Nonlocality and Stochasticity

in micro-Macro & Quantum-Classical Interfaces

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Apologies

1. Unlike most other talks, this one focuses on ideas. Very little mathematics in this talk. So, *not even imprecise*.
2. Many ideas you will find familiar or commonplace.
3. You might find this talk irrelevant to what you are doing.

Familiar, irrelevant or **out of place**

- like Marcel Duchamp's *Fountain*

- Actually this is our starting point (Noise and nonlocality)
- And one of the purposes of this talk:

Have we overlooked something elementary yet important ?

Quantum: micro (Quantum Field Theory)
Gravity: Macro (General Relativity)

- **What is Quantum Gravity**? - A theory of the **microscopic structure of spacetime**

  -- agreement

- But it is not necessarily a theory obtained by **quantizing general relativity**. -- disagreement?

- If it is, like electromagnetism, we are lucky / happy.
- I'm dwelling on the unhappy situation. Sorry.

Quantizing macro variables may not give micro structure

- ***Quantizing macro-variables is not a guaranteed way to obtain micro-structure***.

- Cases which work: **EM → QED**. *Macro and micro variables are the same. [linear theory]* **He 4**

- But**: Difference between He 4 and He 3**.

  For He 3 – to get the macroscopic collective behavior, need details at the atomic level

Q-C versus m-M

Quantum → decoherence, robustness, stability → **Classical**

← **Traditional effort:** quantizing the metric or connection forms

Quantum Gravity (Strings Loops Simplices – micro constituents)

MICRO fluctuations

coarse-|-graining MESO kinetic theory

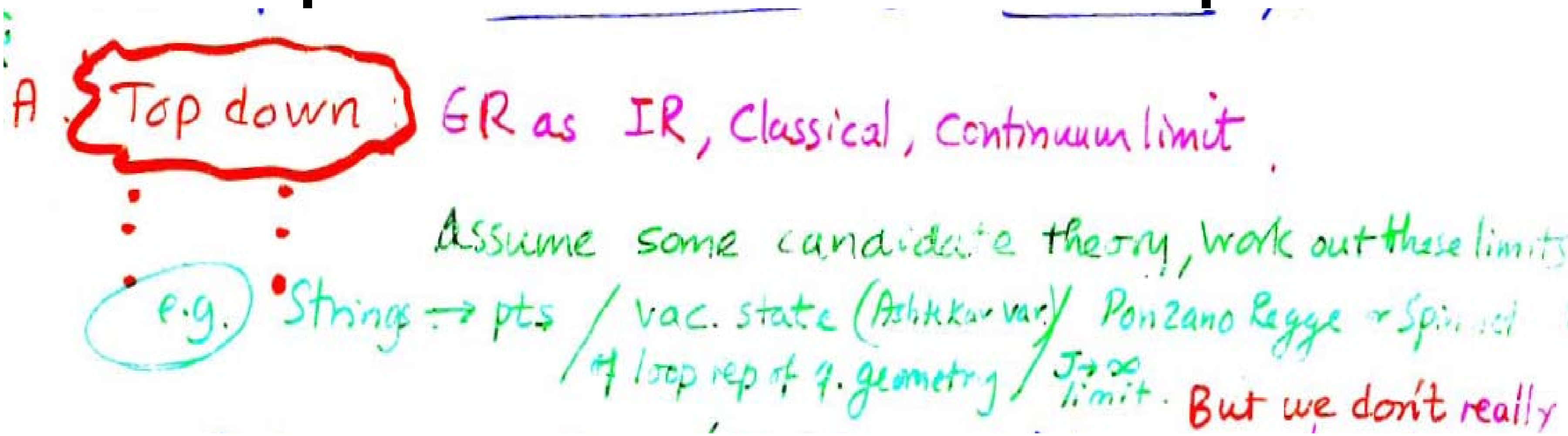
v *Emergent* hydrodynamics

MACRO *spacetime* *General Relativity*

Issues: Coherence, Correlations, Fluctuations, **Stochasticity**;

Collectivity, Variability, Nonlinearity, **Nonlocality**

Top-down or Bottom-up?



String Theory: "Replacing" points in spacetime by strings, closed and open -- closed loop for gravitons.

