Group Field Theories: spacetime from quantum discreteness to an amergent continuum

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Utrecht University

Plan of the talk

- A QFT formulation for Quantum Gravity? GFTs in brief

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- conclusions

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  $\rightarrow\; Z = \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym\Gamma} A(\Gamma)$
  $A(\Gamma) = \prod_v \int dx_v A(\Gamma, x_v) = \prod_e \int dp_e A(\Gamma, p_e)$
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- continuum regime: many quanta $\rightarrow$
  $S(\psi)\,;\; Z = \int D\psi\, e^{iS_\lambda(\psi)}$ or $H(\psi)\,;\; Z = \int D\psi\, e^{-\beta H} \Rightarrow$
  $\Rightarrow\; S_{eff}(\phi)\,;\; Z_{eff} = \int D\phi\, e^{iS_{eff}(\phi)}$ or
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  - quanta of space itself! fundamental excitations of space around the vacuum (nothing)
  - we are not starting from scratch! ideas and results from LQG, matrix models, simplicial QG,...

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- arguments of field have interpretation of geometric data (lengths, areas, gravity connection,...)

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- generalization of matrix models for 2d QG: more complicated combinatorics $+$ group-theoretic data
- both geometry and topology are dynamical

The discrete regime

The Group Field Theory formalism

L. Freidel, hep-th/0505016; D. Oriti, gr-qc/0512103;
D. Oriti, gr-qc/0607032

GFT formalism: kinematics

- consider a complex field ϕ over D copies of a group manifold G (e.g. Lorentz group, for QG): $\phi(g_1, ..., g_D) : \underbrace{G \times ... \times G}_{D} \rightarrow \mathbb{C}$

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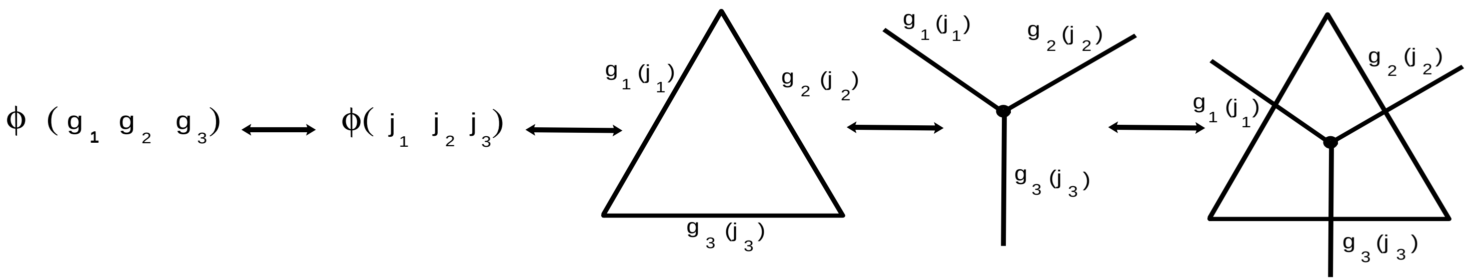

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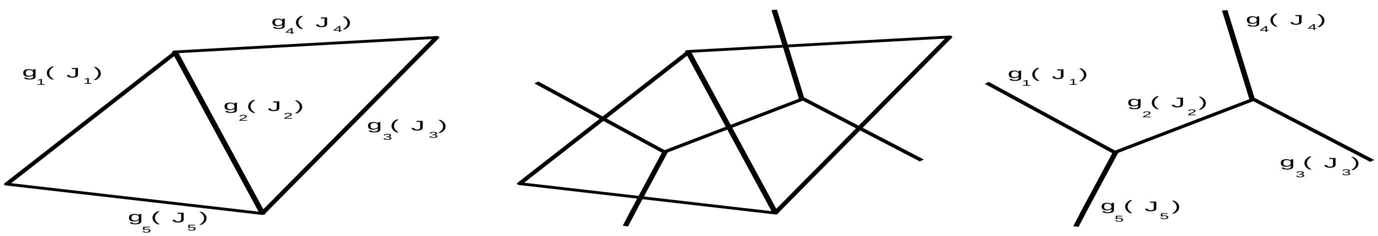
- can consider extended 'pieces'of quantum space:



