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Spherically symmetric space-times in loop quantum gravity

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Plan

- Introduction
- Spherically symmetric gravity
- Loop representation for spherical symmetry exterior.
- Dirac quantization.
- The interior problem.
- Expectations for more general models.

Introduction:

Spherically symmetric space-times include Schwarzschild and therefore the singularity.

They are the “next obvious thing” to try with loop quantum gravity after homogeneous space-times.

Work in progress, we do not have results for the complete space-time.

Models always involve a tradeoff: using special features simplifies treatment but lessens the value as lessons for the full theory.

We will carry out a Dirac quantization in the loop representation and discuss implications for other approaches.

Spherically symmetric canonical quantum gravity:

Previous work on the spherical symmetry with the traditional variables for canonical gravity: Berger, Chitre, Moncrief, Nutku (1973), Lund (1973), Unruh (1976), and the definitive work (in vacuum): Kuchar (1994).

Kuchar does not simply use symmetry-reduced variables and proceed to a Dirac quantization, but makes a careful choice of canonical variables such that the quantization is immediate and the only dynamical variable is the mass. In this sense it can be seen as a "**microsuperspace** quantization".

Such a quantization has so little in common with the full theory that we cannot learn anything about, for instance, the use of loop quantization or singularity elimination.

We would like to use less information about the model in question, i.e. just impose spherical symmetry and then proceed with the usual quantization program.

Midi-superspaces: things complicate

Suppose one considers the usual spherical (spatial) metric,

$$ds^2 = A^2(r)dr^2 + B(r)^2(r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2)$$

And a appropriately spherical conjugate momenta. One will be left with one diffeomorphism constraint C^r and a Hamiltonian constraint H. They will satisfy the usual constraint algebra,

$$\{C, C\} \approx C$$

$$\{C, H\} \approx H$$

$$\{H, H\} \approx gC$$

Remarkably, even this simple model has “the problem of dynamics” of canonical quantum gravity. **This problem is absent in homogeneous cosmologies**.

Spherical symmetry with the new variables

Previous work with the new variables, Bengtsson (1988) Kastrup and Thiemann (1993) and Bojowald and Swiderski (2005, 2006). Choose connections and triads adapted to spherical symmetry,

$$A = A_x(x)\Lambda_3 dr + (A_1(x)\Lambda_1 + A_2(x)\Lambda_2)\, d\theta + ((A_1(x)\Lambda_2 - A_2(x)\Lambda_1)\sin\theta + \Lambda_3\cos\theta)\, d\varphi,$$

$$E = E^x(x)\Lambda_3 \sin\theta \frac{\partial}{\partial x} + \left(E^1(x)\Lambda_1 + E^2(x)\Lambda_2\right)\sin\theta \frac{\partial}{\partial\theta} + \left(E^1(x)\Lambda_2 - E^2(x)\Lambda_1\right)\frac{\partial}{\partial\varphi},$$

Λ's are generators of su(2).

It simplifies the constraints if one introduces a "polar" canonical transformation in the variables $A\varphi$, P^φ, β, P^β

$$\begin{aligned} A_1 &= A_\varphi \cos\beta, \\ A_2 &= -A_\varphi \sin\beta, \\ E^\varphi &= \sqrt{(E^1)^2 + (E^2)^2}. \end{aligned}$$

$$\begin{aligned} P^\varphi &= 2E^1\cos\beta - 2E^2\sin\beta, \\ P^\beta &= -2E^1 A_\varphi \sin\beta + 2E^2 A_\varphi \cos\beta, \end{aligned}$$

To fix asymptotic problems (Bojowald, Swiderski), one does a further canonical change,

$$\begin{aligned} A_\varphi \to \bar{A}_\varphi &= 2\cos\alpha A_\varphi, \\ \beta \to \eta &= \alpha + \beta, \end{aligned}$$

$$P^\beta = P^\eta, \quad P^\varphi = 2E^\varphi \cos\alpha,$$

Leading to the canonical pairs $A_x, E^x, \overline{A_\varphi}, E^\varphi, \eta, P^\eta$.