- Great insight into the relation of spacetime and gauge fields
- But has difficulty producing spacetime itself

Loop Gravity: Quantizing the connection form or holonomy of General Relativity in the Loop (Faraday's) Representation

➔ Spin Network / Foam as the Micro Structure of Spacetime

➔ How to recover the semiclassical limit? " I want to Go Home!" -- *ET*

We need both – physics has always been like that

Happy and not -so- happy situations

If we are confronted with this unfortunate situation, we need statistical mechanics, stochastic processes, probability theory, etc..

If we are blessed with the happy situation we still need statistical mechanics, even condensed matter physics *Uhh!*

Issues all Top-Down models need to deal with:

to introduce stat. concepts:

- Closed → open system : dissipative / noise

Coarse-graining: low energy behavior: Effective Field Theory

Backreaction

statistical

- Decoherence and Quantum → Classical (Gell-Mann Hartle, Griffiths, Omnes

→ what Coarse-graining measures give stable ... give the most appropriate description

- → what Collective Variables,

Emergence of Effective Theories

Collectivity and Emergence

Issues:

$\mu \to M$:
- What constitutes a stable level (e.g. Ren Grp concepts)
- Effective Theory: How good is it?
- What are the most appropriate collective variables at each level of structure & Dynamics.

§ to different microscopic theories
"Hydrodynamics of M-Theory"
(C. Herzog)

$Q \to C$:
- Emergence of quasi-Classical domain: Conditions
- Decoherence: Who chooses the projections?
how
Criteria in coarse-graining:
- stable structures / insensitive to CG
- robustness / variation

Dynamically generated?

• See my article in JJ Halliwell's book on "Time Asymmetry"

Guideposts:
- Coherence
- correlation
- fluctuations
- variability
- collectivity
- nonlocality

nonlinearity

FDR

stochasticity

(e.g. emergence of forms)

B. L. Hu, *in " The Physical Origin of Time Asymmetry" Proceedings of Conference in Huelva, Spain, Oct. 1991. edited by J. J. Halliwell, J. Perez-Mercader and W. H. Zurek (Cambridge University, Cambridge, 1994). gr-qc/9302021.*

Need to Deal with Strongly Interacting and Correlated Systems

B. For deductive emergent behavior, path could be tortuous,

- Usually encounters **nonlinear interactions in strongly correlated systems.**
- Need to identify collective variables at successive levels of structure. Cumbersome to deduce M- with μ-dynamics (e.g., intermediate between μ (molecular) and M (hydro) are **kinetic variables**. Use maximal entropy laws at stages – but how are they related to each other, becomes maximal when?)

And, **nonlocal properties** can emerge. Very involved,

- requires not just hard work of deduction from one level, but new ideas at each level. Interesting challenge.

3. Bottom up — Grass Root Approach

- more realistic depiction; though less complete theory
- rely on phenomenology

— quark gluon
— nucleus
— atoms
— molecules

Some conceptual issues.

1) 'elementary' vs 'compositeness'

- depends on specific energy scale:
- an 'elementary' particle at lower energy behaves like a composite particle at higher energy

2) 'basic laws' vs 'effective descriptions'

QED, QCD, etc.

- emergent symmetries
- collective phenomena

How to find our way up from the grass-root?

Given that observations can only be made at low energies what can one deduce about the progenator theory operative at a higher energy? – 'Tell-tale signs'?

(This was done historically in our advancement of knowledge of 'elementary' particle physics)

(1) → look at junction points between 2 levels in the hierarchy of structures and interactions

objective [Stable phase 1 $\xrightarrow[\text{instability}]{\text{fluctuations}}$ stable phase 2

Phase Transition? different structures

↓ Use this view

→ to go beyond semiclassical gravity: Focus on noise/fluctuations of fields and spacetime

Env. ...

Bottom-up: Macro to Micro

II. Going the reverse way (quest **from macro to micro** structure) is always difficult, if not impossible. BUT,

... that is how physics has progressed through centuries!

Rely on:

A. Topological structures:

More resilient to evolutionary or environmental changes.

See approaches of Volovik (He3 analog, Fermi surface)

Wen (string-nets, emergent light and fermions)

B. Noise-fluctuations: Fluctuations can reveal some substructural contents and behavior (**critical phenomena**).

Information contained in remnants or leftovers. Yet, by reconstructing from corrupted and degraded information one could perhaps have a glimpse of the micro structure.

