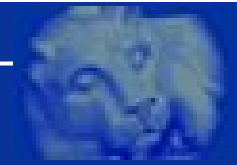




Loop quantum gravity and effective theory

Martin Bojowald

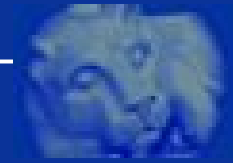
The Pennsylvania State University
Institute for Gravitational Physics and Geometry
University Park, PA



Effective equations [MB, Skrzewski]

Do not work with full states explicitly, rather with finitely many *moments* (expectation values, fluctuations, ...).

Coupled equations of motion capture *back-reaction* of spreading and deforming wave packets as *quantum corrections* to classical equations.



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Dynamical coherent states (interacting vacuum) determined *order by order* in semiclassical (or other) expansions, starting from “free” kinematical coherent states.

Avoid explicit form and representation of states (as well as inner product).

Systematic route to descend from conceptually and technically difficult “fundamental” theory to intuitive description.

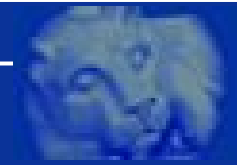


Illustration: anharmonic oscillator

Hamiltonian

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + V(\hat{q}) = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{q}^2 + \frac{1}{3}\lambda\hat{q}^3$$

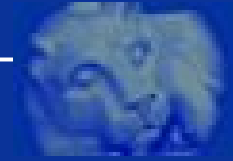


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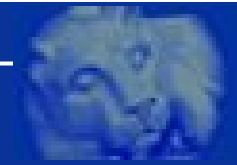
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“Classical” variables given by expectation values, equations of motion (Ehrenfest theorem)

$$\begin{aligned}\frac{d}{dt}\langle\hat{q}\rangle &= \frac{1}{i\hbar}\langle[\hat{q}, \hat{H}]\rangle = \frac{1}{m}\langle\hat{p}\rangle \\ \frac{d}{dt}\langle\hat{p}\rangle &= \frac{1}{i\hbar}\langle[\hat{p}, \hat{H}]\rangle = -m\omega^2\langle\hat{q}\rangle - \lambda\langle\hat{q}^2\rangle \\ &= -m\omega^2\langle\hat{q}\rangle - \lambda\langle\hat{q}\rangle^2 - \lambda(\Delta q)^2 \\ &= -V'(\langle\hat{q}\rangle) - \lambda(\Delta q)^2\end{aligned}$$

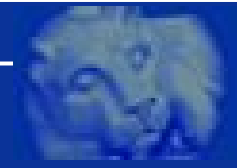
couple expectation values to fluctuation $(\Delta q)^2 = \langle(\hat{q} - \langle\hat{q}\rangle)^2\rangle$,
requiring quantum correction.



Dynamical fluctuations

Fluctuations are themselves dynamical:

$$\begin{aligned}\frac{d}{dt}(\Delta q)^2 &= \frac{d}{dt}(\langle \hat{q}^2 \rangle - \langle \hat{q} \rangle^2) = \frac{1}{i\hbar} \langle [\hat{q}^2, \hat{H}] \rangle - 2\langle \hat{q} \rangle \frac{d}{dt} \langle \hat{q} \rangle \\ &= \frac{1}{m} \langle \hat{q}\hat{p} + \hat{p}\hat{q} \rangle - \frac{2}{m} \langle \hat{q} \rangle \langle \hat{p} \rangle = \frac{2}{m} C_{qp}\end{aligned}$$



Dynamical fluctuations

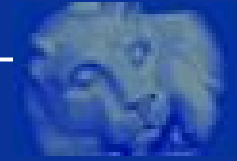
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Requires covariance $C_{qp} = \frac{1}{2} \langle \hat{q}\hat{p} + \hat{p}\hat{q} \rangle - \langle \hat{q} \rangle \langle \hat{p} \rangle$, evolving as

$$\frac{d}{dt} C_{qp} = \frac{1}{m} C_{qp} + m\omega^2 (\Delta q)^2 + 6\lambda \langle \hat{q} \rangle (\Delta q)^2 + 3\lambda G^{0,3}$$

with higher moment $G^{0,3} = \langle (\hat{q} - \langle \hat{q} \rangle)^3 \rangle = \langle \hat{q}^3 \rangle - 3\langle \hat{q} \rangle (\Delta q)^2 - \langle \hat{q} \rangle^3$ of third order (skewness).



