Imperial College London

EU ENRAGE Network
Random Geometries

Can the supercomputer provide new insights into quantum gravity?

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Theoretical Physics Group
Imperial College London

Loops '07, 30 June 2007

Plan of this Talk

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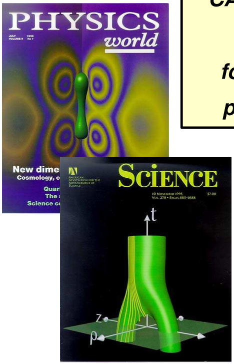
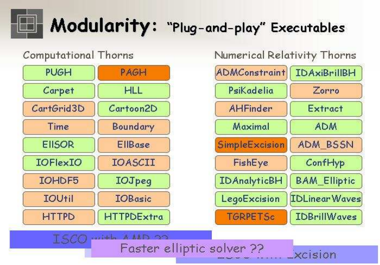
Outline

The Cactus Computational Framework

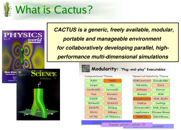
What is Cactus?



CACTUS is a generic, freely available, modular, portable and manageable environment for collaboratively developing parallel, high-performance multi-dimensional simulations



The Cactus Computational Framework



- Began as open source environment for numerical relativity
- Properly designed abstractions & interfaces
- Physics code unchanged as technologies change underneath

- **Collaboration:** passing parameter files, web interface to running codes
- **Cooperation:** inherit developments of others, including computer scientists
- **Community:** build up community code base for addressing difficult problems in QG

- abstract ‘ugly CS issues’ away from physics

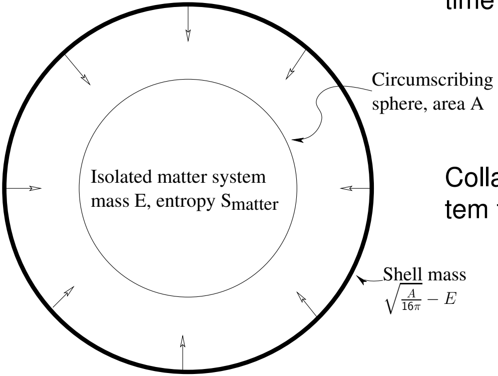
The Cactus Computational Framework

What features are useful for quantum gravity computations?

- Not just for solving PDEs on a fixed lattice...
- Modularity! $\leadsto$ collaboration & community building
  - Language independence
  - Separate computational details from physics
  - Code sharing $\rightarrow$ develop community code base
    - Leverage developments of others, e.g. students, no continual reinventing the wheel
    - Pass around parameter files
    - Works well for numerical relativity, causal set QG: Do same for LQG, spin foams, CDTs?
- Portability
- Automatic parallelism
- Automatic detection and linking to numerical libraries

Outline

Susskind Process



Matter system stable on time scale $> \sqrt{A}$

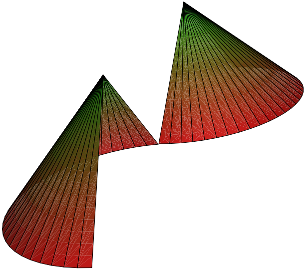
Collapse shell onto system to form black hole

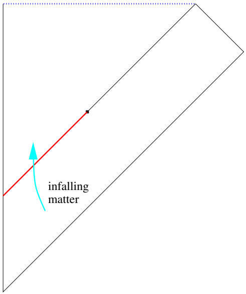
$$\left.\begin{array}{rcl} S_{\text{tot}}^{\text{initial}} & = & S_{\text{matter}} + S_{\text{shell}} \\ S_{\text{tot}}^{\text{final}} & = & S_{\text{BH}} = \frac{A}{4} \end{array}\right\} GSL \Rightarrow S_{\text{matter}} \le \frac{A}{4}$$

$\rightsquigarrow$ Susskind entropy bound

Covariant Entropy Bound

- Consider any spacelike co-dimension 2 surface B, of area A
- Consider congruence of null geodesics emanating from and orthogonal to B, *which is everywhere non-expanding*: a "light sheet"
- Covariant bound: entropy on light sheet $\leq \frac{A}{4}$





- No known violations
- Can recover other bounds under suitable conditions

“Holographic Principle”

$\rightsquigarrow$ “holographic principle”:

$\sim$ Region with boundary of area A described by no more than $\frac{A}{4}$ degrees of freedom

$\rightarrow$ $\exists$ universal relation between geometry and entropy/information

Can we see this emerge from discrete quantum gravity?

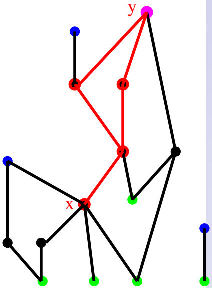
Causal Sets: Fundamentally Discrete Gravity

Based upon two main observations:

- Need for discreteness
- Richness of causal structure

Properties of discrete causal order $\prec$:

- irreflexive ($x \nprec x$)
- transitive ($x \prec y$ and $y \prec z \Rightarrow x \prec z$)
- locally finite ($|\{y|x \prec y \prec z| < \infty$)



Some definitions:

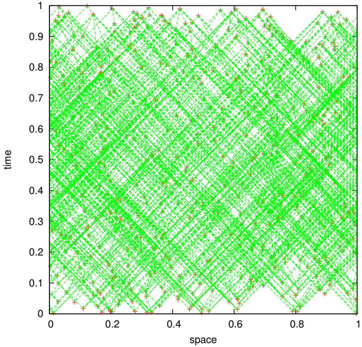
- maximal elements
- (inextendible) antichain

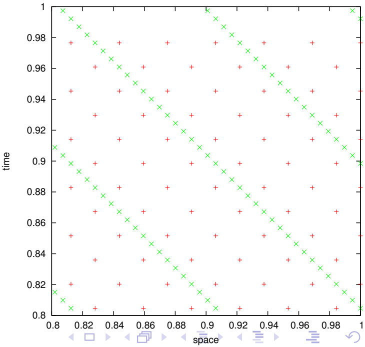
Spacetime Manifold as *Emergent* Structure

- *Embedding* – order preserving map $\phi : \mathcal{C} \to (M, g)$

$$x \prec y \Leftrightarrow \phi(x) \prec \phi(y) \;\; \forall x, y \in \mathcal{C}$$

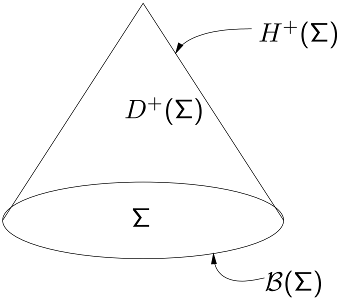
- *Faithful embedding (‘Sprinkling’)*:
  - “preserves number – volume correspondence”
  - scale of geometry change $\gg$ mean spacing of embedded points
- $\exists$ faithful embedding $\Rightarrow (M, g)$ approximates $\mathcal{C}$





An entropy bound from causal sets

- Spherically symmetric spacelike hypersurface Σ of finite volume in strongly causal spacetime of dimension $d \geq 3$.
- Causet $\mathcal{C}$ faithfully embeded into $D^+(\Sigma)$.

