Phenomenological Quantum Gravity

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FOR THEORETICAL PHYSICS

Simplify the Complicated

Simplify the Complicated

Top-down

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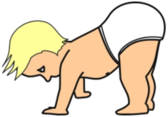
Top-down 

Bottom-up 

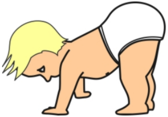
Simplify the Complicated

Top-down

Top-down inspired bottom-up approaches

. . . Extra Dimensions. . . Minimal Length . . .

. . . DSR . . . Holographic Principle . . .

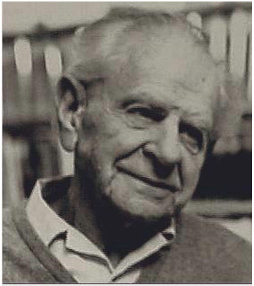


Bottom-up

Simplify the Complicated

Top-down

"Science may be described as the art of systematic over-simplification."



Sir Karl Popper (1902-1994)

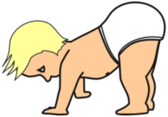
Karl Popper (The Observer, August 1982)

Top-down inspired bottom-up approaches

... Extra Dimensions... Minimal Length ...

... DSR ... Holographic Principle ...

Bottom-up



Top-down inspired bottom-up approaches

- Extra Dimensions:

$\longrightarrow$ KK-excitations, graviton-production, black hole production

- Deformed Special Relativity:

$\longrightarrow$ Shift of reaction-thresholds, energy dependent speed of light

- Generalized Uncertainty:

$\longrightarrow$ Stagnation of cross-section, modifications of loop contributions

- Violation of Lorentz invariance:

$\longrightarrow$ Preferred frame effects, higher oder operators

- Cosmology:

$\longrightarrow$ Imprints of QG fluctuations in the CMB/ν background, spectral index

- Space-time Foaminess, decoherence:

$\longrightarrow$ CPT violation, neutral Kaon systems, stochastic deviations from lightcone

Physics beyond the Standard Model?

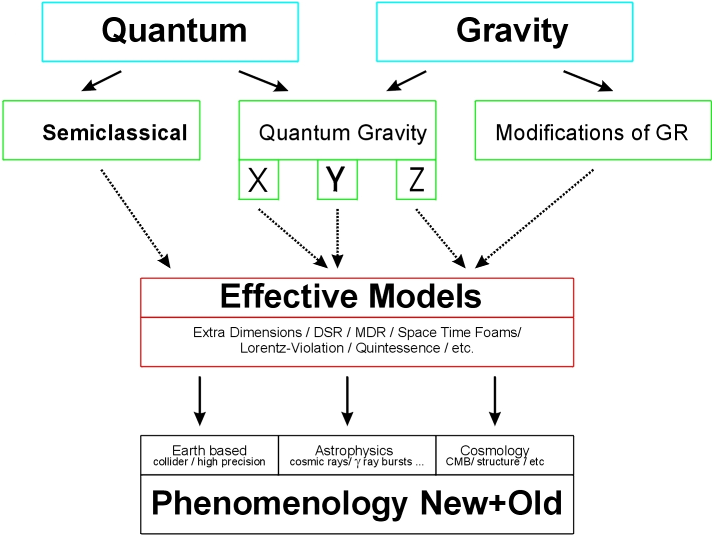
Or quantum gravity?