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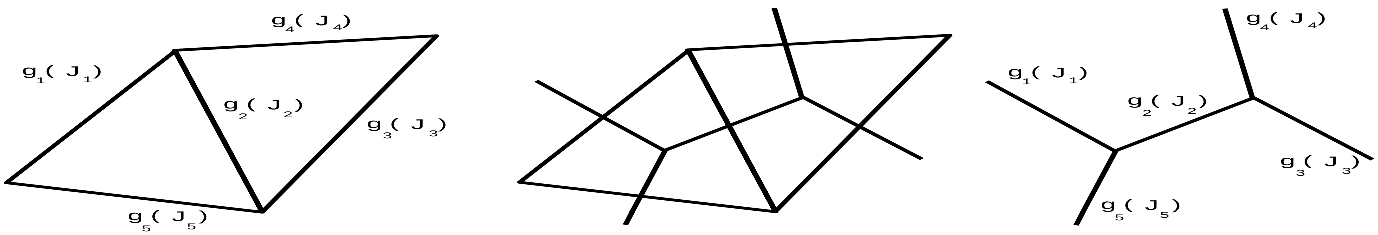

- generic state $\rightarrow$ spin network $\simeq$ simplicial (D-1)-complex, with geometric data attached

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- field action: $S_D(\phi,\lambda)=\frac{1}{2}\prod_{i=1,..,D}\int dg_i d\tilde{g}_i \phi(g_i)\mathcal{K}(g_i\tilde{g}_i^{-1})\phi(\tilde{g}_i)+$
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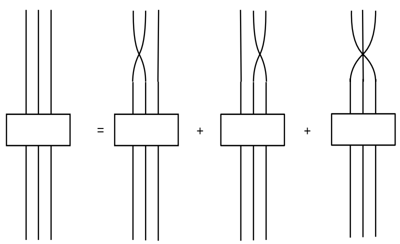
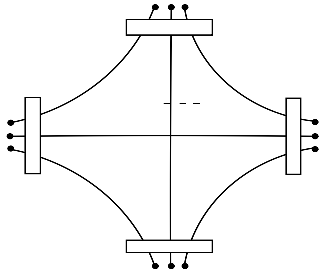
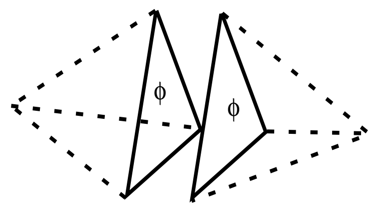
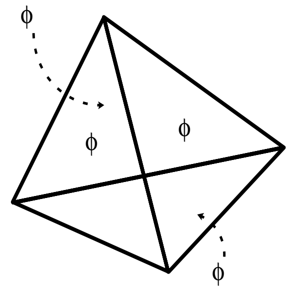
- 



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- no extensive study of classical dynamics carried out yet

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  - a) lines of propagation, with D labelled strands (dual to(D-1)-simplices),

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GFT formalism: quantum dynamics

- the quantum theory is defined by the partition function, in terms of its Feynman expansion:

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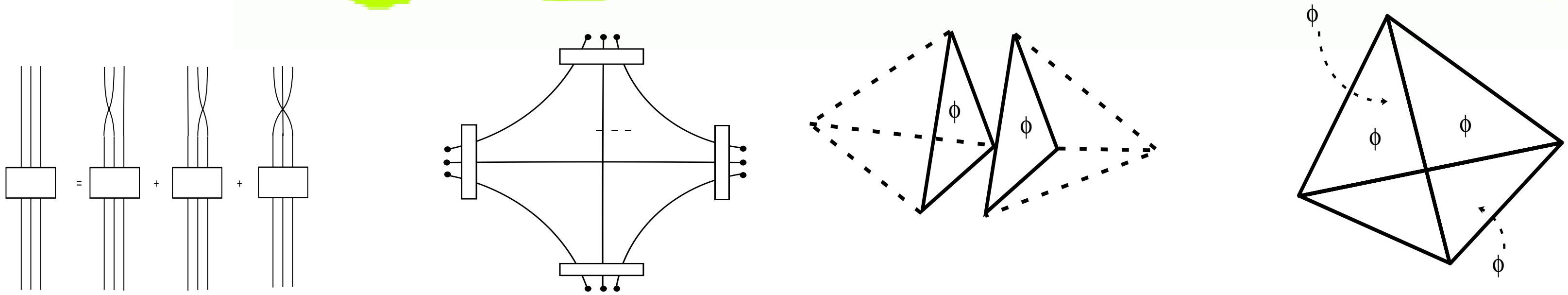
- this produces: c) 2-cells, identified by strands of propagation passing through several vertices, and then closing (for closed FD), dual to (D-2)-simplices; d) 'bubbles': 3-cells bounded by the above 2-cells

GFT formalism: quantum dynamics



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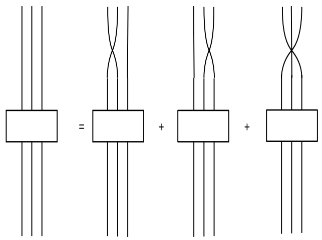
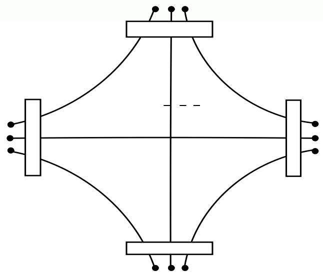
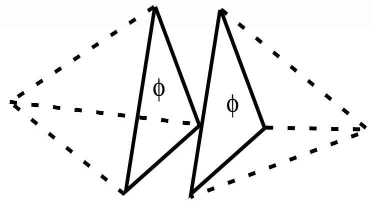
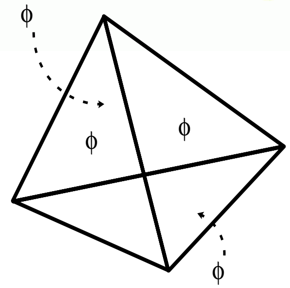
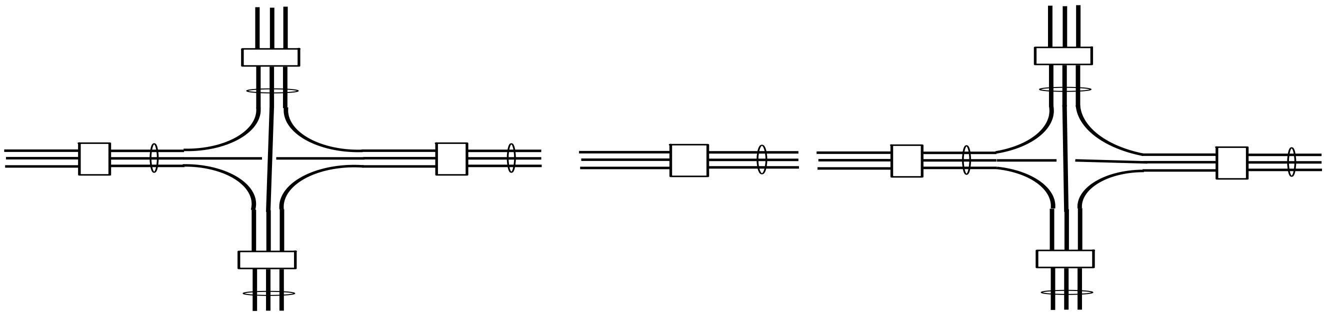
- 





