previous equations in an open $U \subset \mathscr{M}$

$$\begin{array}{c} e = 0, q = X_0 \\ A = 0, p = p_0 \end{array} \Rightarrow \begin{array}{c} e \mapsto e + d_A \eta \\ q \mapsto q - \eta \\ s.t. \;\; \eta^i = X^i \end{array} \Rightarrow \begin{array}{c} e^i_a = d_a X^i = \delta^i_a \\ q^i = -X^i(\tau) + X^i_0 \end{array}$$

The field $e^i_a = \delta^i_a$ provides background Minkowski metric $\eta_{ab} = e^i_a e^j_b \eta_{ij}$.

THE GEOMETRIC INTERPRETATION

Conservation of Ang. Moment.:
Integrability cond. of
$d_A e = [p, q]\delta_\gamma \Rightarrow [p, q] = constant$
$X^i(\tau) \propto p^i$

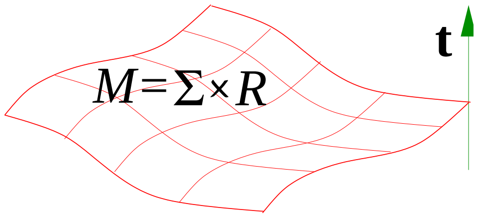


ORBITAL ANGULAR MOMENTUM IS A SOURCE OF TORSION

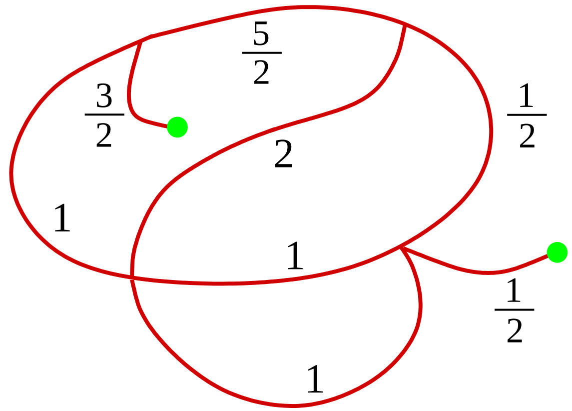
The (effective) action $$S(X_0^i, X_1^i, \tau, N) = \int_0^\tau p_i \dot{X}^i + N(p \cdot p - m^2).$$

THE PREVIOUS ACTION IS OBTAINED BY INTERPRETING THE GAUGE PARAMETERS X^i AT THE LOCATION OF THE PARTICLE AS ITS DEGREES OF FREEDOM

QUANTIZATION



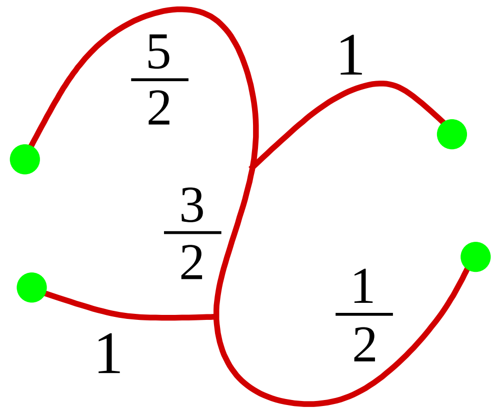
The Kinematical Hilbert Space



The Physical Hilbert Space

$2N-3$ degrees of freedom

(for $M = S^2 \times \mathbb{R}$)



The $2N-3$ quantum numbers encode the information about the relative location of the N particles.

STRINGS COUPLED TO FOUR DIMENSIONAL BF THEORY

The coupling of $(d-3)$-dimensional membranes to d-dimensional BF theory (defined for a large class of structure groups) was recently introduced.The 4d dimensional case with BF structure group $SO(4)$ (or $SO(4)$) is

$$S_{ST-BF} = \int_{\mathscr{M}} B_{IJ} \wedge F^{IJ}(A) + \int_{\mathscr{W}} (B + d_A q)^{IJ} p_{IJ},$$

where $I, J = 1, .., 4$, and if we denote $T_{IJ} \in so(4)$ the generators of the Lie algebra then $q = q^{IJ} T_{IJ}$ is a $so(4)$-valued 1-form on $\mathscr{W}$ and $p = p^{IJ} T_{IJ}$ is a $so(4)$-valued function on $\mathscr{W}$. The gauge symmetries are:

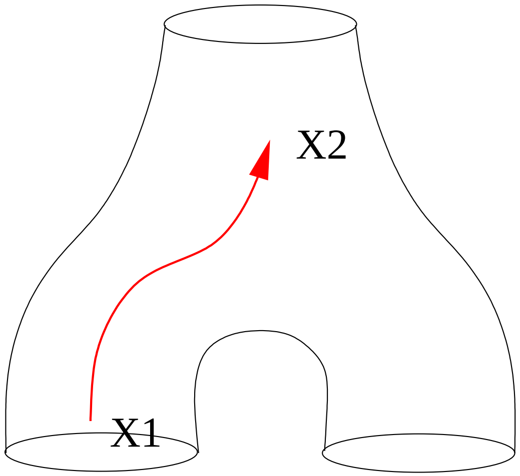
GAUGE SYMMETRIES

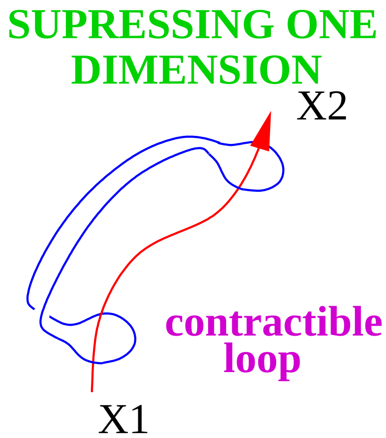
$$\begin{aligned} B &\mapsto gBg^{-1} \\ A &\mapsto gAg^{-1} + gdg^{-1} \\ q &\mapsto gqg^{-1} \\ p &\mapsto gpg^{-1}, \end{aligned} \qquad\qquad \begin{aligned} B &\mapsto B + d_A \eta \\ q &\mapsto q - \eta \end{aligned}$$

with $g \in SO(4)$ and $\eta = so(4)$-valued 1-form

EQUATIONS OF MOTION:

$$F(A) = p\ \delta_{\mathscr{W}}$$
$$d_A B = [q, p]\ \delta_{\mathscr{W}}$$
$$d_A p|_{\mathscr{W}} = 0,\ \ (B + d_A q)|_{\mathscr{W}} = 0$$

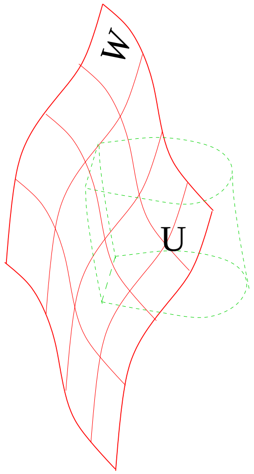




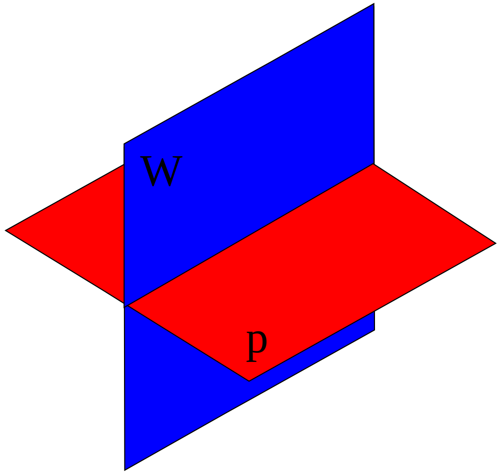
For any connected world-sheet component $p = \tau \lambda v \lambda^{-1}$ and v normalized.
The constant τ
defines the STRING TENSION.

$$\begin{matrix} B = 0, q = d\phi \\ A = 0, p = p_0 \end{matrix} \Rightarrow \begin{matrix} B \mapsto B + d_A \eta \\ q \mapsto q - \eta \\ \eta_a^{IJ} = X^{[I} d_a X^{J]} \end{matrix} \Rightarrow \begin{matrix} B_{ab}^{IJ} = e^I_{[a}\ e^J_{b]} = \delta^I_{[a} \delta^J_{b]} \\ q_a^{IJ} = X^{[I} d_a X^{J]} + d_a \phi^{IJ} \end{matrix}$$