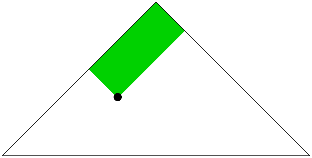


Proposal:
Maximum entropy associated to Σ given by $|\mathrm{max}(\mathcal{C})|$

Claim:
Leads to Susskind's entropy bound in continuum limit

$$\boxed{S_{\mathrm{max}} = \frac{A}{4}}$$

Calculate number of maximal elements



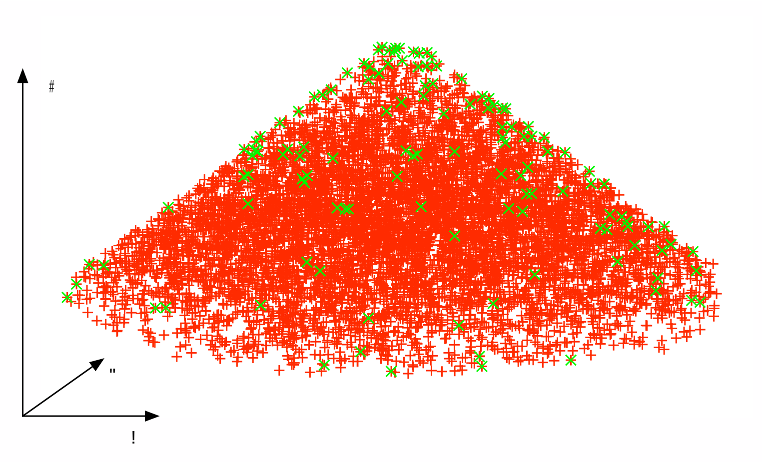
Analytically:
Use properties of Poisson distribution
In $\mathbb{M}^d$

$$< n >= \rho \int_{D^+(S)} dx^d exp\left(J^+(x) \cap D^+(S)\right)$$

Numerically:
Use CausalSets toolkit within Cactus framework:

- Sprinkle via Poisson distribition
- Deduce order relations
- Count maximal elements

Ball in 2+1 dimensional Minkowski space



Ball in 3+1 dimensional Minkowski space

Number of maximal elements:

$$\langle n \rangle = N\ {}_3F_3\left(\frac{1}{2}, 1, 1; \frac{5}{4}, \frac{7}{4}, 2; -\frac{N}{8}\right)$$

Asymptotic behavior $N \gg 1$: $\langle n \rangle = 3\sqrt{2\pi N}$
Define Poisson embedding at unit density in fundamental units:

$$\rho = \frac{N}{V} = \frac{1}{l_f^4} \Rightarrow N = \frac{V}{l_f^4}$$

Volume of cone of radius R: $V = \frac{\pi R^4}{3} \Rightarrow \langle n \rangle = \frac{\sqrt{6}}{l_f^2} \frac{4\pi R^2}{4}$

Choose a fundamental scale $l_f = \sqrt[4]{6} l_p$

Then we have: $\boxed{S_{max} = \frac{4\pi R^2}{4} = \frac{A}{4}}$

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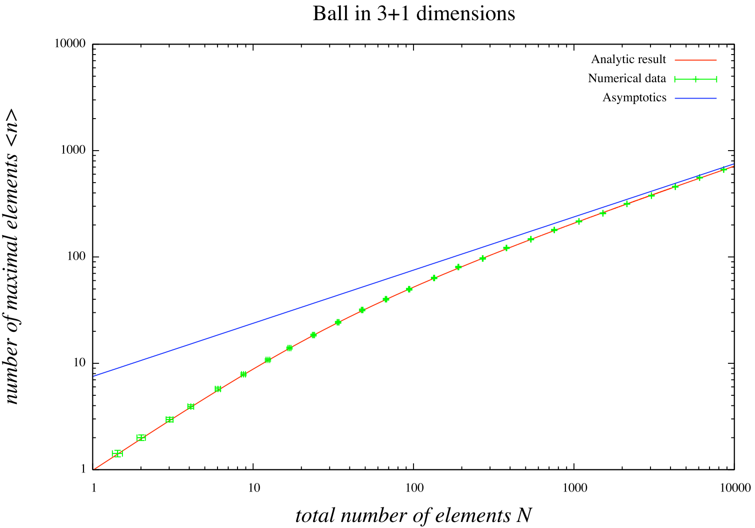
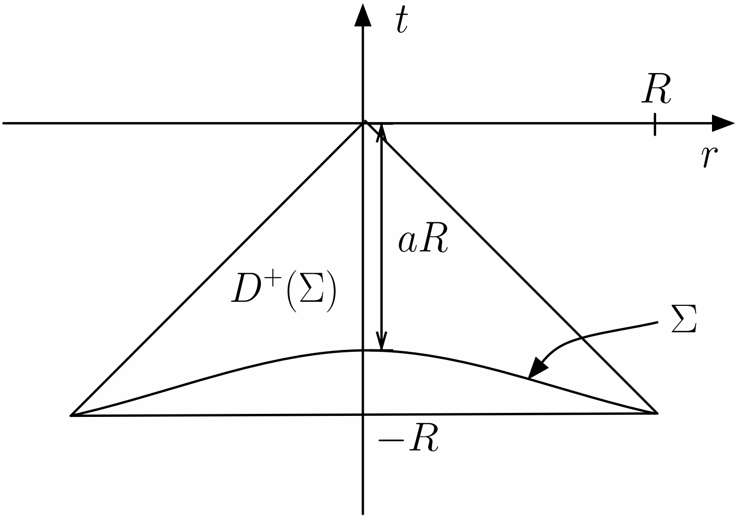


Spherically Symmetric Hyperboloidal Slices

Same boundary sphere



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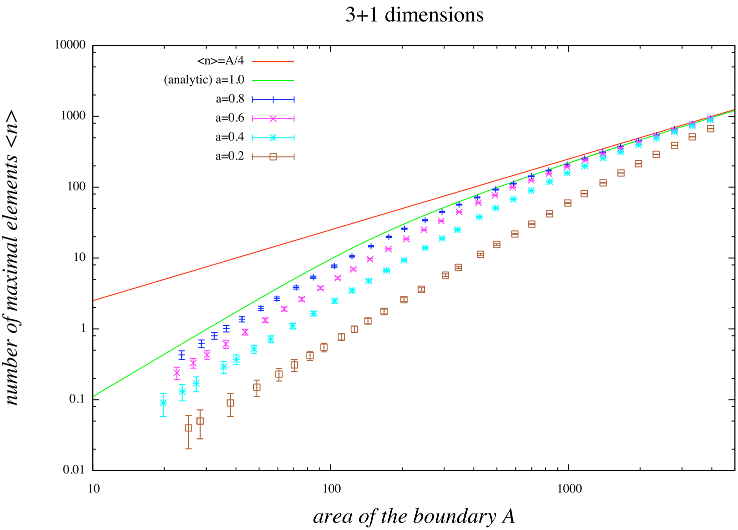

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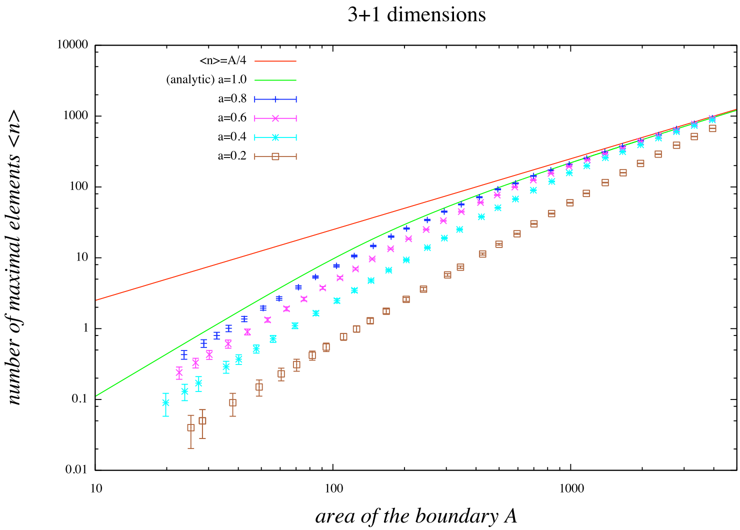
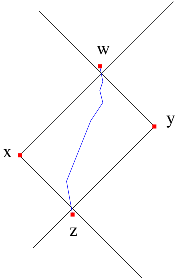


All give same asymptotic form $S_{max} = \frac{4\pi R}{4}$, with same l_f!