Finally, one is left with the following form for the constraints,

$$\begin{aligned} G &= P^\eta + (E^x)' \\ D &= P^\eta \eta' + E^\varphi \bar{A}'_\varphi - (E^x)' A_x. \\ H &= -\frac{E^\varphi}{2\sqrt{|E^x|}} - \frac{A_x \bar{A}_\varphi \sqrt{|E^x|}}{2\gamma^2} - \frac{\bar{A}_\varphi^2 E^\varphi}{8\sqrt{|E^x|}\gamma^2} + \frac{((E^x)')^2}{8\sqrt{|E^x|}E^\varphi} \\ &\quad - \frac{\sqrt{|E^x|}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} - \frac{\bar{A}_\varphi \sqrt{|E^x|}\eta'}{2\gamma^2} + \frac{\sqrt{|E^x|}(E^x)''}{2E^\varphi}. \end{aligned}$$

To simplify matters further, we will fix the spatial coordinate gauge. This eliminates the diffeomorphism constraint, but still leaves a Gauss law and a Hamiltonian constraint with a first class algebra of constraints with structure functions, therefore still a challenging problem.

The choice is $E^x=(x+a)^2$, which in turn puts the horizon at $x=0$. The variable a is a dynamical variable that is related to the mass of the space-time $a=M/2$. One solves $D=0$ for A_x and substituting in H one has an equation for the pair A_φ, E^φ.

The Hamiltonian constraint becomes,

$$H=-\frac{E^{\varphi}}{(x+a)\gamma^2}\left(\frac{\bar{A}_{\varphi}^2(x+a)}{8}\right)'-\frac{E^{\varphi}}{2(x+a)}+\frac{3(x+a)}{2E^{\varphi}}+(x+a)^2\left(\frac{1}{E^{\varphi}}\right)'=0.$$

And the constraint algebra is,

$$\begin{aligned}\{H(x),H(y)\} &= \left(\frac{\bar{A}_{\varphi}(y)}{2\gamma}H(y)\right)'\delta^3(x-y)-\frac{\bar{A}_{\varphi}(y)}{\gamma}H(y)\delta^3_{,x}(x-y),\\ \{G(x),H(y)\} &= 0,\\ \{G(x),G(y)\} &= 0.\end{aligned}$$

So the model, although simplified, is still quite challenging (has structure functions in the constraint algebra). So in principle we cannot treat it with traditional techniques, we could use the “uniform discretizations” or the “master constraint” treatment.

But it turns out that for this example one can introduce a trick that allows for the traditional treatment. Dividing the constraint by E^{φ} turns the Hamiltonian constraint **Abelian!** If one wishes to discretize things to regularize expressions, one can do it in such a way that the constraints remains first class upon discretization. Then one can quantize the discrete theory in the traditional way. But first let us construct a suitable loop representation for these models.

Loop representation for the spherically symmetric case:

Manifold is a line. "Graph" is a set of edges $g = \bigcup_i e_i$. The only variable that behaves as a connnection on the line is A_x. The variables η and A_φ are scalars, so in the loop representation one uses "point holonomies" to represent them.

To avoid presenting too many equations, I will write the states for the "gauge fixed" case we introduced. There the only variables in the bulk are E^φ and $2\gamma K_\varphi = A_\varphi$

$$\mathcal{H} = L^2\left(\otimes_N R_{\text{Bohr}}, \otimes_N d\mu_0\right) \qquad T_{g,\vec{\mu}}[K_\varphi] = \prod_{v\in V(g)} \exp\left(2i\mu_v\gamma K_\varphi(v)\right)$$

$$\hat{E}^\varphi_m = -i\ell^2_{\text{Planck}}\frac{\partial}{\partial K_{\varphi,m}}, \qquad \hat{E}^\varphi_m T_{g,\vec{\mu}} = \sum_{v\in V(g)} \mu_m\gamma\ell^2_{\text{Planck}}\delta_{m,n(v)}T_{g,\vec{\mu}},$$

Volume of an interval I

$$V(I) = 4\pi\sum_{m\in I}|E^\varphi_m|(x_m + a),$$

$$\hat{V}(I)T_{g,\vec{\mu}} = \sum_{v\in I} 4\pi|\mu_v|(x_v + \hat{a})\gamma\ell^2_{\text{Planck}}T_{g,\vec{\mu}}.$$

We will see that this formula has unexpected implications.