Freeman Dyson

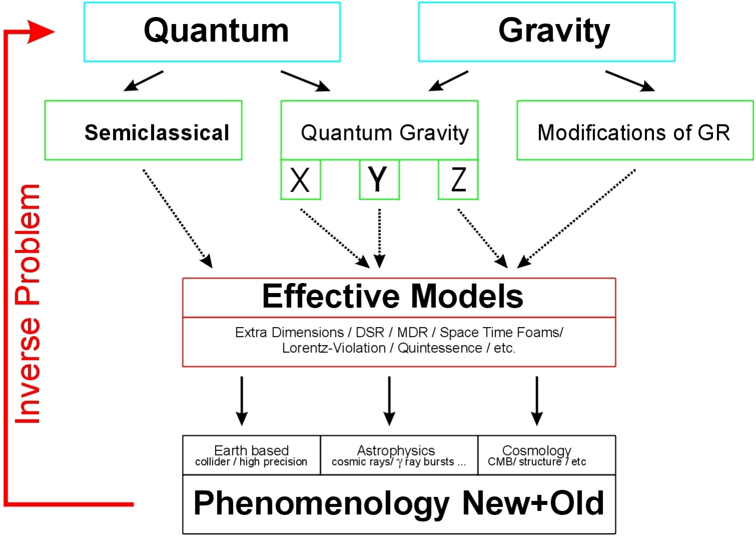
"According to my hypothesis [...] the two theories are mathematically different and cannot be applied simultaneously. But no inconsistency can arise from using both theories, because any differences between their predictions are physically undetectable."

Freeman Dyson
The New York Review of Books Volume 51, Number 8 May 13, 2004
The World on a String By Freeman Dyson
Review of The Fabric of the Cosmos: Space, Time, and the Texture of Reality by Brian Greene

The Inverse Problem



The Inverse Problem



The Planck Scale

The Planck mass m_p is the energy at which a particle causes a significant perturbation of the metric in a volume given by its own Compton wavelength $l_{\rm p} = \hbar / m_{\rm p}$

$$\begin{aligned} 1 = \Delta g &\approx \frac{GM}{rc^2} \to \frac{G m_{\rm p}^2}{c^2\,\hbar} \\ \Rightarrow m_{\rm p} &= \sqrt{\frac{\hbar c}{G}} \approx 10^{16}\mathrm{TeV} \quad , \quad l_{\rm p} = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-20}\mathrm{fm} \end{aligned}$$

And is far, far off the scale we can reach with earth build accelerators.

The Planck Scale

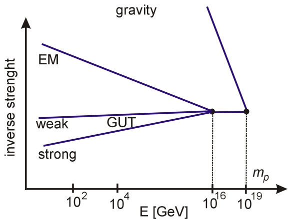
Max Planck,

Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin (1899), p. 479

Dem gegenüber dürfte es nicht ohne Interesse sein zu bemerken, dass mit Zuhülfenahme der beiden in dem Ausdruck (41) der Strahlungsentropie auftretenden Constanten a und b die Möglichkeit gegeben ist, Einheiten für Länge, Masse, Zeit und Temperatur aufzustellen, welche, unabhängig von speciellen Körpern oder Substanzen, ihre Bedeutung für alle Zeiten und für alle, auch ausserirdische und aussermenschliche Culturen nothwendig behalten und welche daher als »natürliche Maasseinheiten« bezeichnet werden können.

It is interesting to note that with the help of the [above constants] it is possible to introduce units [...] which [...] remain meaningful for all times and also for extraterrestial and non-human cultures, and therefore can be understood as 'natural units'.

Extrapolation over 16 orders of Magnitude



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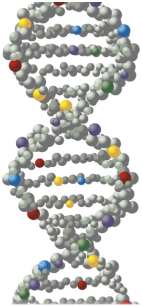
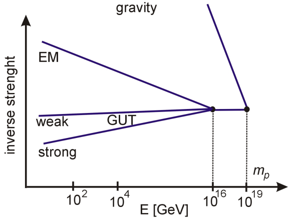
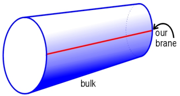




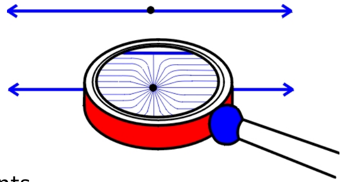
- How does particle physics look like in the Planckian regime?
- For a lowered Planck scale this is important for collider physics
- Concrete scenario to lower Planck scale: large extra dimensions

Models with Extra Dimensions

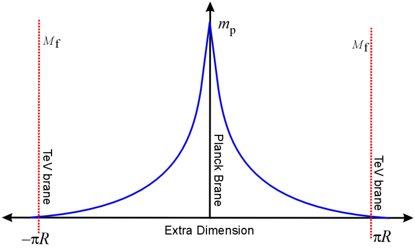
- ADD-model: large extra dimensions $R \gg 1/M_f$
  - + Solves Hierarchy-problem, $m_p^2 = R^d M_f^{d+2}$