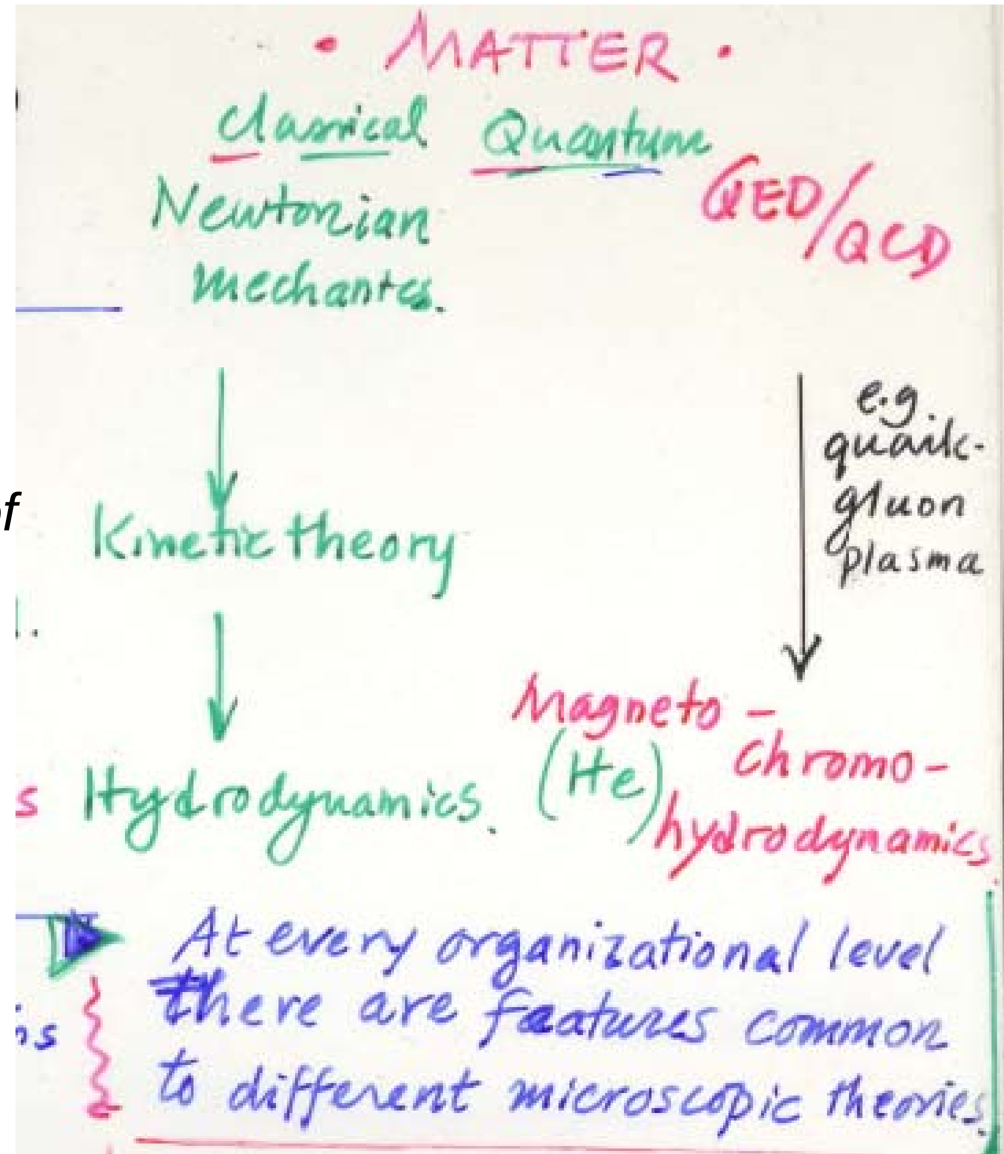
Common Physical Features of Macroscopic Phenomena:

Micro

There are commonalities in the MACroscopic collective behavior of different MICroscopic constituents

Macro

Separate the common features
So as to pinpoint the particulars



General Relativity + Quantum Field Theory → Quantum Field Theory in Curved Spacetime → **Semiclassical Gravity → Stochastic Gravity**

Semiclassical Gravity

1970s, 1980s

- Quantum Fields in ~~Curved~~ classical Space : Test Field approx.
  - Vacuum state (Fulling 71)
  - Particle creation : Cosmological Spacetimes; Black Holes : Hawking Radiation (1974)
  - Regularization of $\langle T_{\mu\nu} \rangle$