Outline

Emergence of Spatial Structure
from Discrete Causal Order



- Temporal notions, such as proper time, are easy to extract from causet
- Spatial notions difficult, because of Lorentz invariance of lattice – has infinite ‘valence’ (nearest neighbors)

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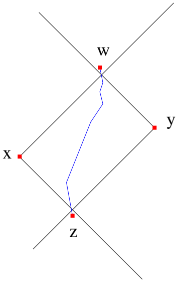

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- Resolution $\rightsquigarrow$ Make local selection of frame $\longrightarrow$ inextendible antichain

  'Edgeless' spacelike hypersurface:
  Every element related to some element of inextendible antichain

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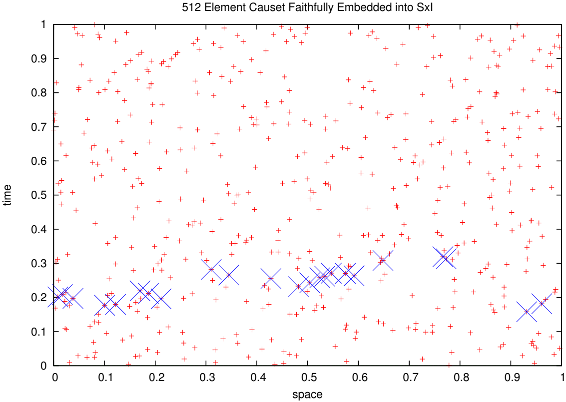

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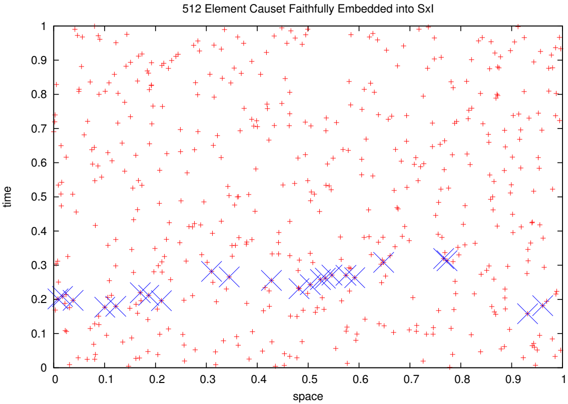

- $\sim$ Cauchy surface, but e.g. does not possess inital data... Only intrinsic information is cardinality
- Can we use neighboring causal structure to deduce which elements are spatial nearest neighbors?

Emergence of Spatial Structure
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- Can we use neighboring causal structure to deduce which elements are spatial nearest neighbors?
- 'Thickened antichain'
  $A_v = \{x | x \in \mathrm{fut}(A) \cup A \text{ and } |\mathrm{past}(x) \setminus \mathrm{past}(A)| \leq v\}\,,$

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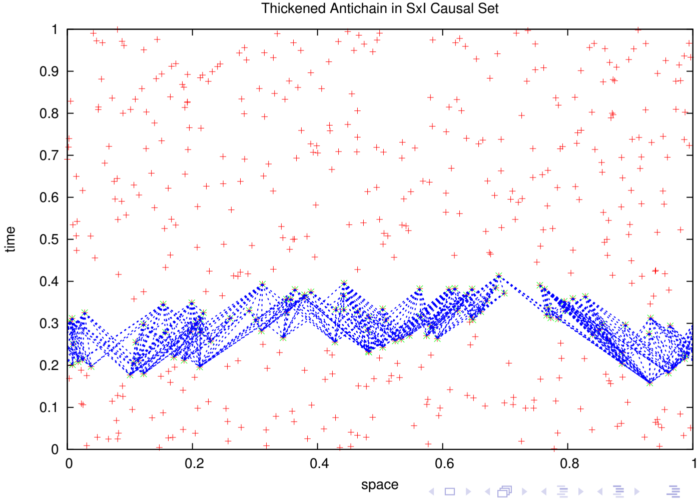

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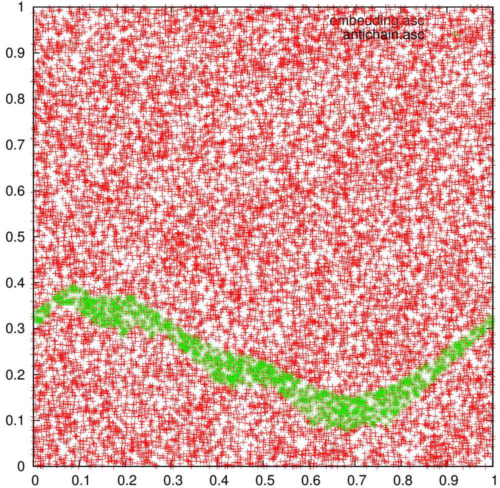
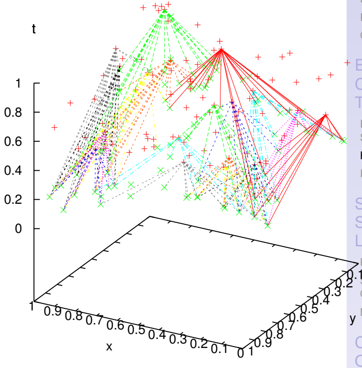


Homology from Thickened Antichain

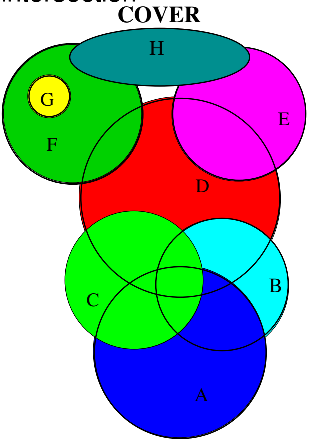
- Can we deduce the continuum homology from the thickened antichain?

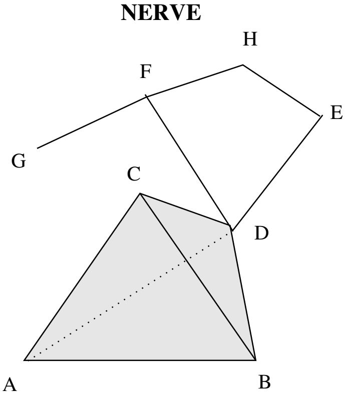
- maximal elements of thickened antichains 'cast shadows' on minimal elements
- provides cover of space
- nerve construction of simplicial complex $\longrightarrow$ homology



Homology from Thickened Antichain

Nerve: Assign vertex to each set U_i in cover
q sets in cover form a $q-1$-simplex if they have non-vanishing intersection

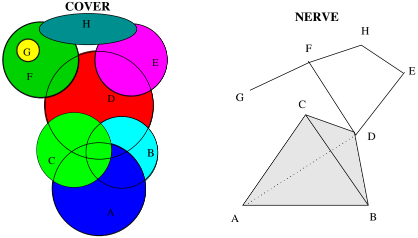




Simplicial homology

- C_k = vector space with generator for each k-simplex α
- α_i = $k-1$-simplex obtained by deleting ith vertex of α
- boundary map $\partial_k : C_k \to C_{k-1} = \sum_{i=0}^{k} (-)^i \alpha_i$
- $\partial^2 = 0$
- Z_k = Ker(∂_k) = k-cycles
- B_k = Im(∂_{k+1}) = k-boundaries
- $H_k = Z_k / B_k$
- Betti numbers $b_k = \dim(H_k)$