We can introduce a basis of loop states $|g, \vec{\mu}>$,

$$\langle K^{\varphi}_m | g, \vec{\mu} \rangle = T_{g,\vec{\mu}}[K],$$

and the Bohr measure guarantees that,

$$\langle g, \vec{\mu} | g', \vec{\mu}' \rangle = \delta_{g,g'} \delta_{\vec{\mu},\vec{\mu}'} .$$

“Transverse point holonomies” and triads are well defined operators,

$$h_{\varphi}(v, \rho) \equiv \exp\left(i\rho A_{\varphi}(v)\right) = \exp\left(2i\rho\gamma K_{\varphi}(v)\right),$$

$$\hat{h}_{\varphi}(v_i, \rho)|g, \vec{\mu}> = |g, \mu_{v_1}, \ldots, \mu_{v_i} + \rho, \ldots >,$$

$$\hat{E}^{\varphi}_m |g, \vec{\mu}> = \sum_{v \in V(g)} \mu_v \gamma \ell^2_{\rm Planck} \delta_{m,n(v)} |g, \vec{\mu}> .$$

And one can do the “Thiemann trick” (calculation omitted) for the non-polynomial portion of the Hamiltonian constraint (as in the full theory and LQC), and that the inverse of the triad is a bounded operator,

$$\widehat{\frac{\mathrm{sgn}\left(E^{\varphi}_m\right)}{\sqrt{E^{\varphi}_m}}} |g, \vec{\mu}> = \frac{2}{\sqrt{\gamma}\ell_{\rm Planck}\rho} \sum_{v \in V(g)} \delta_{m,n(v)} \left(|\mu_v + \frac{\rho}{2}|^{\frac{1}{2}} - |\mu_v - \frac{\rho}{2}|^{\frac{1}{2}}\right) |g, \vec{\mu}> .$$

With this one can represent the Abelian Hamiltonian constraint in the loop representation. We start from a classical discretization that is written in terms of quantities that are easy to promote to operators in the loop representation (i.e. replace connections by "small holonomies", etc)

$$H=\left(\frac{(x+a)^3}{(E^\varphi)^2}\right)'-1-\frac{1}{4\gamma^2}\left((x+a)\bar{A}_\varphi^2\right)'.$$

$$H_m^\rho=\frac{1}{\epsilon}\left[\left(\frac{(x_m+a)^3\epsilon^2}{(E_m^\varphi)^2}-\frac{(x_{m-1}+a)^3\epsilon^2}{(E_{m-1}^\varphi)^2}\right)-\epsilon-\frac{1}{4\gamma^2\rho^2}\left((x_m+a)\sin^2\left(\rho\bar{A}_{\varphi,m}\right)-(x_{m-1}+a)\sin^2\left(\rho\bar{A}_{\varphi,m-1}\right)\right)\right]$$

It turns out that it is relatively easy to solve the constraint in the connection representation. One rewrites it as,

$$E_m^\varphi=\pm\frac{(x_m+a)\epsilon}{\sqrt{1-\frac{a}{x_m+a}+\frac{1}{4\gamma^2\rho^2}\sin^2\left(2\rho\gamma K_m^\varphi\right)}},$$

And imposing it as a quantum operator leads to states,

$$\Psi[K_m^\varphi,\tau,a]=C(\tau,a)\exp\left(\pm\frac{i}{\ell_{\text{Planck}}^2}\sum_m f[K_m^\varphi]\right),$$

And f is an explicit function of elliptic integrals.

The bottom line is that one recovers the same quantization as Kucha_, one has a wave function that depends on the mass C(a,τ), and imposing the constraint on the boundary one gets,

$$C(\tau, a) = C_0(a) \exp\left(-\frac{ia\tau}{2\ell^2_{\text{Planck}}}\right)$$

So one is left with only a function of the mass $C_0(a)$ as the wavefunction of the theory, with no dynamics.