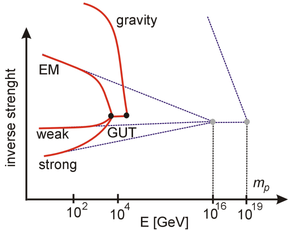


- RS-model (I and II), extra dimension is curved
  - + AdS-CFT Correspondence
  - + Allows non-compact extra dimension



- UXD, TeV-scale dimensions
  - + Accelerated unification of coupling constants

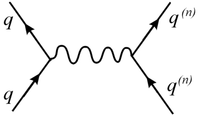




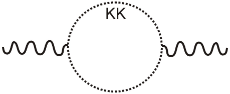
Observables of KK-excitations

- Real production of exitations in UXDs: pair production only

Rizzo and Wells, Phys. Rev. **D61**, 016007 (2000)
Appelquist, Cheng and Dobrescu, Phys. Rev. D **64**, 035002 (2001)



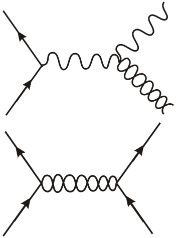
- Modifications due to virtual contributions



- Tightest constraints on UXDs from precision electroweak $1/R > 4$ TeV (for $d = 1$, depends on precise scenario).
- Spacing of excitations distinguishes scenario.

$\longrightarrow$ But that's actually got nothin to do with quantum gravity

Signatures of Gravitons



Collider physics (current bounds on M_f in TeV-range):

- Real gravitons lead to missing energy
- Virtual exchange modifies cross sections

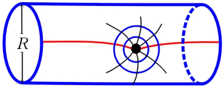
Astrophysics (bounds weak for $d > 4$, strong for $d \leq 4$):

- Enhanced cooling of supernovae/red giants from graviton emmission
- Cooling in early universe and contributions to background from decay of bulk excitations
- Anomalous re-heating of neutron stars by decay of gravitationally trapped massive gravitons

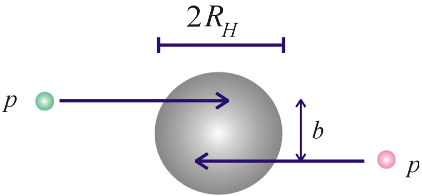
Black Holes in Extra Dimensions

In large extra dimensions (ADD)

- Gravity stronger at small distances $\Rightarrow$ horizon radius R_H larger
- For $M \sim 1$ TeV , R_H increases from $\sim 10^{-38}$ fm to 10^{-4} fm!
- For these black holes it is $R_H \ll R$ and they have approx higher dimensional spherical symmetry

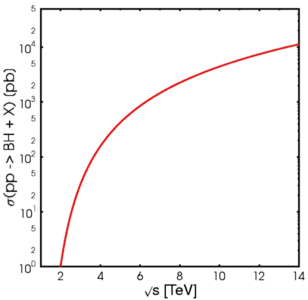


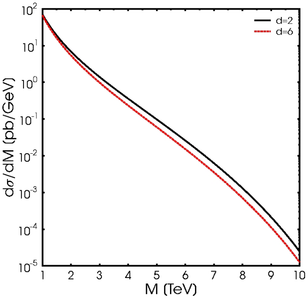
- At the LHC partons can come closer than their Schwarzschild horizon $\longrightarrow$ a black hole can be created!