For example:



$$b_0 = 1 \quad b_1 = 1 \quad b_i = 0 \quad i > 1$$

Cover and thus homology will vary with thickness v

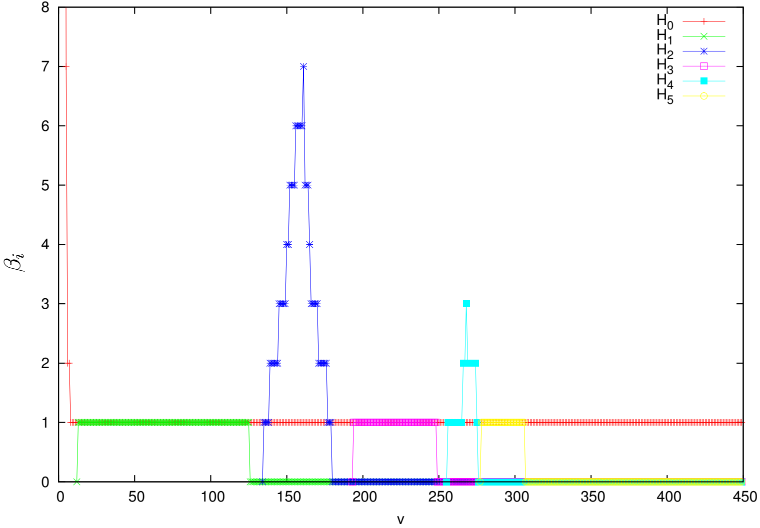
Current Status: Theorem

- Causet $\mathcal{C}$ in globally hyperbolic spacetime with compact spatial slice Σ
- If $\exists$ inextendible antichain A in $\mathcal{C}$ in Σ with appropriate separation of scales, then ...
- A thickened to vol n in this range has homology of space via nerve

- Can likely smooth out bad antichain to get good one, via smoothing (Ricci flow?)

$\Rightarrow$ Conditions on theorem minor.
We explore this numerically.

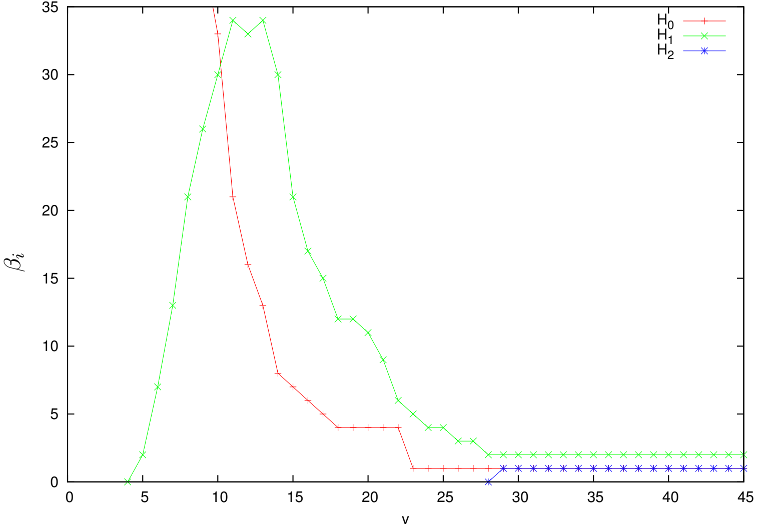
$S \times I$



$N = 2000$

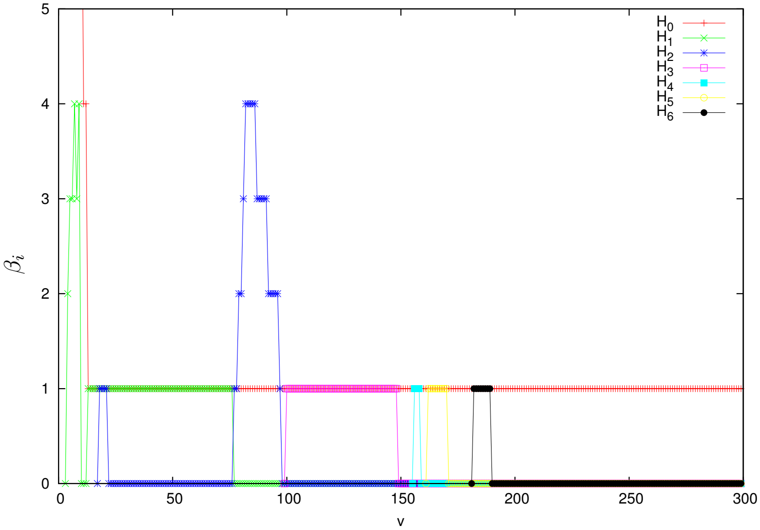
'cosmic scale' at $v = 433$

$T^2 \times I$



$N = 5793$

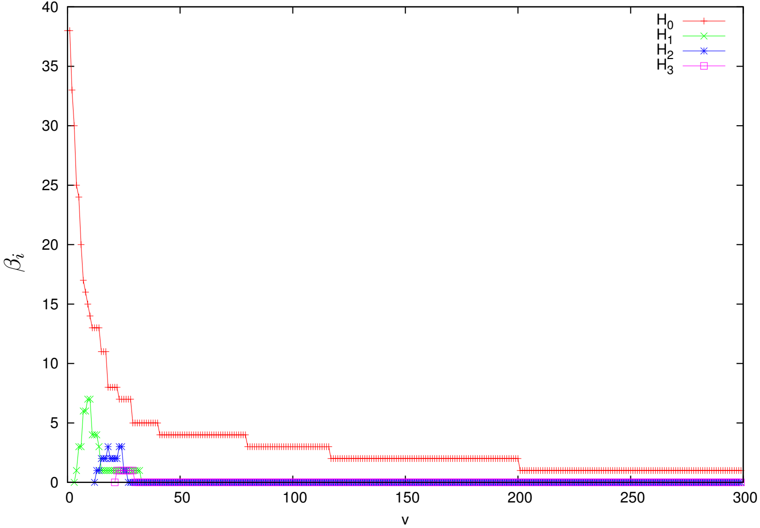
$T^2 \times I$ 'Kaluza Klein'



$b = a/4$

Transitive Percolation

$N = 2000;\ p = .03$

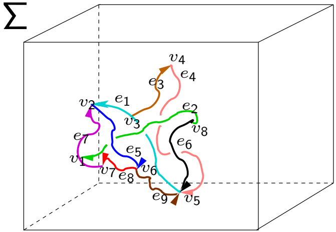


Outline

LQG

Kinematical Hilbert Space

Consider



Canonical variables:
holonomies h_{e_I}, fluxes $E_i(S)$

$E(\gamma)\dots$ edge set of γ
$V(\gamma)\dots$ vertex set of γ

- Kinematical Hilbert space $\boxed{\mathcal{H}_{\mathrm{kin},\gamma} = L^2(\bar{\mathcal{A}}_\gamma, d\mu_\gamma)}$
- Spin Network Functions

$$T_{\gamma\vec{j}\vec{m}\vec{n}}(A) := \prod_{e\in E(\gamma)} \sqrt{2j_e+1}\,[\pi_{j_e}(h(A))]_{m_e n_e}$$

- Basis of $\mathcal{H}_{kin,\gamma}$ (Peter & Weyl): $\sqrt{2j+1}\left[\pi_j(h_e)\right]_{m_e n_e} \sim \left\langle h_e \;\middle|\; j\ m_e\ ;\ n_e \right\rangle$ ← SN
  - Can replace $-\frac{i}{2}E_j \left| j\ m_e\ ;\ n_e \right\rangle \quad = \quad J_j \left| j\ m_e\ ;\ n_e \right\rangle$

Action of operators only containing E_j can be expressed as action of usual angular momentum operators acting on a spin system

Volume Operator: Structure

- Classical Volume Expression

$$V(R) = \int_R d^3x \, \sqrt{\det q}(x) = \int_R d^3x \, \sqrt{|\det E|}(x)$$

$$= \int_R d^3x \, \sqrt{\left|\frac{1}{3!}\varepsilon^{ijk}\varepsilon_{abc}E_i^a(x)E_j^b(x)E_k^c(x)\right|}$$

- Structure of the volume operator

$$\boxed{\hat{V}_\gamma(R)\, f_\gamma \propto \ell_P^3 \sum_{v\in V(\gamma)} \sqrt{\left|\sum_{IJK} \epsilon(I,J,K)\ \hat{q}_{IJK}\right|}\ f_\gamma}$$