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu}(\phi) \rangle_0$$

Expectation value wrt vacuum

- Backreaction :

Spacetime not fixed, but determined by field and vice versa

- Self-consistency

[mean field theory]

SCG is a Mean Field Theory

A (very modest) Bottom-Up route:

Stochastic Semiclassical Gravity : 1990s - 2000

noise-averages

Einstein-Langevin equation (more details later)

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G \langle T_{\mu\nu} \rangle_q + T^{stoch.}_{\mu\nu}$$

noise term from fluctuations of quantum fields

- Backreaction: dissipation

$T^{st}_{\mu\nu} = 2\tau_{\mu\nu}$ classical stochastic source

- Decoherence by quantum field leads to quantum → emergence of classical spacetime

noise

Key ingredient: **NOISE, FLUCTUATIONS**

➔ PHASE TRANSITION, STRUCTURE FORMATION, ENTROPY GENERATION

Stochasticity: nonequilibrium statistical mechanics

Alternative (my) view towards Quantum Gravity – some ideas leading to it: (from Sakharov 1968)

- *Cosmology as `condensed matter' Physics* (1988):
  Phase transition etc. also L. Smolin, W.H. Zurek
- *Semiclassical (mean field) Stochastic (fluctuations) Gravity as Mesoscopic Physics* (1994): **noise and correlations**
- *General Relativity as Hydrodynamics (1996):* conservation laws help **decoherence** of long wavelength modes; Quasiclassical domains
- *From Stochastic Gravity to Quantum Gravity* (1999): Retrieve quantum coherence information from **fluctuations** with C. Anastopoulos, E.Verdaguer
- *Kinetic Theory Approach to Quantum Gravity* (2002): **Correlations**
  find the micro-variables rather than quantize the macroscopic variables.
- *What can we learn about Quantum Gravity from BEC?* (2003)
  Spacetime as Condensates (2005) with A. Roura, See, Volovik: Universe as He3

1 **Quantizing macro-variables =?/= micro theory**
C → Q, M → μ What is quantum gravity?

2. **Emergent theories**: μ → M. Emergence vs Reduction;
Deductible vs Nondeductible.

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3. **Nonlocality** in QM, QFT, spacetime and QG
Some familiar examples from statistical mechanics

4. **Micro nonlocality =/= macro nonlocality**
Ex: micro weave state and nonlocality in macro-spacetime

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5. **Nonlocality and Stochasticity in statistical mechanics**
- Open (Langevin) and Effectively open (Boltzmann) systems
- Coarse-graining and Backreaction

6. **Nonlocality and Stochasticity in Quantum Gravity**
Semiclassical and Stochastic gravity: non-Markovian dynamics
Nonlocality in time (Memory). Nonlocal dissipation and colored noise

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7. **Correlation: Another angle towards nonlocality and stochasticity**
Strongly interacting and correlated systems: Condensed matter models

8. Summary and Tasks.

Nonlocality

Quantum Mechanics: EPR, Bell. We know a little more now

Nonlocality =/= Entanglement (Unruh)

Entanglement measured by correlations (Cirac)

Quantum Field Theory: extended in space vs local observables

Spacetime: Nonlocality versus observance of causal structure

Noncommutative geometry, NC field theory: Nonlocality

Quantum Gravity: String theory, Loop QG, spin-nets …

! micro locality (weave state) =/= macro locality (spacetime) !

Statistical Mechanics: Macro (collective variable) dynamics is often very different from micro dynamics. **Emergent phenomena**

Some familiar examples

Micro: (Molecular) vs **Macro** (Hydro) dynamics:

individual molecules (x_i, p_i) — fluid elements

Newton's second law — Euler equation.

Time scales: Dynamical scale of molecular motion >> Diffusion time

Length scales: Multi-scales appearing, as in Turbulence

Scales can also change drastically as at a Critical point

Molecular to Hydro- dynamics is probably the easiest emergent phenomenon. There are more involved macroscopic emergent phenomena which cannot be predicted, nor so easily deductible, from micro dynamics. E.g. **quantum Hall effect**.

Macrostates are not just superpositions of microstates.

Emergence is more than just repetitive coarse graining

Micro locality =/= Macro locality

A molecule will collide with many others at very high speed for a long time before reaching a macroscopic observer. That molecule's apparent locality (with respect to other molecules arriving at about the same time) in a gas element is very different from its nonlocal history after multiple collisions.