Production of Black Holes

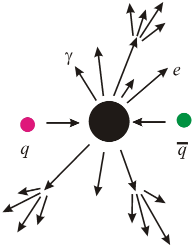
- Semi-classical cross-section $\sigma \sim \pi R_H^2$
- Can be improved by modelling colliding wave packets
- Yields $\sim 10^8$ black holes per year for LHC pp-collisions
- Hawking evaporation results in decay into all particles of the standard model
- Numerical tools available for event simulation

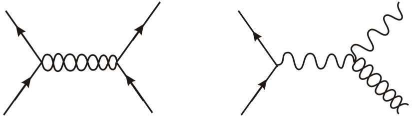




Phenomenology of QG with a lowered Planck Scale

- Black hole production and evaporation
- Real graviton production
- Modified cross-sections from virtual particle exchange





⟶ First grip on the phenomenology of quantum gravity

⟶ Possibility to understand more than perturbative approach

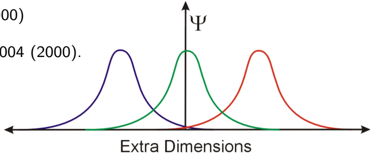
⟶ Interplay between particle physics and general relativity

But...

Split Fermion Scenario

- Localization of fermions at different positions inside 'fat' brane

Arkani-Hamed and Schmaltz, Phys. Rev. D **61**, 033005 (2000)
Mirabelli and Schmaltz, Phys. Rev. D **61**, 113011 (2000)
Arkani-Hamed, Grossman, Schmaltz, Phys. Rev. D **61**, 115004 (2000).



⇒ Couplings on brane depend on overlap: can be very small

- Quick-fix for several problems
  - * Solves proton decay problem with lowered fundamental scale
  - * Explains hierarchies in Yukawa couplings
  - * Suppresses flavor changing decays

Paper Inflation

One can make things more complicated:

... embed UXD into ADD... different radii of extra dimensions... compactification on various topologies... splits... twists... shapes...black strings, rings, things, black saturns etc... various brane configurations and trapping in such... oszillations of branes... recoil effect... potentials to stabilize dimensions...

- Be aware of your model's limitations
- It was never meant to more than a qualitative discription of the first effects of quantum gravity in particle interactions.
- Working out details of models without any experimental evidence whatsoever does little else than increasing the number of published papers.

Committment

Committment

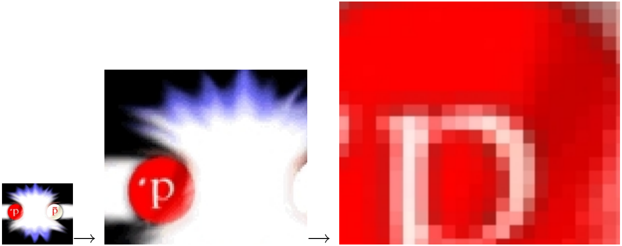
Martin, the physics expert at ScienceForums:

> *The invited talk about QG phenomenology at LOOPS '07 will be given by Sabine Hossenfelder, it is her specialty [...] She will sound pessimistic but that is normal for phenomenologists—they are supposed to be unenthusiastic, uncommitted, cautious, and a bit of the devil advocate who throws the cold water of reality on the theorists [...]*

http://www.scienceforums.net/forum/showthread.php?p=335471

The Minimal Length Scale

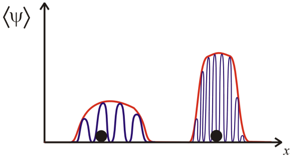
- Very general expectation for quantum gravity: fluctuations of spacetime itself disable resolution of small distances
- Can be found e.g. in string theory, Loop Gravity, NCG, etc.
- Minimal length scales acts as UV cutoff
- Lowering the Planck mass means raising the Planck length



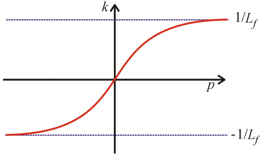
...is there a fundamental limit to the resolution of structures?