Quantum variables

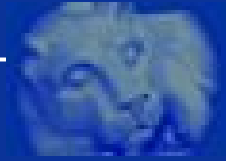
Iteration shows that all infinitely many *quantum variables*

$$G^{a,n} := \left\langle \left((\hat{q} - \langle \hat{q} \rangle_\psi)^{n-a} (\hat{p} - \langle \hat{p} \rangle_\psi)^a \right)_{\text{symm}} \right\rangle_\psi$$

of a state $|\psi\rangle$ are coupled to each other and to expectation values. Whole system of *infinitely many ordinary differential equations* is equivalent to the partial Schrödinger equation.



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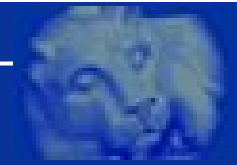
of a state $|\psi\rangle$ are coupled to each other and to expectation values. Whole system of *infinitely many ordinary differential equations* is equivalent to the partial Schrödinger equation.

Effective equations involve only finitely many local degrees of freedom. If $(\Delta q)(q, p)$ is known, inserting it into

$$\frac{d}{dt} \langle \hat{p} \rangle = -V'(\langle \hat{q} \rangle) - \lambda(\Delta q)^2$$

results in effective equations for $q = \langle \hat{q} \rangle$ and $p = \langle \hat{p} \rangle$.

For perturbative potentials around the harmonic oscillator, an adiabatic and semiclassical approximation decouples the equations and allows one to compute Δq order by order.

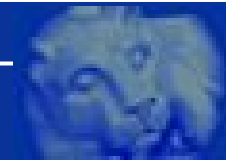


Low energy effective action

To first order in \hbar and second in adiabatic approximation, formulated as a second order equation for q :

$$\left(m + \frac{\hbar U'''(q)^2}{32m^2\omega^5 \left(1 + \frac{U''(q)}{m\omega^2}\right)^{\frac{5}{2}}} \right) \ddot{q} + \frac{\hbar \left(4m\omega^2 U'''(q) U''''(q) \left(1 + \frac{U''(q)}{m\omega^2}\right) - 5U'''(q)^3 \right)}{128m^3\omega^7 \left(1 + \frac{U''(q)}{m\omega^2}\right)^{\frac{7}{2}}} \dot{q}^2 + m\omega^2 q + U'(q) + \frac{\hbar U'''(q)}{4m\omega \left(1 + \frac{U''(q)}{m\omega^2}\right)^{\frac{1}{2}}} = 0$$

with general anharmonic potential $U(q)$.



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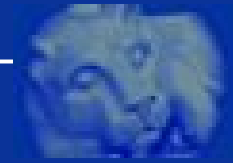
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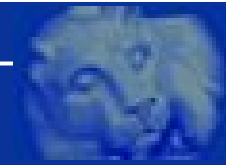
Agrees with *1-particle irreducible low energy effective action*

$$\Gamma_{\text{eff}}[q] = \int dt \left(\left(m + \frac{\hbar (U''')^2}{32m^2 \left(\omega^2 + \frac{U''}{m} \right)^{\frac{5}{2}}} \right) \frac{\dot{q}^2}{2} - \frac{1}{2} m\omega^2 q^2 - U - \frac{\hbar\omega}{2} \left(1 + \frac{U''}{m\omega^2} \right)^{\frac{1}{2}} \right)$$



Properties

- *General states* can be used through initial values of quantum variables, not tied to vacuum state. (Adiabatic regimes may not exist for any choice.)
- Generalizable to constrained systems, *physical inner product implemented through reality conditions* for classical type functions without explicit integral (or other) form for normalization of states.
- (Off-shell) *anomaly issue* addressed order by order e.g. for cosmological perturbations.
- *Graph-changing operators* seem tractable. Moments and correlations to other edges arise only once new edge degrees of freedom are excited. Keep finite reservoir: include correlations between all edges, but have only finitely many present in Hamiltonian. (Difficult in edge-wise coherent states.)