How does the constraint look like in the loop representation? Start from classically rewriting the constraints

$$E^\varphi_m = \pm \frac{(x_m + a)\epsilon}{\sqrt{1 - \frac{a}{x_m+a} + \frac{1}{4\gamma^2\rho^2}\sin^2\left(2\rho K^\varphi_m\right)}},$$

as O_m =1. Then the quantum version of O is,

$$\hat{O}_m = \left(\sqrt{1 - \frac{a}{x_m + a} + \frac{1}{4\gamma^2\rho^2}\sin^2\left(2\rho \hat{K}^\varphi_m\right)}\,\frac{\hat{E}^\varphi_m}{(x_m + a)\epsilon}\right)^2$$

This can be immediately represented in the loop representation as,

$$\left(1-\frac{a}{x_m+a}+\frac{1}{8\gamma\rho}\right)\frac{\mu_m^2\gamma^2\ell_{\rm Planck}^4}{(x_m+a)^2\epsilon^2}\Psi\left(\mu_m\right)-\left[\frac{\mu_m^2\gamma^2\ell_{\rm Planck}^2}{(x_m+a)^2\epsilon^2}+\frac{2\mu_m\gamma\ell_{\rm Planck}^4}{(x_m+a)^2\epsilon^2}\right]\frac{\Psi\left(\mu_m+4\rho\right)}{16\gamma\rho}$$
$$-\left[\frac{\mu_m^2\gamma^2\ell_{\rm Planck}^2}{(x_m+a)^2\epsilon^2}-\frac{2\mu_m\gamma\ell_{\rm Planck}^4}{(x_m+a)^2\epsilon^2}\right]\frac{\Psi\left(\mu_m-4\rho\right)}{16\gamma\rho}=\Psi\left(\mu_m\right).$$

Notice the parallels with the expression that arises the Loop Quantum Cosmology case.

This is suggestive, since it might imply a similar resolution for the Schwarzschild singularity as one had for the cosmological one. However, detailed calculations in horizon penetrating coordinates would be needed to confirm this.

The above recursion relation can be explicitly solved and one can show that the solution is the "loop transform" of the solution we found in the connection representation,

$$\Psi_r\left(\mu_m\right)=\int_0^{\pi/(\rho\gamma)}dK^\varphi\Psi\left(K\right)\exp\left(2\rho K^\varphi\gamma\mu_m(r)\right).$$

r identifies super-selection sector.

Use of uniform discretizations:

What if we had not made use of the trick of Abelianizing the constraints? Then the only approach we know is to use the "uniform discretizations".

Briefly recalling, the uniform discretizations are defined by the following canonical transformation between instants n and n+1.

$$A_{n+1} = e^{\{\bullet, H\}}(A_n) \equiv A_n + \{A_n, H\} + \frac{1}{2}\{\{A_n, H\}, H\} + \cdots$$

Where A is any dynamical variable and H is a "Hamiltonian". It is constructed as a function of the constraints of the continuum theory. An example could be,

$$H(q,p) \ = \ 1/2 \sum_{i=1}^{N} \phi_i(q,p)^2$$

(More generally, any positive definite function of the constraints that vanishes when the constraints vanish and has non-vanishing second derivatives at the origin would do) Notice also that parallels arise with the "master constraint program".

These discretizations have desirable properties. For instance H is automatically a constant of the motion. So if we choose initial data such that $H<\varepsilon$, such statement would be preserved upon evolution.

The main challenge is to implement H as a quantum operator and checking that zero is in the spectrum. One can make use of the ambiguity of discretization to either have zero in the spectrum or to minimize the eigenvalue of the fundamental state. We use this as criterion for choosing the best discretization possible.

In this model the best discretization would be the Abelian one we presented. One could then construct H and show that zero is an eigenvalue. Since the method coincides with the Dirac method for Abelian constraints, there is no need to do this.

In order to illustrate what is expected to happen in more general models where one cannot find a vanishing eigenvalue for H, one can construct H for a different discretization and evaluate its expectation value on the eigenstates for the Abelian model. One generically gets,

$$\langle\Psi|\hat{\mathbb{H}}|\Psi\rangle = C_1\frac{\epsilon^3}{a^3} + C_2\frac{\ell^2_{\rm Planck}}{a^3} + C_3\frac{\ell^4_{\rm Planck}}{a\epsilon^3} + \sum_{n=3}^{8} C_{n+1}\frac{\ell^{2n}_{\rm Planck}}{a\epsilon^{n+1}},$$

The operator therefore does not have a quantum continuum limit, $\varepsilon \to 0$, l_{Planck} finite. On the other hand it does have a classical continuum limit, $\varepsilon \to 0$, $l_{Planck} \to 0$.

Solving the eigenvalue problem becomes a (hard) problem in quantum mechanics, akin to those in solid state physics. It is encouraging that there exist established methods to deal with these problems.