LOCALLY PLANAR WORLD-SHEETS



$$B = 0, q = d\phi \qquad\qquad B \mapsto B + d_A\eta \qquad\qquad B^{IJ}_{ab} = e^I_{[a}\, e^J_{b]} = \delta^I_{[a}\delta^J_{b]}$$
$$A = 0, p = p_0 \quad \Rightarrow \quad q \mapsto q - \eta \quad \Rightarrow \quad q^{IJ}_a = X^{[I} d_a X^{J]} + d_a\phi^{IJ}$$
$$\eta^{IJ}_a = X^{[I} d_a X^{J]}$$

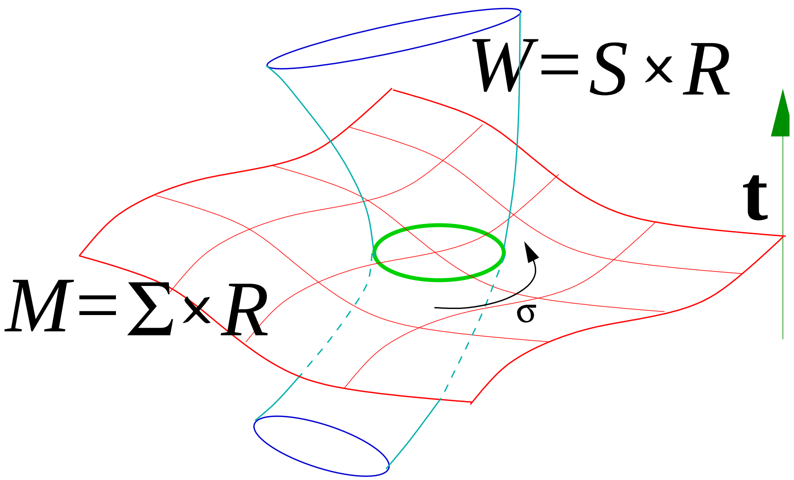


$$dB = [q, p]\delta_{\mathscr{W}} \Rightarrow d[p, q] = 0 \Rightarrow [p, q] = d\alpha,$$
$$q^{IJ}_a = X^{[I} d_a X^{J]} + d_a\phi^{IJ} \text{ s.t. } \phi = \alpha,$$
and $[XdX, p] = 0$

this implies $X^I p_{IJ} = 0$.

Can chose $\phi^{IJ} = C^{[I} X^{J]}$ to translate plane

CANONICAL STRUCTURE:



CONSTRAINTS

$$L_{IJ} := D_\mu E^\mu_{IJ} - 2\delta_{\mathscr{S}}[q_{1[I|M|}p^M{}_{J]}] \approx 0$$

$$K^{\mu\, IK} := \epsilon^{\mu\nu\rho} F^{IJ}_{\nu\rho}(x) + \delta_{\mathscr{S}}[p^{IJ}(\partial_\sigma)^\mu] \approx 0$$

$$\mathrm{tr}[T_{IJ}\lambda z\lambda^{-1}]J^{IJ} \approx 0, \text{ and } \mathrm{tr}[p\lambda z\lambda^{-1}] \approx \tau \mathrm{tr}[vz]$$

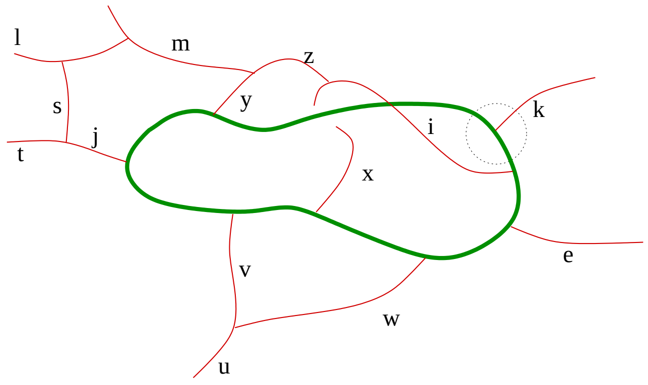
where $\delta_{\mathscr{S}}[\phi] := \int_{\mathscr{S}} \phi\ \delta^{(3)}(x - x_{\mathscr{S}}(s))$, and $[z, v] = 0$

DIRAC BRAKETS defining $J_{IJ} := [q_1, p]_{IJ}$

$$\{p_{IJ}(s), J_{KL}(s')\}_D = c^{ST}_{IJKL} p_{ST}(s)\delta^{(1)}(s - s')$$
$$\{J_{IJ}(s), J_{KL}(s')\}_D = c^{ST}_{IJKL} J_{ST}(s)\delta^{(1)}(s - s')$$
$$\{J_{IJ}(s), \lambda(s')\}_D = -T_{IJ}\lambda(s)\delta^{(1)}(s - s')$$
$$\{E^\mu_i(x), A^j_\nu(y)\}_D = \delta^\nu_\mu \delta^j_i \delta^{(3)}(x - y)$$