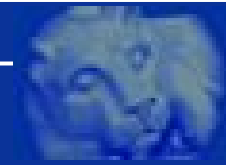


Solvable cosmological model [gr-qc/0608100]

Friedmann equation $c^2 \sqrt{p} = \frac{1}{2} p^{-3/2} p_\phi^2$ provides loop Hamiltonian $\hat{p}_\phi = \hat{H} = |\widehat{p \sin c}|$ for evolution in ϕ .

Describes whole class of quantizations: apply canonical transformation from (c, p) to $(p^{1-k} c, p^k)$. Examples: “ μ_0 ” for $k = 0$, “ $\bar{\mu}$ ” for $k = 3/2$ as limiting cases of range $0 < k < 3/2$ parameterizing behavior of inhomogeneous constraint operator.

Large- H description of *loop quantum cosmology constraint*.



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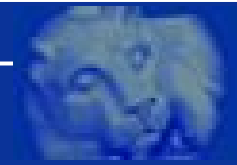
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Large- H description of *loop quantum cosmology constraint*.

Procedure: Possible to solve for wave function [see also Ashtekar, Corichi, Singh], but *solvable* in much stronger sense. Moments decouple from expectation values; *no quantum back-reaction*.

Then, much more *efficient* to solve for expectation values/moments directly; road to *effective equations in perturbation theory* for more general models.

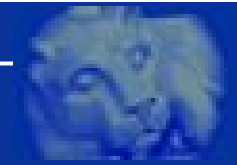


Equations of motion

For solvability, introduce $\hat{J} = \hat{p}e^{ic}$: *linear* $\hat{H} = -\frac{1}{2}i(\hat{J} - \hat{J}^\dagger)$.
(Absolute value not relevant for large- H solutions.)

Solvable “free” system, but non-canonical variables (\hat{p}, \hat{J}) ,
centrally extended $\mathfrak{sl}(2, \mathbb{R})$ *algebra*

$$[\hat{p}, \hat{J}] = \hbar \hat{J} \quad , \quad [\hat{p}, \hat{J}^\dagger] = -\hbar \hat{J}^\dagger \quad , \quad [\hat{J}, \hat{J}^\dagger] = -2\hbar \hat{p} - \hbar^2$$



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Equations of motion for $p := \langle \hat{p} \rangle$, $J := \langle \hat{J} \rangle$

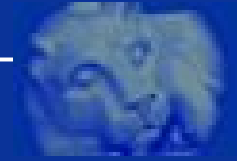
$$\dot{p} = -\frac{1}{2}(J + \bar{J}) \quad , \quad \dot{J} = -\frac{1}{2}(p + \hbar) = \dot{\bar{J}}$$

with general solution

$$p(\phi) = \frac{1}{2}(c_1 e^{-\phi} + c_2 e^{\phi}) - \frac{1}{2}\hbar$$

$$J(\phi) = \frac{1}{2}(c_1 e^{-\phi} - c_2 e^{\phi}) + iH$$

“Bounce” since $|p| \rightarrow \infty$ for $\phi \rightarrow \pm\infty$, but *could enter deep quantum regime* if $c_1 c_2 < 0$; solvable model would break down.



Reality condition

Classically we have $J\bar{J} = p^2$ for $J = p \exp(ic)$. This is related to the *physical inner product*: $\exp(ic)$ becomes *unitary operator*.