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graph structure

$$\epsilon(I,J,K) = \mathrm{sgn}\big(\det(\dot{e}_I, \dot{e}_J, \dot{e}_K)(v)\big)$$

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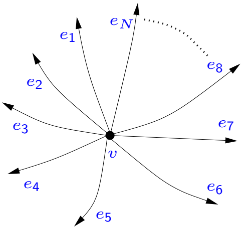
edge spin

$$\hat{q}_{IJK} := \left[\underbrace{(J_{IJ})^2}_{\boxed{(J_I+J_J)^2}}, (J_{JK})^2\right] \propto \varepsilon_{ijk}J_I^i J_J^j J_K^k$$

Volume Operator
Edge Spins & Recoupling Theory



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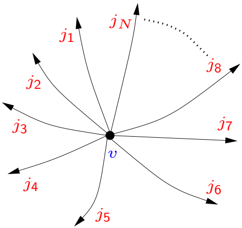

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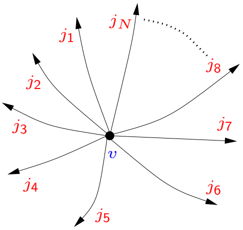
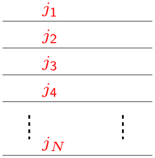


- Tensor Basis



$$T_{v\vec{j}\vec{m}\vec{n}} = \bigotimes_{k=1}^{N} \mid j_k \, m_k \, ; \, n_k >$$

Volume Operator

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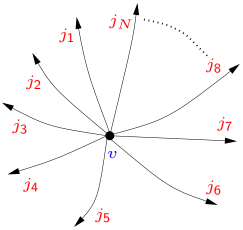

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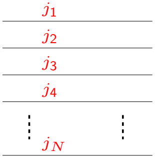

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- Decomposition

$$\pi_{j_1} \otimes \pi_{j_2} = \bigoplus_{j_{12}=|j_1-j_2|}^{j_1+j_2} \pi_{j_{12}}$$

Volume Operator

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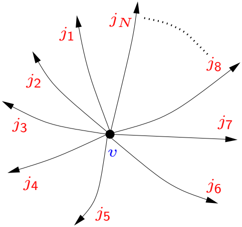

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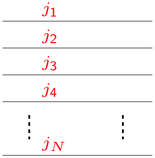

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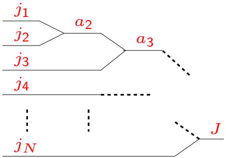
- Recoupling Basis



$$T_{v\;\vec{I}\;J\;M\;\vec{j}\;\vec{n}} = \mid \vec{a}\;J\;M\,;\,\vec{j}\;\vec{n}\;\; >$$

Edge Spins & Recoupling Theory

$$< \vec{a}|\hat{q}_{IJK}|\vec{a}' > = \frac{1}{4}(-1)^{j_K+j_I+a_{I-1}+a_K}(-1)^{a_I-a'_I}(-1)^{\sum_{n=I+1}^{J-1} j_n}(-1)^{-\sum_{p=J+1}^{K-1} j_p}\times$$

$$\times X(j_I,j_J)^{\frac{1}{2}}X(j_J,j_K)^{\frac{1}{2}}\sqrt{(2a_I+1)(2a'_I+1)}\sqrt{(2a_J+1)(2a'_J+1)}\times$$

$$\times\left\{\begin{matrix} a_{I-1} & j_I & a_I \\ 1 & a'_I & j_I \end{matrix}\right\}\left[\prod_{n=I+1}^{J-1}\sqrt{(2a'_n+1)(2a_n+1)}(-1)^{a'_{n-1}+a_{n-1}+1}\left\{\begin{matrix} j_n & a'_{n-1} & a'_n \\ 1 & a_n & a_{n-1} \end{matrix}\right\}\right]\times$$

$$\times\left[\prod_{n=J+1}^{K-1}\sqrt{(2a'_n+1)(2a_n+1)}(-1)^{a'_{n-1}+a_{n-1}+1}\left\{\begin{matrix} j_n & a'_{n-1} & a'_n \\ 1 & a_n & a_{n-1} \end{matrix}\right\}\right]\left\{\begin{matrix} a_K & j_K & a_{K-1} \\ 1 & a'_{K-1} & j_K \end{matrix}\right\}$$

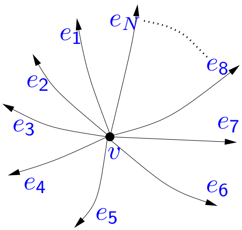
$$\times\Bigg[(-1)^{a'_J+a'_{J-1}}\left\{\begin{matrix} a_J & j_J & a'_{J-1} \\ 1 & a_{J-1} & j_J \end{matrix}\right\}\left\{\begin{matrix} a'_{J-1} & j_J & a'_J \\ 1 & a_J & j_J \end{matrix}\right\}$$

$$-(-1)^{a_J+a_{J-1}}\left\{\begin{matrix} a'_J & j_J & a'_{J-1} \\ 1 & a_{J-1} & j_J \end{matrix}\right\}\left\{\begin{matrix} a_{J-1} & j_J & a'_J \\ 1 & a_J & j_J \end{matrix}\right\}\Bigg]\times$$

$$\times\prod_{n=2}^{I-1}\delta_{a_n a'_n}\prod_{n=K}^{N}\delta_{a_n a'_n}$$

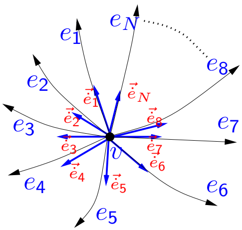
where $X(a,b) = 2a(2a+1)(2a+2)2b(2b+1)(2b+2)$

Graph Structure



- N edges at vertex v

Graph Structure



- N edges at vertex v
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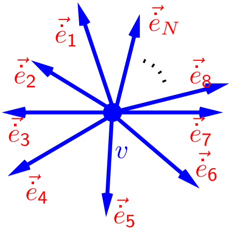
Graph Structure



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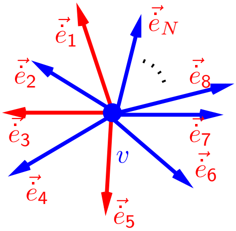

- N edges at vertex v
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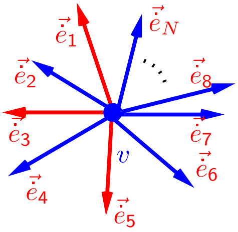
- edge triple e_1, e_3, e_5

Graph Structure



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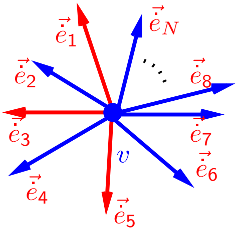

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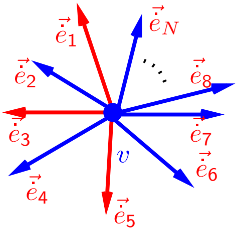

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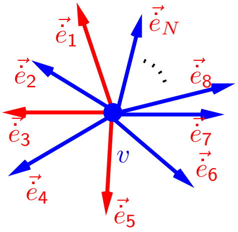

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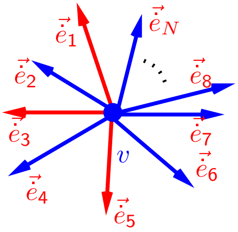

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What sign combinations will occur at all?