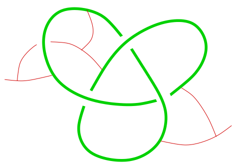
QUANTIZATION[10]

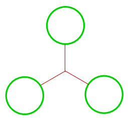
The KINEMATICAL HILBERT SPACE



The PHYSICAL HILBERT SPACE:

Knotting





Relational position

All the constraint can be implemented.

NO REGULATOR DEPENDENCE[11].

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[10] J. Baez and AP, gr-qc/0605087, to appear in ATMP.

[11] W. Fairbairn and AP, in preparation (see Winston's talk tomorrow)

Coupling world-sheet Yang-Mills fields

Work in progress in collaboration with M. Montesinos

YANG-MILLS IN 2d

$$S_{YM} = \int_{\mathscr{W}} \left[\mathcal{E}_a F^a(\mathcal{A}) + \lambda \mathcal{E}_a \mathcal{E}^a\right],$$

where $A = \left(A^a_\mu dx^\mu\right) \otimes J_a$, for $a = 1, \ldots, \dim(\mathfrak{g})$ and generators J_a such that $[J_a, J_b] = f^c{}_{ab} J_c$. The field $\mathcal{E}_a$ is a collection of $\dim(\mathfrak{g})$ many 0-forms. One can show that if λ is non degenerate (i.e., a volume form) the previous action is equivalent to the standard Yang Mills action

$$S_{YM} = \int_{\mathscr{W}} \sqrt{g} g^{\mu\nu} g^{\rho\rho} F^a_{\mu\nu} F_{\rho\rho\, a},$$

where the 2d metric $g_{\mu\nu}$ is such that $\lambda = \sqrt{g} dx^1 \wedge dx^2$.

The theory has no local excitations; however, is not background independent (e.g. The Hamiltonian is not weakly vanishing due to the presence of the (non-dynamical) background structure λ)

RESTORING BACKGROUND INDEPENDENCE IN 4d: Combine the B field and the world sheet variable p to produce a volume 2-form $\lambda = B^{IJ} p_{IJ}$ on the world sheet. The result is given by the following action:

$$S_{BFYM} \quad = \quad \int_{\mathscr{M}} B_{IJ} \wedge F^{IJ}(\omega) + \int_{\mathscr{W}} \left(\left[B^{IJ} \mathcal{E}^a \mathcal{E}_a - d_\omega q^{IJ} \right] p_{IJ} + \mathcal{E}_a F^a(A) \right)$$

GAUGE SYMM.

$$\begin{array}{ccc} B \mapsto gBg^{-1} & & \\ A \mapsto gAg^{-1} + gdg^{-1} & B \mapsto B + d_A \eta & \mathcal{A} \mapsto \alpha \mathcal{A} \alpha^{-1} + \alpha d \alpha^{-1} \\ q \mapsto gqg^{-1} & q \mapsto q + \mathcal{E}^a \mathcal{E}_a \eta & \mathcal{E} = \alpha \mathcal{E} \alpha^{-1} \\ p \mapsto gpg^{-1} & & \end{array}$$

with $g \in SO(4)$ and $\eta = so(4)$-valued 1-form, and $\alpha \in G$.

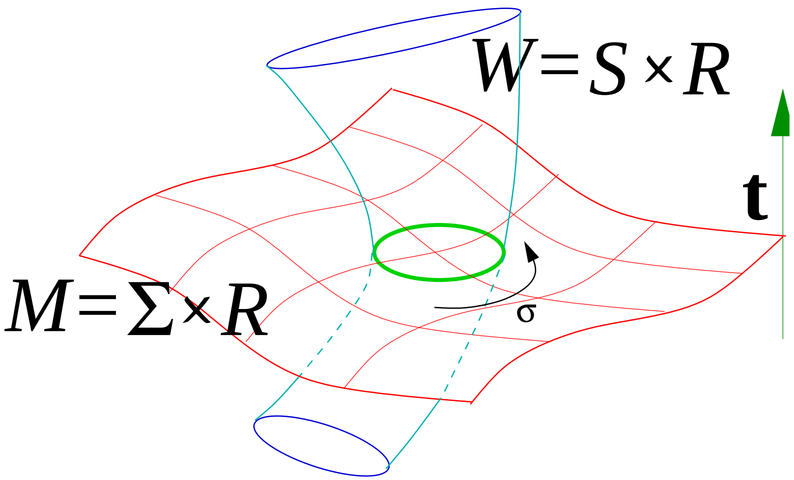
Equations of motion: Back reaction of YM on the BF bulk:

$$F(\omega) + \delta_{\mathscr{W}} [\mathcal{E}^a \mathcal{E}_a p] = 0, \qquad d_A B + \delta_{\mathscr{W}} [qp] = 0,$$

$$B \cdot p \; \mathcal{E}^a + F^a(A) = 0, \qquad d_{\mathcal{A}} \mathcal{E}^a = 0.$$

The Yang-Mills fields back react on the BF bulk: The strength of the curvature singularity is proportional to $\mathcal{E}^2$. There is a $\mathcal{E}$-field strength dependent effective string tension $\tau_{eff} = \tau \mathcal{E}^a \mathcal{E}_a$.

CANONICAL STRUCTURE:



CONSTRAINTS

$$L_{IJ} := D_\mu E^\mu_{IJ} - 2\delta_{\mathscr{S}}[q_{1[I|M|}p^M_{\ J]}] \approx 0$$

$$K^{\mu\, IK} := \epsilon^{\mu\nu\rho}F^{IJ}_{\nu\rho}(x) + \delta_{\mathscr{S}}[\mathcal{E}^a\mathcal{E}_a p^{IJ}(\partial_\sigma)^\mu] \approx 0$$

$$\mathrm{tr}[T_{IJ}\lambda z\lambda^{-1}]J^{IJ} \approx 0, \text{ and } \mathrm{tr}[p\lambda z\lambda^{-1}] \approx \tau\mathrm{tr}[vz]$$

$$d_{\mathcal{A}}\mathcal{E}^a \approx 0$$

where $\delta_{\mathscr{S}}[\phi] := \int_{\mathscr{S}} \phi\ \delta^{(3)}(x - x_{\mathscr{S}}(s))$, and $[z, v] = 0$

PROPERTIES OF THE CLASSICAL THEORY

The Gauss constraint implies $\mathcal{E}_a \mathcal{E}^a = constant$ on each string.

The constraint algebra closes with FIELD DEPENDENT structure CONSTANTS

It is easy to see that the constraint algebra closes forming a first class system of $6 + 18 + \dim(\mathfrak{g})$ first class constraints for the same number of configuration variables

$$\{q_1^{IJ}, A_\mu^{IJ}, \mathcal{A}_1^a\}.$$

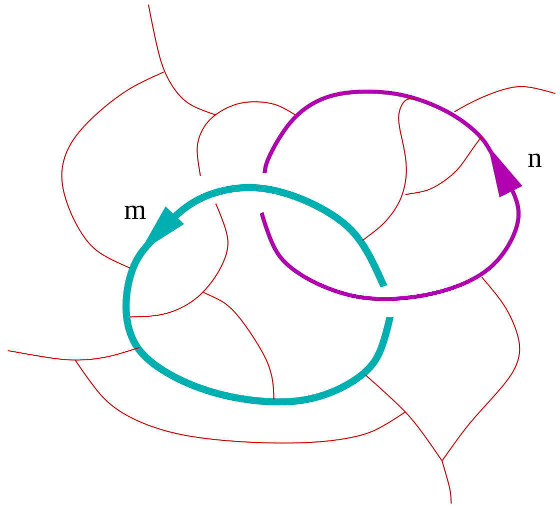
The model is genuinly TOPOLOGICAL, i.e., its background independent and there are no local degrees of freedom

The strings on Σ are flux lines of Yang-Mills electric field which back react on the environment producing a conical singularity whose strength is modulated by the Yang-Mills 'energy density'

$$\rho = \delta_{\mathscr{S}}[\mathcal{E}_a \mathcal{E}^a p_{IJ}].$$

QUANTIZATION

The KINEMATICAL HILBERT SPACE

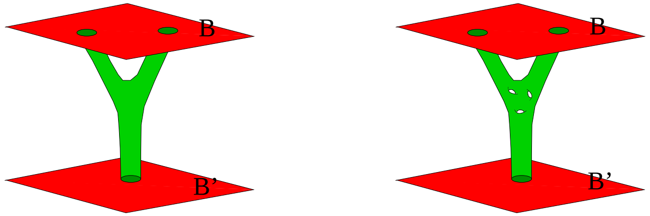


The quantum numbers $n, m \in \mathbb{N}$, where n labels the n-th eigenvalue ϵ_n of the square of the electric field $\widehat{\mathcal{E}^a \mathcal{E}_a}$.