Implies $c_1 c_2 = H^2 + O(\hbar)$, *bouncing* solution ($e^{2\delta} = c_2/c_1$)

$$p(\phi) = H \cosh(\phi - \delta) - \hbar \quad , \quad J(\phi) = -H(\sinh(\phi - \delta) + i)$$



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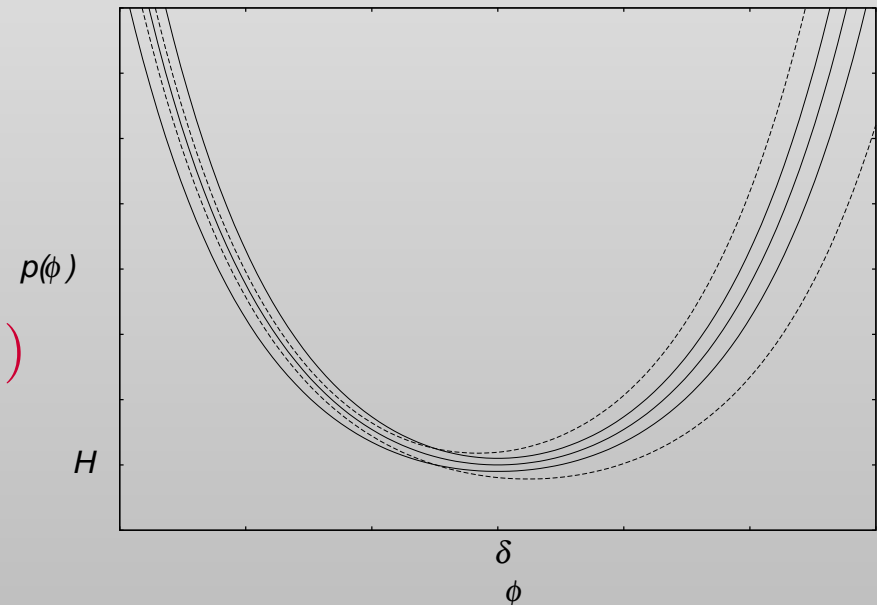
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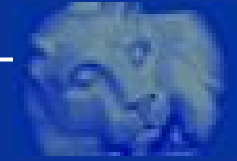
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Uncertainties: $\dot{G}^{0,2} = -2G^{1,2}$, $\dot{G}^{2,2} = -2G^{1,2}$ and

$$\dot{G}^{1,2} = -\frac{1}{2}G^{2,2} - \frac{3}{2}G^{0,2} - \frac{1}{2}(p^2 - J\bar{J} + \hbar p + \hbar^2/2)$$

For $H \gg \hbar$ solution given by
 $(\Delta p)^2 = G^{0,2} \approx \hbar H \cosh(2(\phi - \delta_2))$



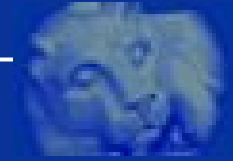


Effective equations without back-reaction

Same equations of motion for expectation values follow from *effective Hamiltonian*

$$H_{\text{eff}} = \langle \hat{H} \rangle = \frac{1}{2i} (J - \bar{J}) = p \sin c$$

Only difference to classical Hamiltonian is replacement of c by $\sin c$. *Proves validity of effective equations* in this model.



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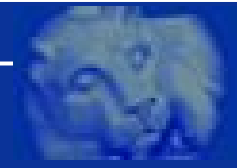
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Consequence of *solvability* in this specific factor ordering. Implies *decoupling of higher moments* rather than just closed solutions for wave functions.

Generalizing the model introduces *quantum back-reaction effects* and additional terms in effective equations. Then, just using $\sin c$ for c does not provide reliable effective equations, rather phenomenological ones.