- 

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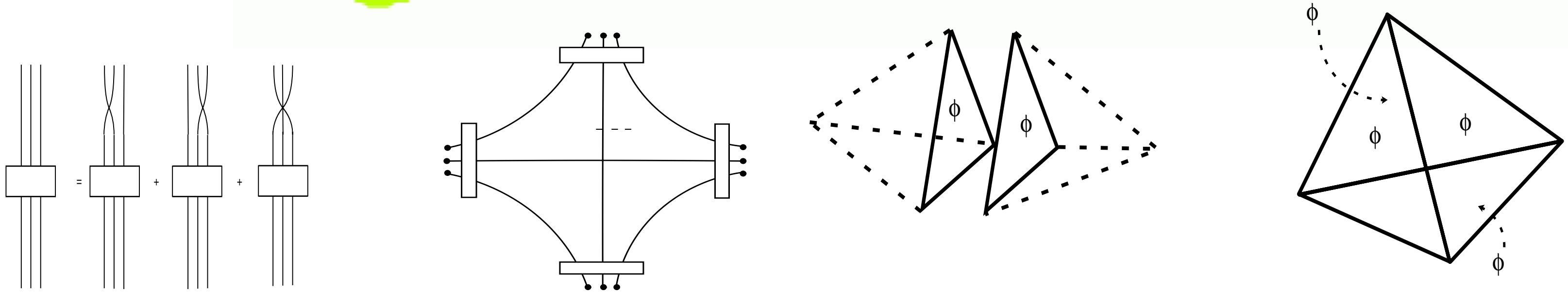
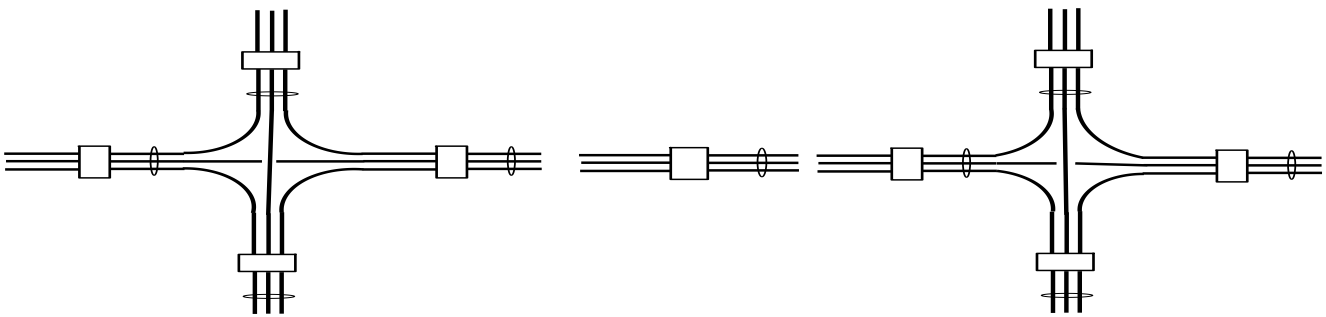
- 

- 

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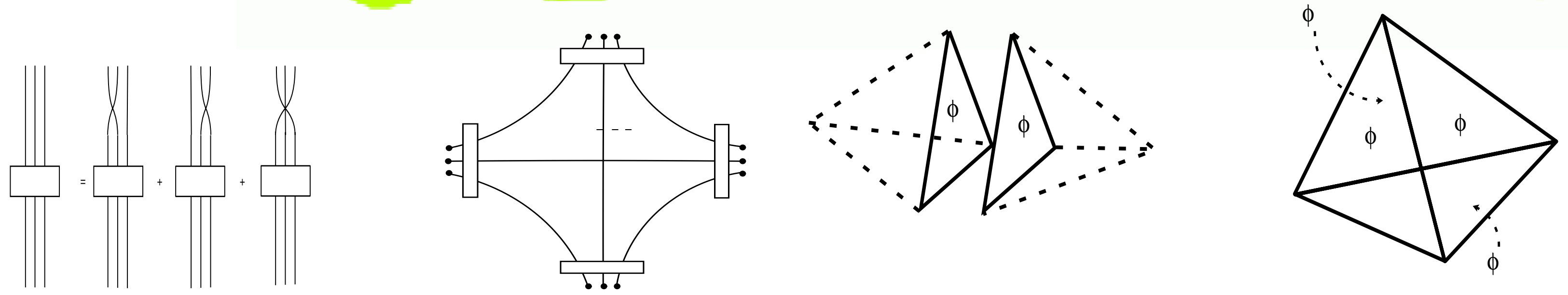