An Effective Model for the Minimal Length*

- For large momenta, p, Compton-wavelength $\lambda = 1/k$ can not get arbitrarily small $\lambda > L_{\rm f} = 1/M_{\rm f}$



- Model by modifing relation between wave-vector k and momentum p. Results in modified commutation relations
$k = k(p) = \hbar p + a_1 p^3 + a_2 p^5 \ldots \Rightarrow [p_i, x_j] = i\partial p_i / \partial k_j$

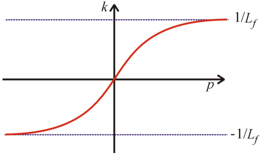


* SH *et al*, Phys. Lett. B598 (2004) 92-98; SH, Phys. Rev. D **73**, 105013 (2006)

An Effective Model for the Minimal Length

Requirements on $k(p)$

- For small $p \ll M_f$ the relation is linear as usual $p = k$
- The function is odd to preserve parity $k(-p) = -k(p)$
- It is monotonically increasing (i.e invertible and has exactly one zero)
- For large $p \gg M_f$ it reaches an asymptotically constant value that is the inverse of the minimal length $p \to 1/M_\mathrm{f} = L_\mathrm{min}$
- Note: this means it can not be a polynomial of finite order
- Often used: first order approximation $p \approx k + a_1 k \, (k L_\mathrm{min})^2$



Consequences of the Minimal Length

- Implies a generalized uncertainty principle, first correction

$$\Delta x \Delta p \geq \frac{1}{2}\hbar\left(1 + b_1 \frac{\Delta p^2}{M_{\rm f}^2}\right) \quad ,$$

- A squeezed phase space at high energies

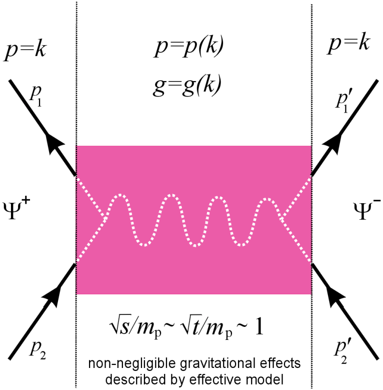
$$\langle p|p'\rangle = \frac{\partial p}{\partial k}\delta(p-p') \Rightarrow dk \to \frac{dp}{\hbar}\frac{\partial k}{\partial p} = \frac{dk}{\hbar}e^{-p^2 L_{\rm min}^2} \quad ,$$

- And a modified dispersion relation

$$\omega^2 - k^2 - \mu^2 = \Pi(k,\omega)$$

- Can but need not have a energy dependent speed of light $d\omega/dk \neq 1$.

The Collision Region



Quantisation with a Minimal Length

- Lagrangian for free fermions

$$\mathcal{L}_f = \mathrm{i}\overline{\Psi}(\not{p}(k) - m)\Psi \qquad \mathcal{L}_f = \mathrm{i}\overline{\Psi}(g^{\nu\kappa}(k)\gamma_\nu k_\nu - m)\Psi$$

- Coupling of the gauge field via $\partial_\nu \to D_\nu := \partial_\nu - \mathrm{i}eA_\nu$ yields the gauge- and Lorentz-invariant higher order derivative interaction

$$\mathcal{L} = \bar{\Psi}\not{p}(D)\Psi \qquad \mathcal{L} = \bar{\Psi}\gamma_\nu g^{\nu\kappa}(D)D_\kappa\Psi$$

- To first order one finds the usual $\mathcal{L} = \mathcal{L}_f - e\overline{\Psi}\eta^{\kappa\nu}\gamma_\kappa A_\nu\Psi + \mathcal{O}(eL_{\mathrm{min}}^2)$ and the dominant modification comes from the propagators

$$(\not{p}(k) - m)^{-1} \qquad (g^{\nu\kappa}(k)\gamma_\nu k_\kappa - m)^{-1}$$
$$(p^\nu(k)p_\nu(k) - m^2)^{-1} \qquad (g^{\nu\kappa}(k)k_\nu k_\kappa - m^2)^{-1}$$

- Recipe: replace p with $p(k) \longrightarrow$ higher order derivative Lagrangian

The Locality Bound*

From the commutator

$$[a_p, a^\dagger_{p'}] = \delta(p - p') \left| \frac{\partial p}{\partial k} \right|$$

And the field expansion

$$\phi(x) = \int \mathrm{d}^3 p \left| \frac{\partial k}{\partial p} \right| \left[v_p(x) a_p + v^*_p(x) a^\dagger_p \right]$$

One finds the equal time commutator for $x = (\mathbf{x}, t), y = (\mathbf{y}, t)$.

$$[\phi(x), \pi(y)] = \mathrm{i} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left| \frac{\partial k}{\partial p} \right| \mathrm{e}^{\mathrm{i}k(x-y)} \to \mathrm{i} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{e}^{\mathrm{i}k(x-y) - \varepsilon p^2}$$

where $\varepsilon \sim L^2_{\mathrm{min}}$. I.e.

$$[\phi(x), \pi(y)] \neq \delta(x - y)$$

*Giddings and Lippert, Phys. Rev. D **65**, 024006 (2002), Phys. Rev. D **69**, 124019 (2004).