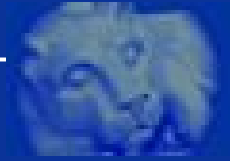
Interactions [MB, Hernández, Skrzewski]

Effective equation with non-zero potential:

$$\begin{aligned} \dot{p} = & -\frac{J + \bar{J}}{2} + \frac{J + \bar{J}}{(J - \bar{J})^2} p^3 V(\phi) \\ & + 3 \frac{p^3 (J + \bar{J})}{(J - \bar{J})^4} (G^{JJ} + G^{\bar{J}\bar{J}} - 2G^{J\bar{J}}) V(\phi) \\ & - 6 \frac{p^2 (J + \bar{J})}{(J - \bar{J})^3} (G^{pJ} - G^{p\bar{J}}) V(\phi) + 3 \frac{p (J + \bar{J})}{(J - \bar{J})^2} G^{pp} V(\phi) \\ & - \frac{2p^3}{(J - \bar{J})^3} (G^{JJ} - G^{\bar{J}\bar{J}}) V(\phi) + \frac{3p^2}{(J - \bar{J})^2} (G^{pJ} + G^{p\bar{J}}) V(\phi) \end{aligned}$$

together with equations for \dot{J} and $\dot{G}^{a,2}$ as *independent variables*.

No regime of adiabatic quantum variables found so far: *essential quantum degrees of freedom* in presence of interactions or anisotropy and inhomogeneity, unlike in free bounce model.



General effective equations

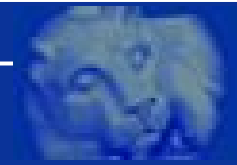
Three types of corrections in constraints of loop quantum gravity:

—→ Use of holonomies implies *higher powers of connection components*, e.g. $\sin c$ instead of c in free bounce model.

Moreover, spatial discretization effects.

—→ Genuine quantum corrections due to *back-reaction* of spreading wave packet.

—→ *Inverse powers of triad components* corrected for small triads. Ignored in solvable model, but *more important in inhomogeneous situations*.



General effective equations

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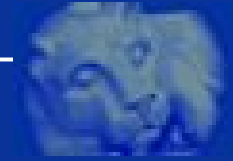
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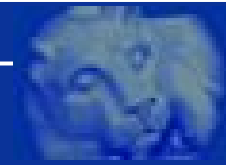
Inhomogeneous constraints give rise to anomaly issue in presence of quantum corrections.



Anomalies

Effective constraints can be computed without paying attention to constraint algebra. *Anomaly freedom* implementable subsequently, order by order e.g. in perturbative inhomogeneities.

Test if specific quantum corrections can be compatible with *covariance* and how strongly *ambiguities are restricted*.



Anomalies

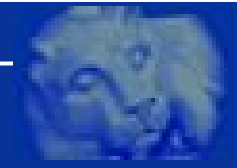
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Non-trivial quantum corrections possible for *anomaly-free effective constraints*. Results in well-defined set of equations in terms of gauge invariant observables.

[More in Mikhail Kagan's talk this afternoon]

[Work in collaboration with Hossain, Kagan, Mulryne, Nunes, Shankaranarayanan]