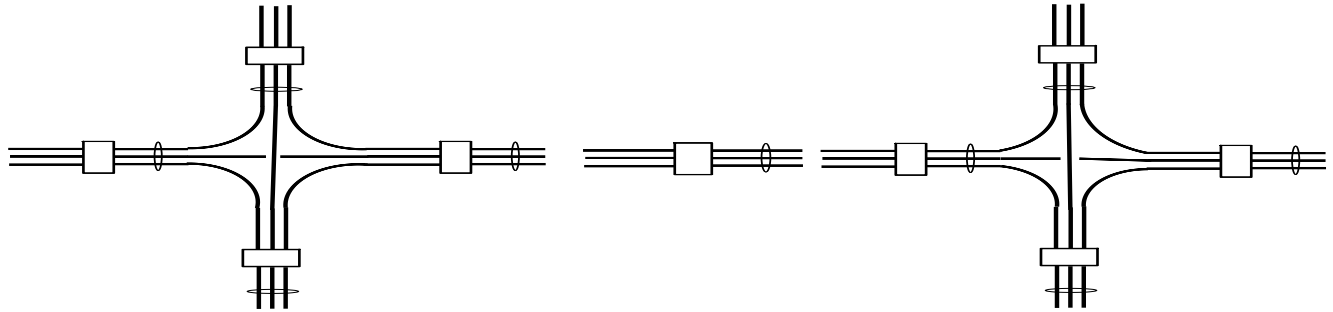

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- Quantum Gravity formulated as a sum over simplicial complexes of all topologies, as interaction processes

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- same result from path integral quantization of 3d Riemannian gravity on triangulation Δ dual to Γ

$$S_{\mathcal{M}}(e, \omega) = \int_{\mathcal{M}} tr(e \wedge F(\omega)) \rightarrow S_{\Delta}(X_e, g_{e*}) = \sum_e tr\,(X_e G_e)$$

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- $\left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\} \propto_{j\rightarrow\infty} e^{i\, S_R(j_e)} + e^{-i\, S_R(j_e)}$,

  $S_R =$ Regge action, $j_e =$ edge lengths

GFTs as a general framework for (discrete) Quantum Gravity approaches

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  (S. Drappeau, E. Livine, D.O., in progress)

2nd quantization of spin nets

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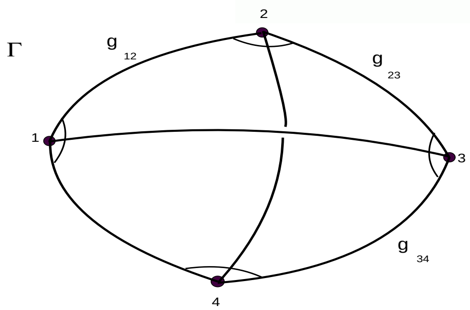

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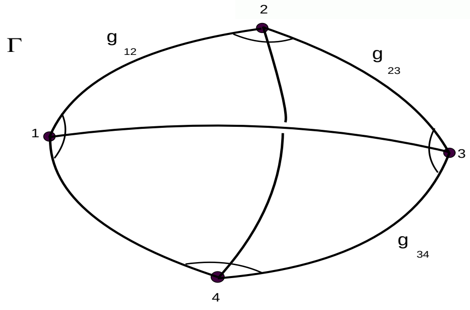

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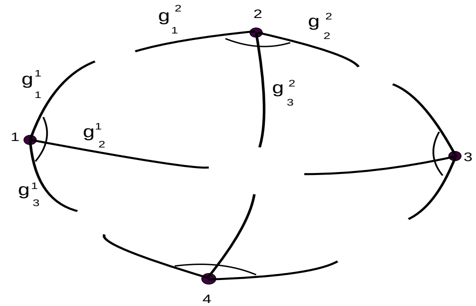