Graph Structure $\leftrightarrow$ Edge Spins

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$$=: \sqrt{\left| \sum_{I,J,K<N} \sigma(IJK)\, \hat{q}_{IJK} \right|} \qquad \text{where } -4 \le \sigma(IJK) \le 4$$

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$\rightsquigarrow$ Contact: recoupling of spins $\leftrightarrow$ properties of space

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N	$\binom{N}{3}$	$N_{\vec{\epsilon}}^{(max)}$	$N_{\vec{\epsilon}}$	$\frac{N_{\vec{\epsilon}}}{N_{\vec{\epsilon}}^{(max)}}$	$N_{\vec{\sigma}}$	$N_{\vec{\sigma}=0}$
4	4	16	16	1	5	6
5	10	1024	384	0.375	171	24
6	20	2^{20}	23,808	0.023	8,207	120
7	35	2^{35}	2,324,832	$6.766\cdot 10^{-5}$	1,912,373	108

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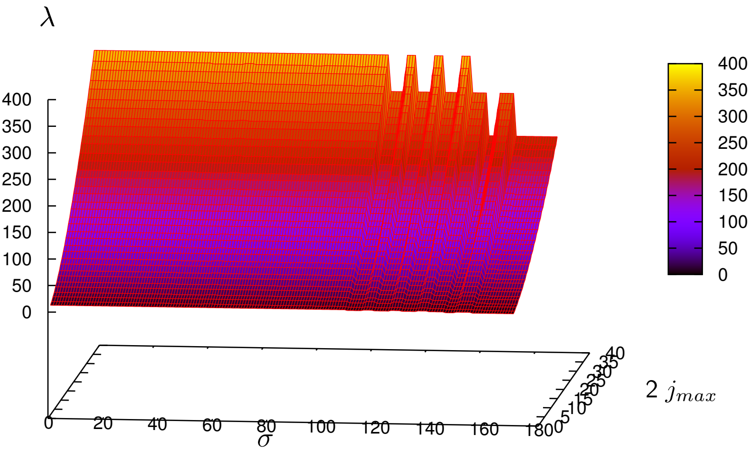
Zero Volume states $\rightarrow$ property independent from spins

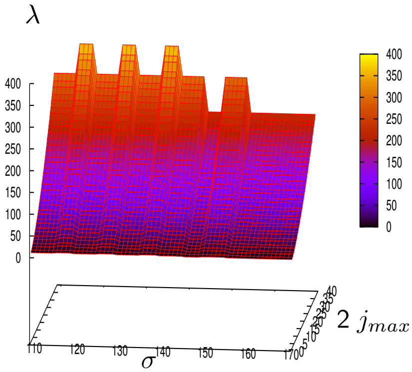
Gauge Invariant 5-Vertex: Numerical Results

- Largest Eigenvalue



Gauge Invariant 5-Vertex: Numerical Results

- Largest Eigenvalue: σ-dependence



$\#\sigma$	$\sigma(123)$	$\sigma(124)$	$\sigma(134)$	$\sigma(234)$
109	-2	2	2	-2
110	2	-2	-2	2
111–114 have same absolute values				
115	-4	-4	-2	0
119	0	-2	-2	2
120	0	2	2	-2
125	-2	4	2	0
129	-2	0	2	-2
135	-4	-2	-2	0
118	-2	2	0	-2
170	2	0	0	0

Gauge Invariant 5-Vertex: Numerical Results

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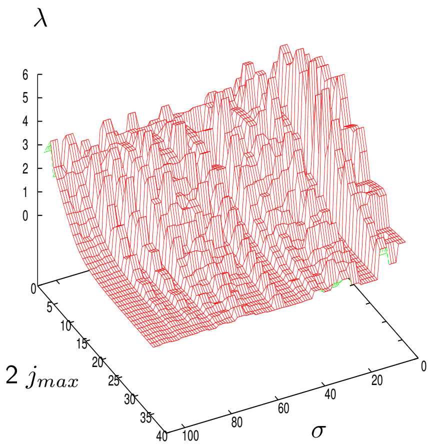
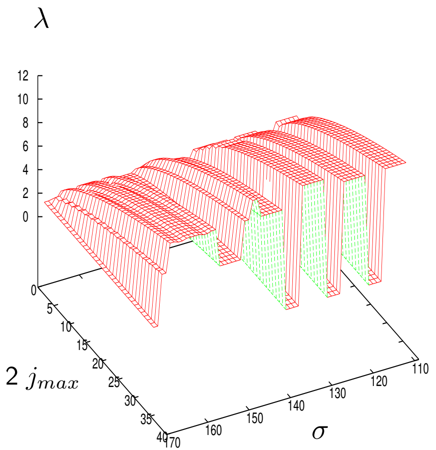
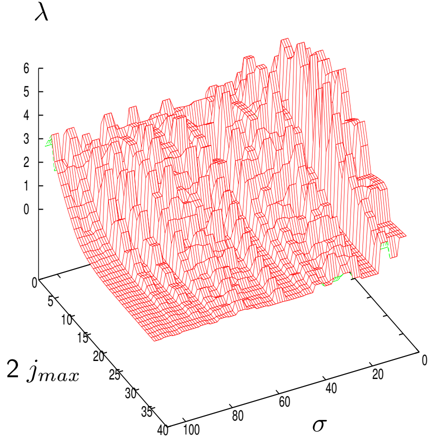




Gauge Invariant 5-Vertex: Numerical Results

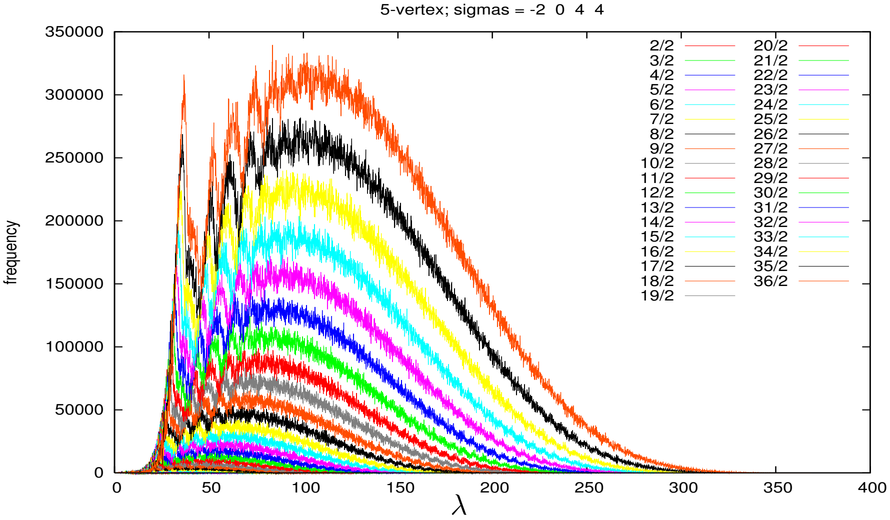
- Smallest Eigenvalue: σ-dependence



$\#\sigma$	$\sigma(123)$	$\sigma(124)$	$\sigma(134)$	$\sigma(234)$
0	-2	-2	-4	-4
1–3 have same absolute values				
4	0	-2	4	4
8	-2	0	4	4
12	-2	4	2	-4
16	0	-4	2	4
20	-4	2	2	-4
24	-2	-4	0	4
28	-4	0	2	-4
32	0	-2	-2	4
36	-4	2	0	-4

Gauge Invariant 5-Vertex: Numerical Results

- Histograms for each sigma config



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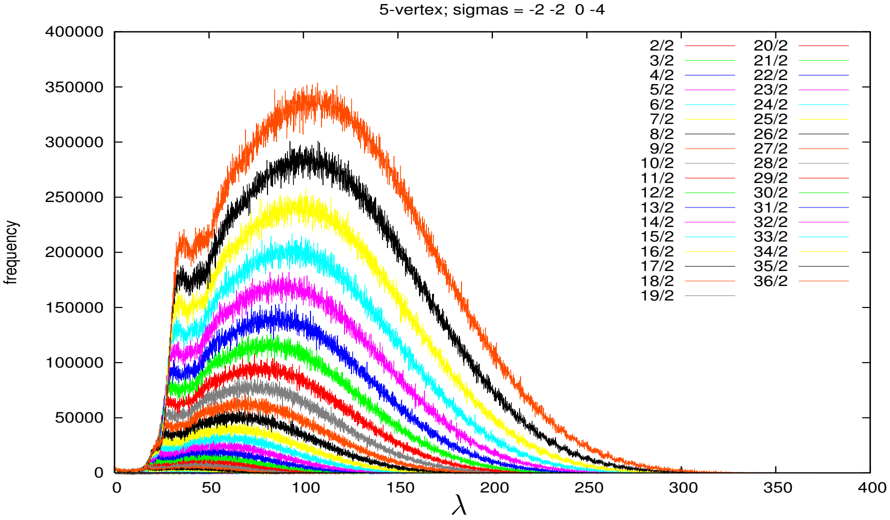