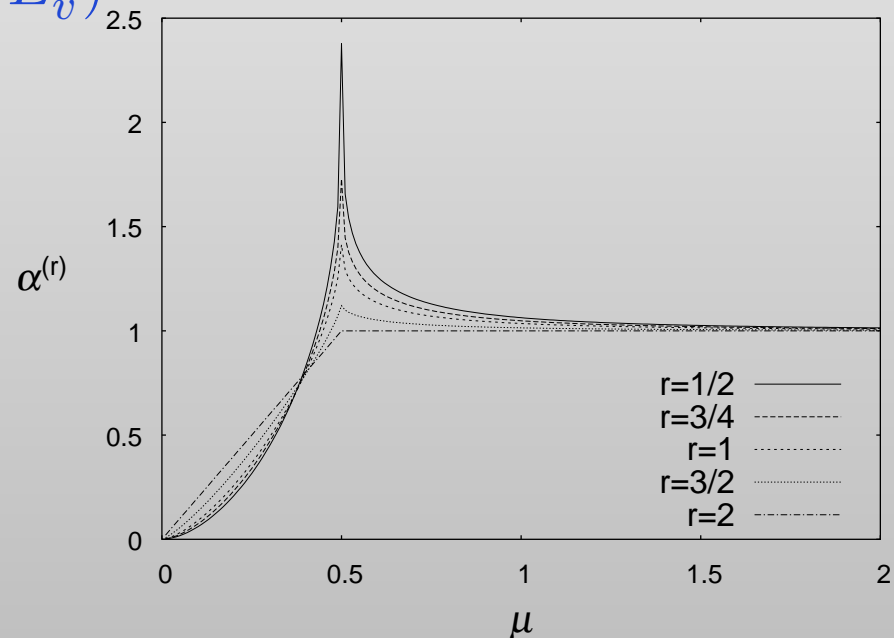
Example: scalar cosmological modes

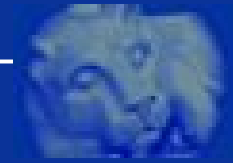
Inverse power corrections computed for *diagonal metric perturbations* (scalar mode $E_i^a = E\delta_i^a$ in longitudinal gauge).
Correction factor from Thiemann's quantization in terms of flux
 $E_v = \int_{S_v} d^2y E(y)$:

$$\alpha(E_v) = (2\pi\gamma\ell_P^2)^{-1} \sqrt{|E_v|} \left(\sqrt{|E_v + 2\pi\gamma\ell_P^2|} - \sqrt{|E_v - 2\pi\gamma\ell_P^2|} \right)$$

$$= 1 + \frac{1}{4}\pi\gamma\ell_P^4/E_v^2 + O(\ell_P^8/E_v^4)$$

More complicated for off-diagonal perturbation which requires non-Abelian calculations.





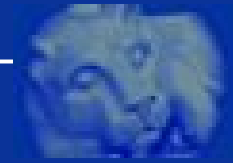
Anomaly cancellation

Insert correction functions in effective constraints, e.g.

$$\int d^3x N \frac{(\epsilon_{ijk} F_{ab}^i - 2(1 + \gamma^2) K_{[a}^j K_{b]}^k) E_j^a E_k^b}{\sqrt{|\det(E_l^c)|}} \alpha(E_i^a)$$

and compute Poisson brackets with diffeomorphism constraint.

Absence of anomalies relates dependence of correction functions on (known) diagonal and (unknown) off-diagonal metric modes.



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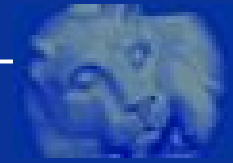
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Provides *conjectures* to be tested by direct computations of full function $\alpha(E_i^a)$ from $\langle E_j^a E_k^b / \sqrt{|\det(E_l^c)|} \rangle$ without mode assumption. Non-trivial *internal consistency* test whether loop effects can be compatible with covariance, and of how strongly ambiguities will be restricted.



Possible quantum effects

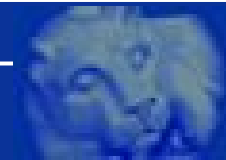
[MB, Hernández, Kagan, Singh, Skirzewski]

Use (partial) effective constraint

$$\int d^3x N \frac{(\epsilon_{ijk} F_{ab}^i - 2(1 + \gamma^2) K_{[a}^j K_{b]}^k) E_j^a E_k^b}{\sqrt{|\det(E_l^c)|}} \alpha(E)$$

specialized to scalar mode where α is known.

Constraint surface as well as gauge transformations change.