The Propagator

$$\frac{1}{p^{\nu}(k)p_{\nu}(k) - m^2}$$

- Since $p(k)$ has exactly one zero, there are no additional poles on the real axis

$\longrightarrow$ This goes wrong in the first order approx (signs of coefficients are not fixed)

- For the same reason, the characteristic polynomial of the wave-equation has only one (real) zero.

Applications of the Model

The model is useful to examine effects of a minimal length scale

- Modified quantum mechanics:

$\longrightarrow$ Schrödinger's equation, levels in hydrogen atom, g-2, Casimir-effect

- Derivation of modified Feynman-rules:

$\longrightarrow$ General prescription for calculations

- Tree-level cross-sections (e.g. $e^+e^- \rightarrow f^+f^-$):

$\longrightarrow$ Show overall suppression relative to SM-result

- Loop-contributions (e.g. running coupling):

$\longrightarrow$ Finite, minimal length acts as UV-regulator

Deformed Special Relativity

- Minimal length $L_{\rm min}$ requires new Lorentz-transformations
- New transformations have 2 invariants: c and $L_{\rm min}$
- Generalized Uncertainty $\Longleftrightarrow$ Deformed Special Relativity
  - When relation $k(p)$ is known and p's (usual) transformation, then also the transformation of k is known.
  - When the new transformation on k is known, then one gets $k(p)$ by boosting in and out of the restframe where $k = p$.

SH, Class. Quantum Grav. 23 (2006) 1815.

Deformed, Non-linear Action on Momentum Space

- Lorentz-algebra remains unmodified

$$[J^i, K^j] = \varepsilon^{ijk} K_k \ , \ [K^i, K^j] = \varepsilon^{ijk} K_k \ , \ [J^i, J^j] = \varepsilon^{ijk} J_k$$

- But it acts non-linearly on momentum space, e.g.*

$$e^{-iL_{ab}\omega^{ab}} \to U^{-1}(p_0) e^{-iL_{ab}\omega^{ab}} U(p_0) \quad \text{with} \quad U(p_0) = e^{L_{\min} p_0 p_a \partial p^a}$$

- Leads to Lorentz-boost (z-direction)

$$p_0' = \frac{\gamma(p_0 - v p_z)}{1 + L_{\min}(\gamma - 1) p_0 - L_{\min} \gamma v p_z}$$

$$p_z' = \frac{\gamma(p_z - v p_0)}{1 + L_{\min}(\gamma - 1) p_0 - L_{\min} \gamma v p_z}$$

which transforms $(1/L_{\min}, 1/L_{\min}) \to (1/L_{\min}, 1/L_{\min})$

*Magueijo and Smolin, Phys. Rev. Lett. **88**, 190403 (2002).

Interpretation of an Invariant Minimal Length

Besides c there is a second invariant $L_{\rm min}$ for all observers

- DSR approach (from SR)
  * Deformed transformation applies to free particles
  * Physical momentum is subject to deformed transformation
  ? If caused by quantum gravity effects what sets the scale?
- GUP approach (from particle physics)
  * Two observers can not compare lengths without interaction
  * The strength of gravitational effects sets the scale for the importance of quantum gravity
  * Free particles do not experience any quantum gravity or DSR
  * Effects apply for virtual particles in the interaction region only
  * Physical momentum transforms under standard Lorentz transformation