The physical Hilbert space is obtained by requiring the holonomy of loops around the string carrying Yang-Mills quantum flux number $n \in \mathbb{N}$ to be in the conjugacy class of $\exp(-\epsilon_n v)$.

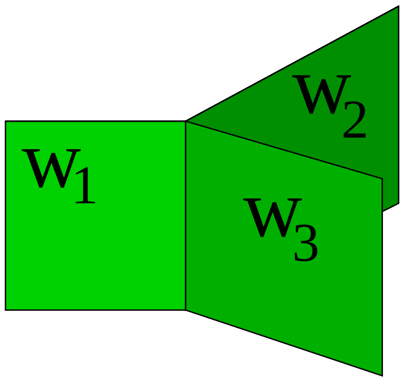
The techniques developed in (Fairbairn-AP) can be simply extended to treat this case. All the constraint can be implemented and REGULATORS can be removed without ambiguities.

We know the theory is well defined, we need to study its structure in detail (e.g. compute amplitudes for Riemann surfaces, dependence on the genus?)



World sheet amplitudes $A_{\mathscr{W}} \approx \exp\left(i\ A_p[\mathscr{W}]\epsilon_n\right)$ where $A_p[\mathscr{W}]$ is the area of the world sheet computed with the area form $(e \wedge e)^{*IJ} p_{IJ}$. This is precisely the functional dependence of the Yang-Mills amplitude in any dimension (Conrady)

World sheet intersection



$$\Rightarrow \mathcal{E}_1^a + \mathcal{E}_2^a + \mathcal{E}_3^a = 0$$

SUMMARY

- **Starting from the basic theory of conical singularities coupled to BF theory physically interesting degrees of freedom can be included in a background independent fashion (e.g. Yang-Mills fields).**
- **World-sheet Topological Models are 4d models: there are physical observables encoding the relative location of strings (in addition to knottiness).**
- **For a given space-time manifold $\mathscr{M}$ and embedded world-sheet $\mathscr{W}$ there are finitely many degrees of freedom.**
- **These models are 'RENORMALIZABLE' in the sense that there are no ambiguities in their quantization.**
- **Now we would like to explore the possibility of summing over world-sheets to construct background independent quantum field theories with 'LOCAL' degrees of freedom.**

There is a world of possible world-sheet topological models.

- **For instance, in addition to YM fields, certain kinds of particles on the world-sheet can be added. 2d BF theories, etc...**
- **Another example is the addition of a tetrad field: which bears intriguing resemblances with GR** [12]

$$
\begin{aligned}
S = & \int_{\mathscr{M}} B_{IJ} \wedge F^{IJ}(\omega) + \\
+ & \int_{\mathscr{W}} \left(\left[B^{IJ} \mathcal{E}_a \mathcal{E}^a - d_\omega q^{IJ} + *(e^I \wedge e^J) \right] p_{IJ} + \pi_I \wedge de^I + \mathcal{E}_a F^a(A) \right),
\end{aligned}
$$

On the world sheet $\epsilon^{\mu\nu\rho\tau} \epsilon_{IJKL} e^J_\nu \bar{F}^{KL}_{\rho\tau} = \epsilon^{\mu\nu} (d_A \pi_I)_\nu \mathcal{E}^2$**.**

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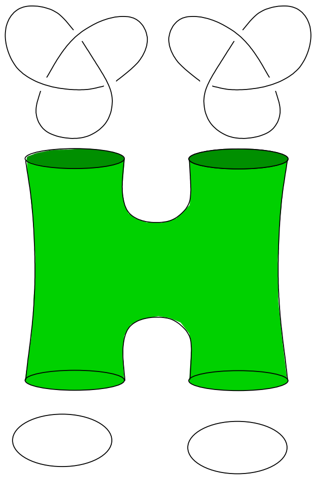
[12] See M. Montesinos and AP, to appear soon.

NEED TO STUDY QUANTUM AMPLITUDES

$$\langle 00, TT^* \rangle =$$



$$\langle 00, TT \rangle =$$