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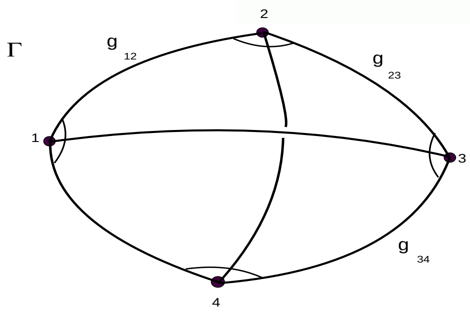

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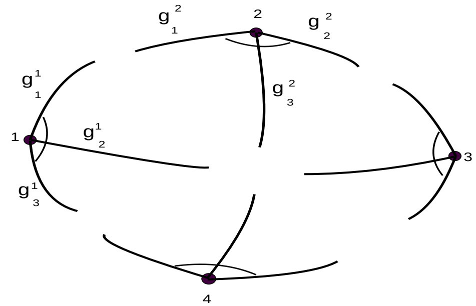

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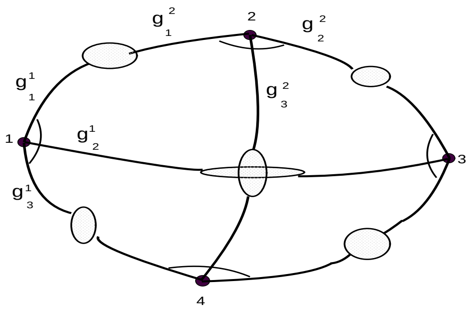

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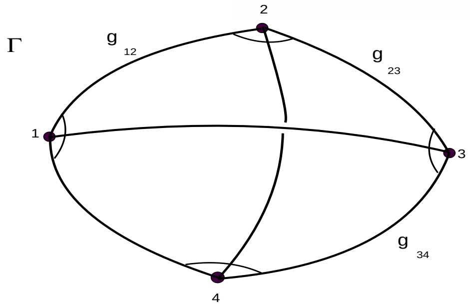

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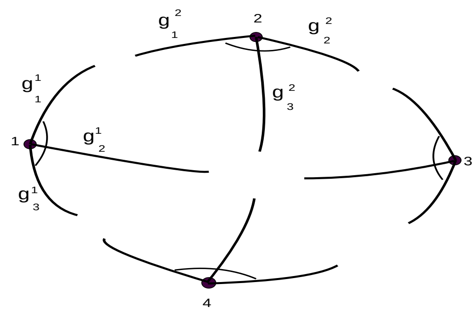

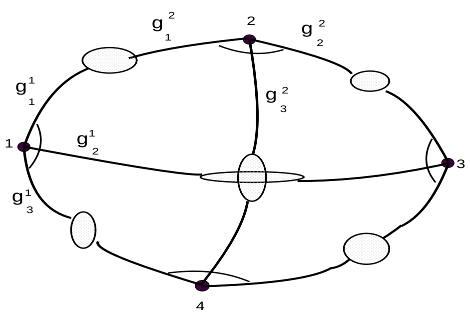

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GFT and Loop Quantum Gravity

- GFT (boundary) states are Spin Networks, based on full Lorentz group (not $SU(2)$ in general)
- GFTs are 2nd quantization of SpinNets, re-written as multi-particle states, with vertices as 'particles'
- Any operator from 1st quantization (LQG) has a 2nd quantized (GFT) counterpart
  (S. Drappeau, E. Livine, D.O., in progress)
- GFTs provide 2nd quantized dynamics: GFT classical eqns are quantum eqns of spin net functionals
- GFT perturbative expansion at tree level (trivial topology) defines physical scalar product of LQG !!! (L. Freidel)

$$\langle \Psi_1 \mid \Psi_2 \rangle_{phys} = \sum_{\Gamma | tree/\partial\Gamma = \gamma_1 \cup \gamma_2} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

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  - beyond tree level: there is much more than that in a QFT, $\longrightarrow$ there is much more in GFT than in LQG....