$\longrightarrow$ Propagator of exchange particles is modified

Features of DSR

- Non-linear transformation of physical momenta results in unusual addition law

$$\begin{aligned}\widetilde{\Lambda}(p_1+p_2) &\neq \widetilde{\Lambda}(p_1)+\widetilde{\Lambda}(p_2)\\ p_1\oplus p_2 &= p(k_1+k_2)\neq p(k_1)+p(k_2)\end{aligned}$$

$\longrightarrow$ Modification of interaction thresholds (description of particle interactions missing)

- Modified dispersion relation for free particles

$$m^2 \approx E^2 - \vec{p}^2 + \eta\left(\frac{E}{m_{\mathrm{p}}}\right)^n$$

$\longrightarrow$ Energy dependend speed of light (position space description missing)

Bugs of DSR

- Non-linear transformation of physical momenta results in unusual addition law

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Multi-Particle Limit

Problem: Multi particle states (e.g. soccer-balls) have no limit on total energy E. How to recover the standard behaviour?

- Want theory for matter fields

$\longrightarrow$ Should deal with energy densities ($T_{\mu\nu}$) instead of integrated quantities like total energy E

- Want theory that effectively describe quantum gravitational effects

$\longrightarrow$ Should become important at high densities, not high total energies

- Instead of E being bounded by m_p require ρ being bounded by m_p/l_p^3 ? (rspt. require deformed trafo for $T_{\mu\nu}u^\nu$ instead)

$\longrightarrow$ Solves soccerball problem

Observables of DSR

- Modifications of interaction thresholds: GZK cutoff...

Aloisio, Blasi, Ghia and Grillo, Phys. Rev. D **62**, 053010 (2000)
Amelino-Camelia and Piran, Phys. Rev. D **64**, 036005 (2001)
J. Alfaro and G. Palma, Phys. Rev. D **67**, 083003 (2003)

R. Abbasi *et al.* [HiRes Collaboration] astro-ph/0703099
"The High Resolution Fly's Eye (HiRes) experiment has observed the GZK cutoff. HiRes' measurement of the flux of cosmic rays shows a sharp suppression at an energy of 6×10^{19} *eV, exactly the expected cutoff energy."*

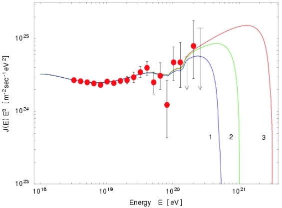

Figure by Alfaro and Palma

- Energy dependent speed of light
  Modifications in the time of flight for γ-ray bursts

Amelino-Camelia, Phys. Rev. D **64** (2001) 036005
Magueijo and Smolin, Phys. Rev. Lett. **88** (2002) 190403
Judes and Visser, Phys. Rev. **D 68** (2003) 045001

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$\longrightarrow$ Conclusion: Develop models that can be applied to various effects, and combine predictions to solve inverse problem.

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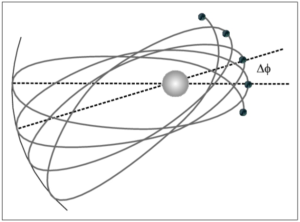

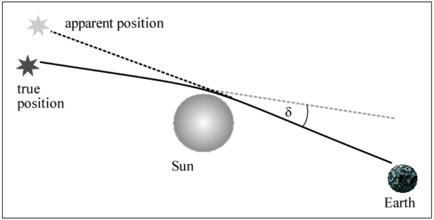

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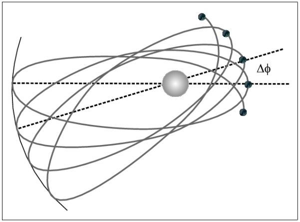

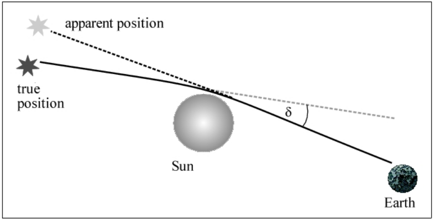

... a factor two mattered ...

Summary

"The problem is all inside your head, she said to me
The answer is easy if you take it logically
I'd like to help you in your struggle to be free
There must be fifty ways to [quantum gravity] "

Paul Simon, (*50 Ways to leave your Lover*)