Variational Monte Carlo

From Wikipedia, the free encyclopedia

Variational Monte Carlo(VMC) is a quantum Monte Carlo method that applies the variational method to approximate the ground state of the system.

The expectation value necessary can be written in the x representation as

$$\frac{\langle\Psi(a)|H|\Psi(a)\rangle}{\langle\Psi(a)|\Psi(a)\rangle} = \frac{\int \Psi(X,a)^2 \frac{H\Psi(X,a)}{\Psi(X,a)} dX}{\int \Psi(X,a)^2 dX}.$$

Following the Monte Carlo method for evaluating integrals, we can interpret $\frac{\Psi(X,a)^2}{\int \Psi(X,a) dX}$ as a probability distribution function, sample it, and evaluate the energy expectation value $E(a)$ as the average of the local function $\frac{H\Psi(X,a)}{\Psi(X,a)}$, and minimize $E(a)$.

Although an obvious over-kill for the vacuum problem, the complexity of the situation changes little if one couples gravity to matter.

One could envisage in the near future solving the “quantum Choptuik problem” of the gravitational collapse of a scalar field or the CGHS black hole using variational Monte Carlo.

The interior problem:

The interior of Schwarzschild is isometric to a Kantowski-Sachs universe. The ansatz for the three geometry we considered is general enough to include this metric, so we can also use it for the interior. The Gauss and diffeomorphism constraints read as before,

$$
\begin{aligned}
G &= P^\eta + (E^x)' \\
D &= P^\eta \eta' + E^\varphi \bar{A}'_\varphi - (E^x)' A_x .
\end{aligned}
$$

We now rewrite things in terms of gauge invariant combinations of variables K_ϕ and $K_x=(A_x+\eta')/\gamma$ that guarantee that Gauss' law is satisfied. The diffeo and Hamiltonian constraint take the form,

$$
\begin{aligned}
D &= E^\varphi K'_\varphi - (E^x)' K_x \\
H &= -\frac{E^\varphi}{2\sqrt{|E^x|}} - 2K_x K_\varphi \sqrt{|E^x|} - \frac{K_\varphi^2 E^\varphi}{2\sqrt{|E^x|}} + \frac{((E^x)')^2}{8\sqrt{|E^x|}E^\varphi} \\
&\quad - \frac{\sqrt{|E^x|}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} + \frac{\sqrt{|E^x|}(E^x)''}{2E^\varphi} .
\end{aligned}
$$

And are first class. We now fix a gauge $(E^x)'=0$, which in turn determines the lapse $N'=0$. One is left with a super-Hamiltonian,

$$
H = \int dx N \left(\frac{E^\varphi}{2\sqrt{|E^x|}} + 2K_x K_\varphi \sqrt{|E^x|} + \frac{K_\varphi^2 E^\varphi}{2\sqrt{|E^x|}} \right) - 2\gamma N^r E^\varphi K'_\varphi
$$

And the diffeomorphism constraint becomes $E^\varphi K'_\varphi = 0$ which implies that K is independent of r and is determined by the ODE, together with the form of N that preserves that $(E^x)'=0$,

$$\dot{K}_\varphi = \{K_\varphi, H\} = \frac{1}{2}\frac{2N(1+K_\varphi^2)}{t},$$

with solution

$$K_\varphi = \sqrt{\frac{2M}{t} - 1},$$

The usual form of the Kantowski-Sachs metric arises with the additional gauge condition $(E^\varphi)' = 0,$ which implies the shift is a function of t only yielding an ODE for the triad with solution,

$$E^\varphi = \frac{\sqrt{t(2M-t)k}}{k}$$

$$g_{xx} = \frac{(E^\varphi)^2}{E^x} = \frac{2M}{t} - 1.$$

Starting from a rescaled version of the the Hamiltonian constraint we had,

$$H = \frac{E_\varphi}{2K_\varphi} + 2|E^x|K_x + \frac{1}{2}K_\varphi E_\varphi$$

We proceed to "holonomize" it (it is slightly easier to implement its square),

$$\frac{4(E^x)^2 \sin^2(\rho K_x)}{\rho^2} - \left(\frac{1}{2}\frac{\sin(\rho K_\varphi)}{\rho} + \frac{1}{2}\frac{\rho}{\sin(\rho K_\varphi)}\right)^2 E_\varphi^2 = 0$$