GFT and Simplicial Quantum Gravity

- GFT seems to incorporate formalism, insights and ideas from both main approaches to simplicial quantum gravity:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} \sum_{\{J_i\}} A_\Gamma(J_i)$$

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  - Dynamical Triangulations: freeze the geometric data, then GFT gives QG as sum over triangulations

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} \curvearrowright Z_{DT} = \sum_{\Gamma} \frac{1}{sym[\Gamma]} A_\Gamma(\lambda) \approx \text{“} \int Dg \, e^{iS_{GR}(g)} \text{”}$$

(too much included? need for further restriction to 1) trivial topology, 2) Causal Dynamical Triangulations? QFT meaning of CDT restrictions?)

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  - may be useful to introduce extra variables (not g-only or j-only) to reproduce (e,ω) or (L_e, g_{e*})

A GFT for simplicial Quantum Gravity

(D.O., T. Tlas, in preparation)

- consider the GFT, for $D = 3$, both Riemannian and Lorentzian, based on the complex field $\phi^{\alpha}(g_1, s_1; g_2, s_2; g_3, s_3) : [(G \times \mathbb{R})^{\times 3}/G] \rightarrow \mathbb{C}$, with $\phi^{\alpha=+1} = \phi\ ,\ \phi^{\alpha=-1} = \phi^{*}$ (orientation dependence)

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- usual model(s) = SUL restriction: $\prod_i (i\partial_i + \Box_i) \to \prod_i (\delta(g_i, \tilde{g}_i) \delta(s_i - \tilde{s}_i))$

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- DT restriction (fixed geometric data): $\phi(g_i, s_i)$ eigenstate of $i\partial_i$, trivial gauge invariance (D.O., in progress)

The problem of the emergence of the continuum (and of GR) from GFTs

Some current strategies (very much simplified!) seen from GFT

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- GFT translation: identify appropriate hugely populated (10^{24}? 10^{51}?) “multi-particle states”(each with its own “momentum”), that can be approximated by a continuum, characterized by a smooth “momentum field”. Study their dynamics in perturbative expansion around the vacuum, analyzing hugely complicated FD amplitudes (complexity of the FD $\simeq$ complexity of the states)

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- if continuum is many-particle physics, NO!

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  - in general, (modified) GR does not coincide with hydrodynamics, but is reproduced only in some limits
- questions from CM perspective: what are the atoms of space? what is the microscopic theory? which CM system reproduces full GR?

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- hypothesis: continuum is coherent many-particles physics for GFT atoms of space at very low temperature (hydrodynamic approx)

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- identify GFT phases for $T \rightarrow 0$, large number of quanta, for different GFT models

  microscopic details mostly irrelevant (e.g. exact form of Feynman amplitudes/SF), what is relevant: variables, symmetries, statistics, general form of interaction

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- look for geometric interpretation of hydrodynamic variables and dynamics (does it give (modified) GR?)

  GFTs contain topology change......is the effective continuum theory going to be GR on a fixed topology, or rather the classical continuum theory on superspace behind 3rd quantized gravity?

Hamiltonian analysis - Fock structure

(D.O., J. Ryan, A. Youssef, in preparation)

- consider model with kinetic term:

$$S = \int dg_i \int_{\mathbb{R}} ds_i \, \phi^\dagger(g_1, s_1; ...; g_D, s_D) \prod_i (i\partial_{s_i} + \Box_i) \, \phi(g_1, s_1; ...; g_D, s_D) + h.$$

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- basis of solutions spanned by 'plane wave' solutions of $(i\partial_2 + \Box_2)\phi = 0$ (times generic plane wave in '1' variables), $+$ 'plane wave' solutions of $(i\partial_1 + \Box_1)\phi = 0$ (times plane wave in '2' variables), plus complex conjugate

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- can define appropriate Poisson Brackets, conserved quantities, etc

Hamiltonian analysis - Fock structure

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- upon quantization, in order to preserve positivity of the Hamiltonian, it turns out that GFTs of this type have to be quantized using Bose statistics in Riemannian case, and Fermi statistics in Lorentzian case

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(D.O., J. Ryan, A. Youssef, in preparation)

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- only problem: there is plenty of work still to be done! :(