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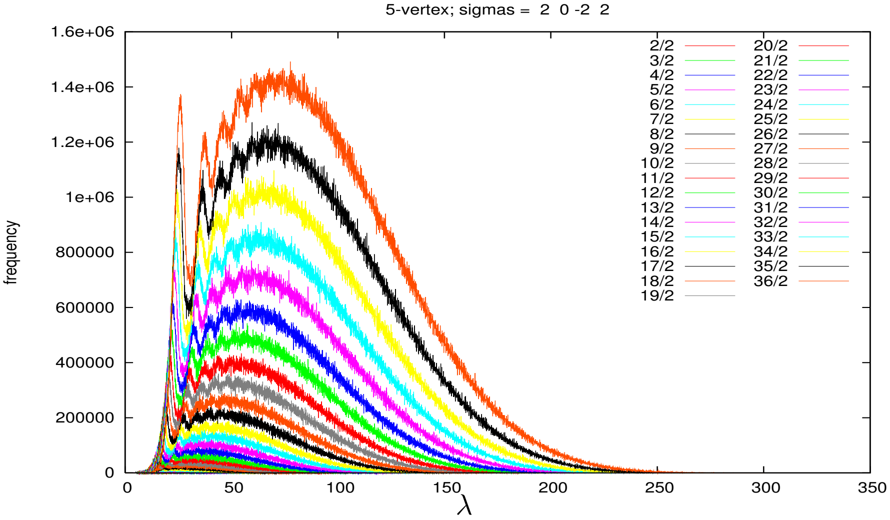

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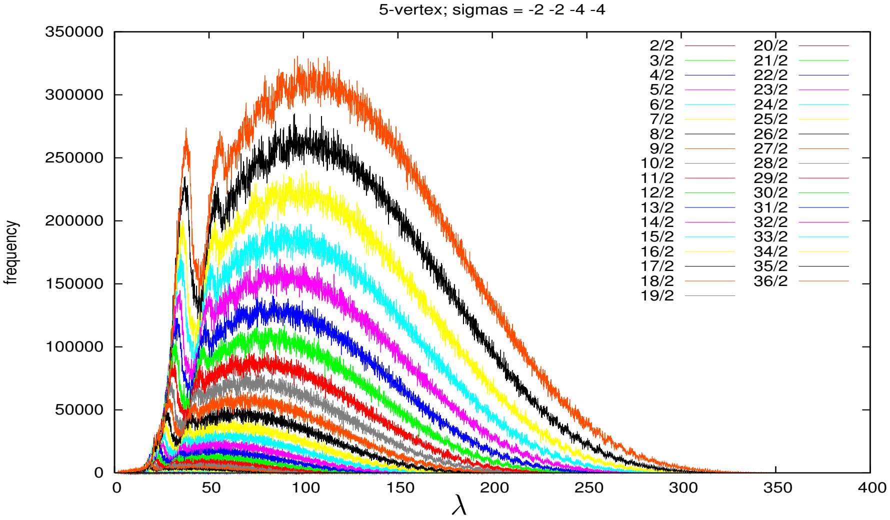
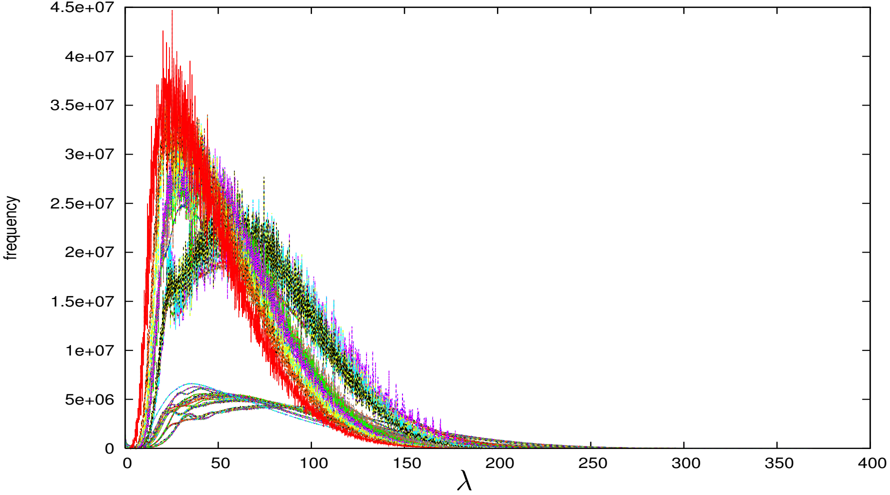


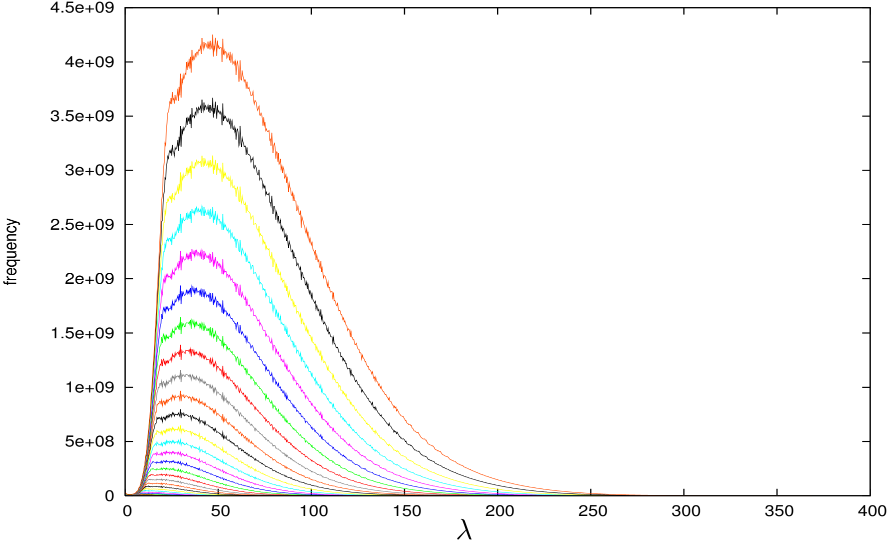
Gauge Invariant 5-Vertex: Numerical Results

- Histograms for each sigma config
- Histograms for all sigma configs together



Gauge Invariant 5-Vertex: 'Full Spectrum'

- Cumlative histogram for each j_{max}



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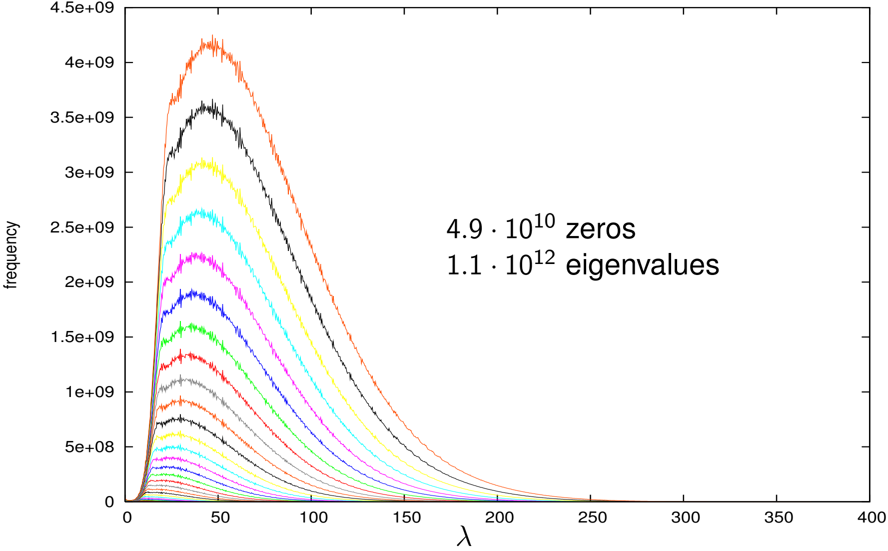