Quantizing, one would have have, for a symmetric factor ordering,

$$\frac{1}{2}\left[\left(\hat{E}^x\right)^2 A(K_x)^2 + A(K_x)^2\left(\hat{E}^x\right)^2 + B(K_\varphi)^2\left(\hat{E}^\varphi\right)^2 + \left(\hat{E}^\varphi\right)^2 B(K_\varphi)^2\right]\Psi\left(K_x, K_\varphi\right) = 0.$$

And one can try a solution of the form $\Psi = C\exp\left(\frac{i}{\ell^2_{\text{Planck}}} S\left(K_x, K_\varphi\right)\right),$

Assuming $C = \sum_{k=0}^\infty C_k \left(\ell^2_{\text{Planck}}\right)^k$ one is led to a solution scheme in powers of Planck's length.

To zeroth order,

$$A(K_x)^2\left(\frac{\partial S(K_x,K_\varphi)}{\partial K_x})\right)^2 - B(K_\varphi)^2\left(\frac{\partial S(K_x,K_\varphi)}{\partial K_\varphi})\right)^2 = 0$$

To first order

$$\begin{aligned} B(K_\varphi)^2\frac{\partial^2}{\partial K_\varphi^2}S(K_x,K_\varphi) + 2\frac{B(K_\varphi)^2}{C_0(K_x,K_\varphi)}\frac{\partial S(K_x,K_\varphi)}{\partial K_\varphi}\frac{\partial C_0(K_x,K_\varphi)}{\partial K_\varphi} + \frac{\partial B(K_\varphi)}{\partial K_\varphi}\frac{\partial S(K_x,K_\varphi)}{\partial K_\varphi} \\ +A(K_x)^2\frac{\partial^2 S(K_x,K_\varphi)}{\partial K_x^2} - 2\frac{A(K_x)^2}{C_0(K_x,K_\varphi)}\frac{\partial S(K_x,K_\varphi)}{\partial K_x}\frac{\partial C_0(K_x,K_\varphi)}{\partial K_x} - \frac{\partial A(K_x)}{\partial K_x}\frac{\partial S(K_x,K_\varphi)}{\partial K_x} = 0. \end{aligned}$$

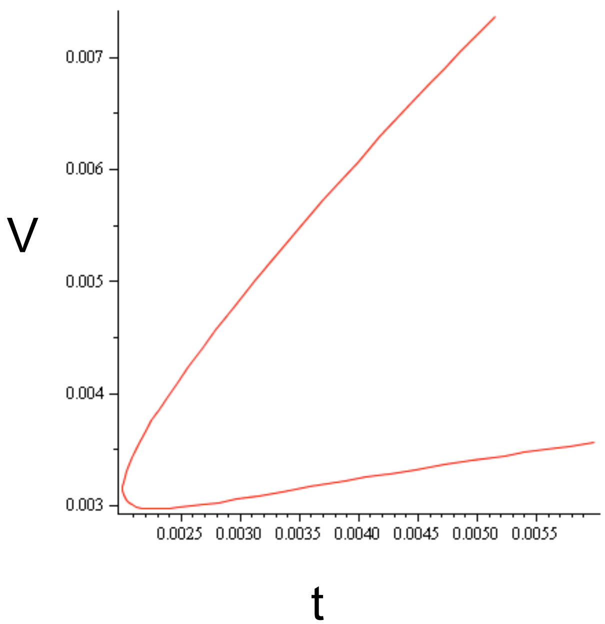
The zeroth order is (the product) of quasilinear equations for S. The first order is a quasilinear equation for C_0. Similar equations arise for the C_i's at higher orders.