Possible quantum effects

[MB, Hernández, Kagan, Singh, Skirzewski]

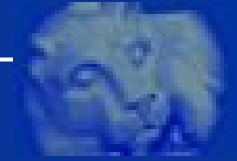
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Constraint surface as well as gauge transformations change.

$$\begin{aligned} \alpha^2 \nabla^2 \psi - 3 \frac{\dot{a}}{a} \dot{\psi} - 3 \left(1 - \frac{\alpha'}{\alpha} a^2 \right) \left(\frac{\dot{a}}{a} \right)^2 \psi &= -4\pi G \alpha a^2 \delta T_0^0 \\ \ddot{\psi} + 2\psi \frac{d}{d\eta} \left(\frac{\dot{a}}{a} \right) \left(1 - \frac{\alpha'}{\alpha} a^2 \right) + 3\dot{\psi} \frac{\dot{a}}{a} \left(1 - \frac{2}{3} \frac{\alpha'}{\alpha} a^2 \right) - \frac{4}{3} \alpha \alpha' a^2 \nabla^2 \psi \\ + \psi \left(\frac{\dot{a}}{a} \right)^2 \left(1 - 5 \frac{\alpha'}{\alpha} a^2 + 4 \left(\frac{\alpha'}{\alpha} \right)^2 a^4 - 2 \frac{\alpha''}{\alpha} a^4 \right) &= 4\pi G \alpha a^2 \delta T_{(a)}^a \\ \partial_a \left(\dot{\psi} + \frac{\dot{a}}{a} \psi (1 - 2a^2 \alpha' / \alpha) \right) &= -4\pi G a^2 \delta T_a^0 \end{aligned}$$

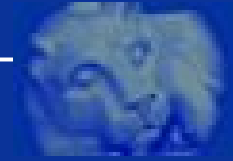


Implications

Combine to Poisson equation

$$\nabla^2 \psi - \mu(E) \psi = 4\pi G \alpha^{-1} a^2 \left(\delta\rho + 3\alpha^{-1} \frac{\dot{a}}{a} (\bar{\rho} + \bar{P}) u \right)$$

with $\mu(E) = 3\dot{a}^2 \alpha' / \alpha^3 < 0$: Small *correction to Newton potential*.



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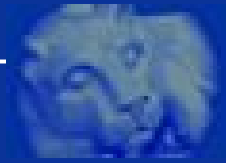
Cosmology:

$$\ddot{\psi} + 3(1 + w + \epsilon_1) \frac{\dot{a}}{a} \dot{\psi} - (w + \epsilon_2) \nabla^2 \psi + \epsilon_3 \left(\frac{\dot{a}}{a} \right)^2 \psi = 0$$

with $\epsilon_3 = -2\alpha'' a^4 / \alpha < 0$. Large scale solutions (constant w):

$\psi(\eta) = \eta^\lambda$ with $\lambda = -\frac{\nu}{2} \pm \frac{1}{2} \sqrt{\nu^2 - 4\epsilon_3}$. One solution *increasing*; only slightly but over long evolution times.

Estimate during inflation: correction factor in power up to $e^{-60\epsilon_3} \approx 1 - 10^2 \epsilon_3$ and $\epsilon_3 > 10^{-6}$. *Potentially visible*.

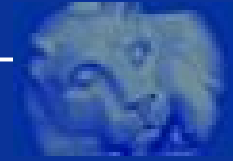


Comparison with boundary propagator

Loop quantum gravity now provides two examples for Newton potential plus corrections.

Comparison:

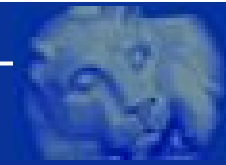
- Properties of states required, but only *finite number of parameters* such as expectation values, spread. Not as unique as low energy effective actions unless distinguished state available (“vacuum”).
- Detailed semiclassical analysis can rule out some possibilities in absence of fundamental arguments for unique (class of) state(s).
- Choices can be studied systematically by effective means.



Conclusions

Effective theories provide means to extract phenomenological information from fundamental quantum theories, focusing on few essential parameters.

Now available for *canonical quantizations*, can take into account requirements of loop quantum gravity.

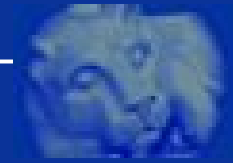


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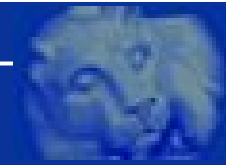
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Different approaches to the same question available, giving valuable comparisons.