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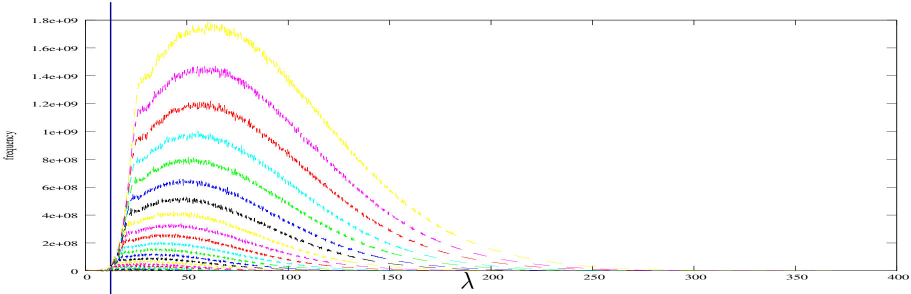
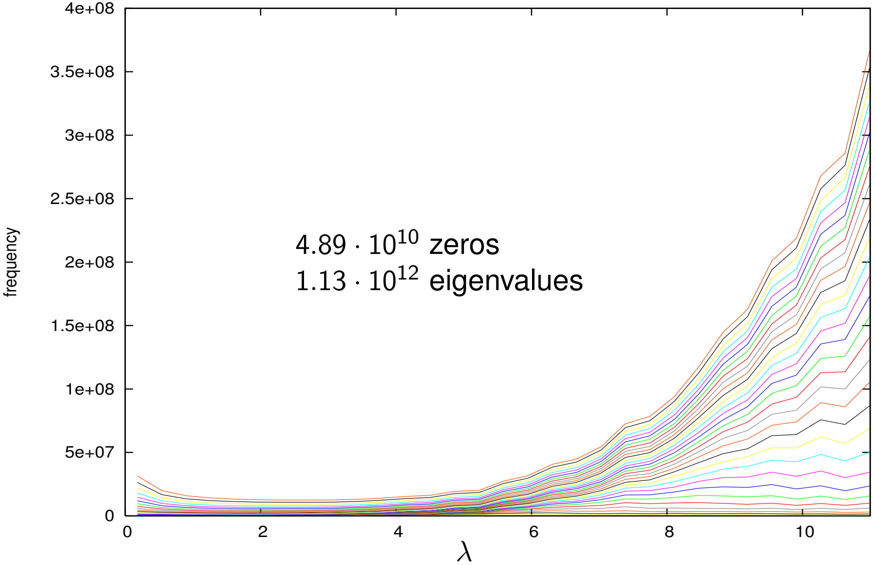


Expand region with λ<11

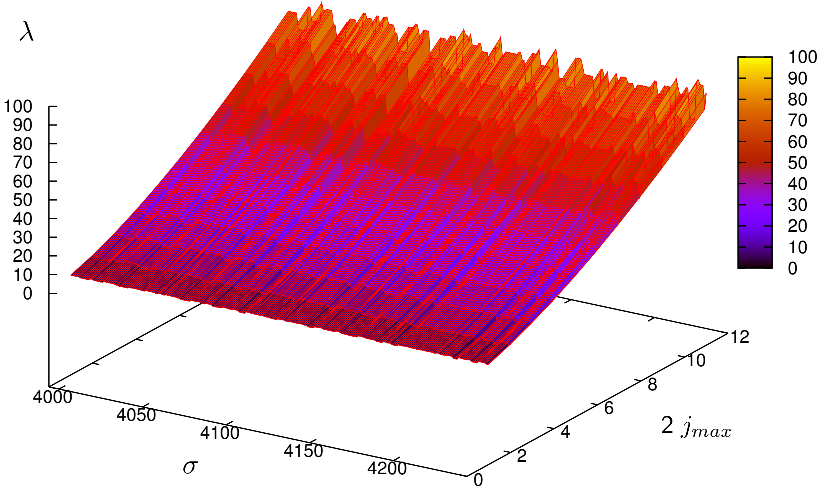
Gauge Invariant 5-Vertex: ‘Full Spectrum’

- Cumlative histogram for each j_{max} — for $\lambda < 11$



Gauge Invariant 6-Vertex: Numerical Results

- Largest Eigenvalue



Gauge Invariant 6-Vertex: Numerical Results

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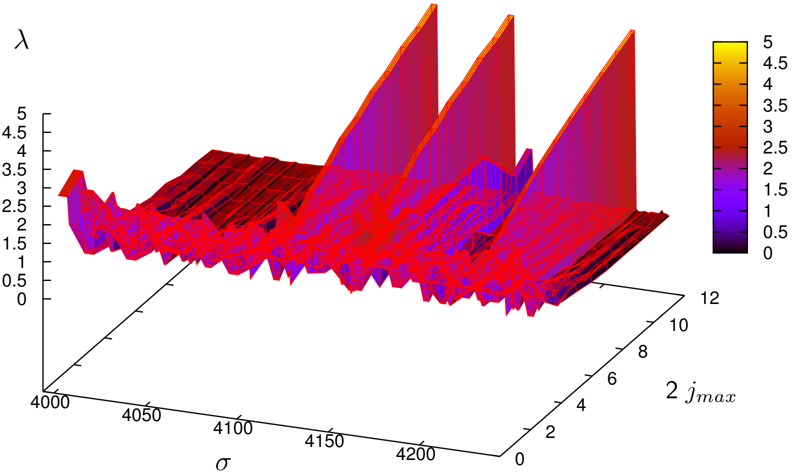

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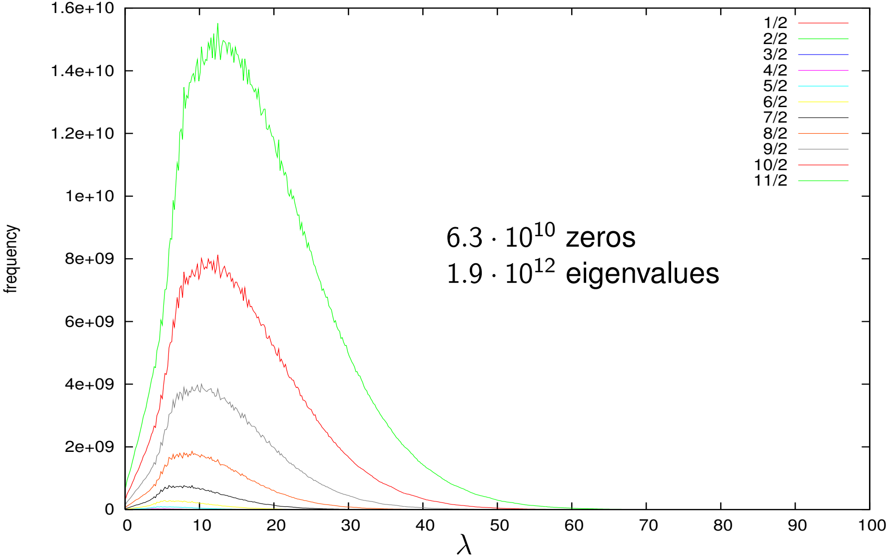

Outline

Conclusions & Outlook

Entropy Bounds from Discrete Gravity:

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- Susskind bound may arise via counting maximal elements

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QG Supercomputing & Cactus:

- Numerical computing useful for gaining insight into QG
- Cactus is excellent tool to facilitate this
- $\rightsquigarrow$ Develop community code base to address problems in QG