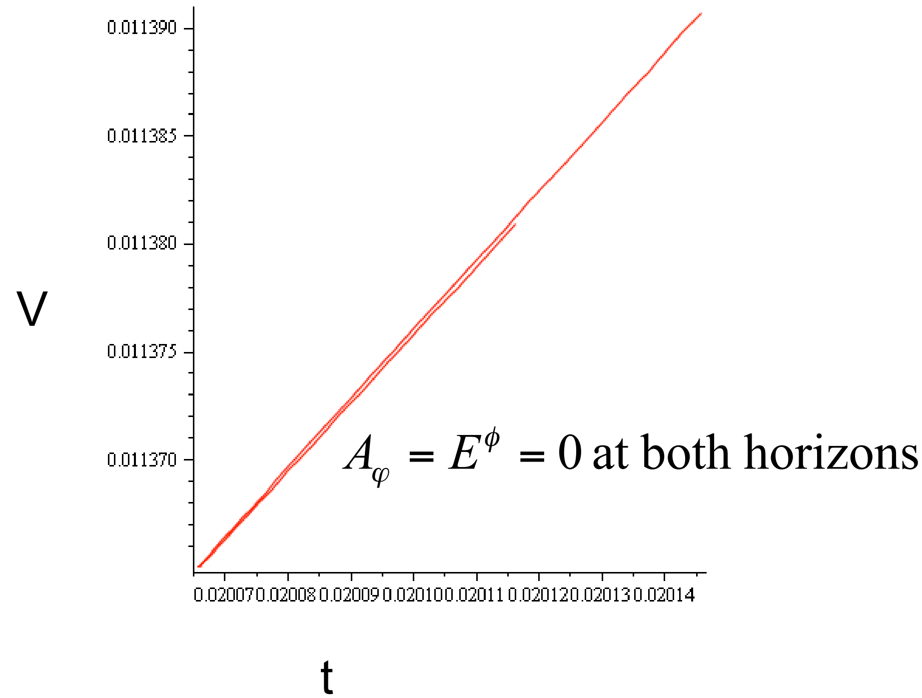
One can solve exactly for S,

$$S(K_x,K_\varphi) = \frac{k(1-\cos(\rho K_x))}{\sin(\rho K_x)\exp\left(\frac{1}{\sqrt{1+\rho^2}}\tanh^{-1}\left(\frac{\cos(\rho K_\varphi)}{\sqrt{1+\rho^2}}\right)\right)}$$

This technique can be used to provide a zeroth order approximation for the variational Monte Carlo techniques we mentioned.

It is interesting to study the classical solutions of the action we obtained. One can see that indeed they are singularity free, but they require careful tuning of boundary conditions for the tunneling through the singularity to be symmetric.





This requires further study, but it might be a sign of the instabilities that have been observed in the recursion relations that appear in the loop quantization by Cartin, Khanna, Bojowald, etc.

It would be interesting to connect this quantization with the more traditional loop approach, Ashtekar & Bojowald, Modesto, etc.

One last intriguing observation: holography?!

As in LQC, one expects the value of the parameter of the “transverse point holonomy”, ρ, to take a finite minimum value, $h_\varphi(v,\rho) \equiv \exp\left(i\rho A_\varphi(v)\right) = \exp\left(2i\rho\gamma K_\varphi(v)\right)$

If one acts with this holonomy on a spin network state, one adds an element of volume, $\Delta V = 4\pi\rho\gamma(x_v + a) l^2_{\mathrm{Planck}}$ Ng, Lloyd, etc

Therefore for any model based on this kinematical structure, the volume grows in discrete increments that take as minimum value the element of volume mentioned. This statement is independent of the details of the dynamics of the model considered (e.g. it would survive coupling the theory to a scalar field, for instance).

If one considers the volume of a shell of width Δx asymptotically one has N elements of volume per shell,

$$N = \frac{V}{\Delta V} = x\frac{\Delta x}{2\gamma\rho l^2_{\mathrm{Planck}}}, \quad \text{so for a finite shell} \quad \boxed{N \approx \frac{b^2 - a^2}{l^2_{\mathrm{Planck}}} \qquad \text{Bekenstein bound ?}}$$

Where we used the usual values $\gamma = c_A/(\pi\sqrt{3})$ with $\mathrm{c_A} \approx 1$ and $\rho = \sqrt{3}/2$

Notice that it makes sense that one obtains the (spatial) Bekenstein bound in spherical symmetry, since it is known not to hold if one is in a more general situation.

Summary:

- One can study spherically symmetric space-times using loop quantum gravity.
- One needs to use special features of spherical symmetry to apply the traditional Dirac quantization technique.
- The setup is ready, we need to extend it to horizon penetrating coordinates to make better statements about the singularity.
- Without using special tricks the problem is hard, and it offers a promising arena to test new ideas for handling the problem of dynamics in canonical quantum gravity, like the “uniform discretization” or the “master constraint” approaches.
- Holography and the Bekenstein bound can be connected with the basic elements of loop quantum gravity, irrespective of the details of the dynamics.