Though each collision is local (contact potential, short ranged, unlike Vlasov dynamics of a long ranged interaction, or collisionless Boltzmann), however, when viewed at the collective level, each particle has a highly nonlocal history.

[Cause: divergent congruences in chaotic dynamics, Liapunov exponent --- Distinction between chaotic system, mixing system and ergodic system]

- **The smoothness and continuity** of the fluid element's movement to a macroscopic observer described by the Navier-Stoke equation (diffusion, transport) belie **the stochastic and nonlocal nature** of molecular collisions.

Molecular to hydrodynamics is one example where one can deduce the macro time scales from the micro.   But many systems are NOT like that.

Similar situation in quantum gravity

Example: in **loop quantum gravity**, a **weave state** is a kinematical state designed to match a given slowly varying classical spatial metric. [Ashtekar, Rovelli, Smolin 92]

The concept of **quantum threads** (spin-network) **weaved into a fabric** (manifold) of classical spacetime already **tacitly assumes a particularly simple kind of μ-M transition**, where there is a simple correspondence or even equivalence between **locality at the micro AND the macro levels**.

This is not the case for even simple examples of emergence like molecular to hydro- dynamics.

Think about the **sense of locality** at the level of M theory for **string theory** or simplices for **dynamical triangulation** versus locality in our macroscopic spacetime, presumably emergent.

<< nonlocal weave states >>

leave marks on the fabrics of spacetime

- **Bombelli** [gr-qc/0101080]; **Bombelli, Corichi and Winker**, gr-qc/ 0409006] use combinatorial methods to turn random weave (micro) states into (macro) states of spacetime manifold.
- **Markopoulou & Smolin** [gr-qc/0702044] recently pointed out:

  - most weave states including Bombelli et al's assume that these weave states all satisfy an unstated condition of locality (edges connect two nodes of metric distance ~ Planck length)

  - there are plenty of weave states which do not satisfy this condition: **existence of nonlocal weave states**.
- **Question**: How does one weave from these nonlocal micro states a fabric (manifold) with the familiar macroscopic locality?
  Fotini & Lee's answer : Disordered locality can be tolerated, even useful in the production of dark energy.

***My attitude*:**

Take this **inequivalence of micro and macro locality** seriously. Need to depart, even radically, from familiar concepts in our macro world. There is **new physics** to be uncovered**!**

A good example: Recent discoveries that basic laws of non-equilibrium thermodynamics (like 2nd law) can be understood or derived from chaotic dynamics. [Gaspard, Dorfman et al]

Our conception and construct of the macro world may not bear any resemblance to the micro world.

- (Non) locality at one level may have little to do with (Non) locality at other levels.
- The easy ways of $\mu \rightarrow M$ (weaving) or $c \rightarrow Q$ (quantizing) may not be the true ways.

Nonlocality and Stochasticity from statistical mechanics

A simple way to see how nonlocality emerges in dynamics [Zwanzig] :
One closed system divided into two subsystems A and B:

- Dynamics of each subsystem obeys 2nd order ODE

-Can transform this set of coupled equations into an *integral differential equation* governing only variables of A (or B)

- Nonlocal in time because it contains the natural time scales of subsystem B's dynamics. Information is subsumed (not erased)

-**Back-reaction** is the source of nonlocality in time (memory)

Coarse-Graining and Backreaction

If subsystem B has many degree of freedom
– call it environment E

- **Coarse-graining** of subsystem B (or the environment E) –
  - can begin to talk in terms of mean field and fluctuations
  - renders the combined system an open system
  – information lost by choice (or limitations)

- **Backreaction** of coarse-grained environment on the system engenders *nonlocal dissipation* in open system dynamics

- Nonlocal dissipation is accompanied by nonlocal fluctuations (*colored noise*)

Q. Closed System

Closed System: Quantum deterministic Dynamics

Subsystems A + B

$\boxed{\text{System}}$ / + reversible unitary evolution — many dof: $\boxed{\text{Environment}}$

· particle
· spacetime
(macro)

· field *
· fluctuations

* COARSE GRAINING
(micro)

$S(x)$ $E(q)$

→ Open System

(Effective Theory)

$\mathcal{F}[x, x']$ after integrating out q, q'
nonunitary, dissipative
irreversible

Quantum Open System→

• Influence Functional / action $\mathcal{F} = e^{iS_{IF}}$

(Closed-time-path coarse-grained effective action)

S_{IF} : $\boxed{\mu + i\nu}$

Backreaction

$\boxed{\text{DISSIPATION}}$ kernel μ ← $\boxed{\text{NOISE}}$ / Fluctuation kernel $\nu(t, t')$ $= \frac{1}{\hbar}\langle \xi(t)\xi(t')\rangle$

renormalization Q
UV divergences

Decoherence

issues in M_{maco} meso μ_{ico} Issues in

Quantum Field Theory ◁ C ▷ N Eq. Stat. Mechanics

Quantum
Open System

• Influence Functional $\mathcal{F} = e^{iS_{IF}}$ / action

(Closed-time-path coarse-grained effective action)

S_{IF} : $\boxed{\mu + i\nu}$

Backreaction

DISSIPATION ← kernel μ

NOISE / Fluctuation kernel $\nu(t,t') \equiv \frac{1}{\hbar}\langle \xi(t)\,\xi(t')\rangle$

Decoherence

renormalization

UV divergences

Q

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issues in Quantum Field Theory ◁

▷ Issues in NEq. Stat. Mechanics

Quantum → Classical via (environment-induced) decoherence

Master Eqn., Fokker-Planck Eqn.

Langevin Eqn. Classical Stochastic Dynamics

e.g.

$$M \frac{d^2}{ds^2} \underbrace{X_c(s)}_{} + 2M \int_0^s ds' \underbrace{\gamma(s-s')}_{} \frac{d}{ds'} X_c(s') + \underbrace{M\omega_0^2}_{\Omega^2} X_c(s) = F_\xi(s)$$

Noise

CM of classical paths

$\frac{d\gamma}{ds} = \mu$ Dissipation

ξ for linear cplg

Quantum Open System

Closed System: **Density Matrix** $\hat{\rho}(t) = \mathcal{J}(t, t_i)\hat{\rho}(t_i).$

$\mathcal{J}(x, \mathbf{q}, x', \mathbf{q}', t \mid x_i, \mathbf{q}_i, x'_i, \mathbf{q}'_i, t_i)$ is the (**unitary**) **evolutionary operator** of the system from initial time t_I to time t.

OPEN SYSTEM: System (s) interacting with an Environment (e) or Bath (b): Integrate out (coarse-graining) the bath dof renders the system open. Its evolution is described by the Reduced Density Matrix

$$\rho_r(x, x') = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \rho(x, \mathbf{q}; x', \mathbf{q}')\delta(\mathbf{q} - \mathbf{q}')$$

$$\rho_r(x, x', t) = \int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty} dx'_i \; \mathcal{J}_r(x, x', t \mid x_i, x'_i, t_i) \; \rho_r(x_i, x'_i, t_i \;).$$

Quantum Brownian Motion

System (S): quantum oscillator with time dependent natural frequency
Environment (E) : n-quantum oscillators
with time-dependent natural frequencies = Scalar Field
Coupling: c_n F (x) q_n.

$$S[x,\mathbf{q}] = S[x] + S_E[\mathbf{q}] + S_{\text{int}}[x,\mathbf{q}]$$
$$= \int_0^t ds \left[\frac{1}{2} M(s)[\dot{x}^2 + B(s)x\dot{x} - \Omega^2(s)x^2] \right.$$
$$\left. + \sum_n \left\{ \frac{1}{2} m_n(s)[\dot{q}_n^2 + b_n(s) q_n \dot{q}_n - \omega_n^2(s) q_n^2] + \sum_n \left(-c_n(s) F(x) q_n \right) \right. \right]$$

Influence Functional

Assume **factorizable condition** between the system (s) and the bath (b) initially

$$\hat{\rho}(t=t_i) = \hat{\rho}_s(t_i) \times \hat{\rho}_b(t_i),$$

:

Evolutionary operator for the reduced density matrix is

$$\mathcal{J}_r(x_f, x'_f, t \mid x_i, x'_i, t_i) = \int_{x_i}^{x_f} Dx \int_{x'_i}^{x'_f} Dx' \, \exp\left(\frac{i}{\hbar}\left\{S[x] - S[x']\right\}\right) \, \mathcal{F}[x, x']$$

Influence Functional

$$\mathcal{F}[x, x'] = \int_{-\infty}^{+\infty} d\mathbf{q}_f \int_{-\infty}^{+\infty} d\mathbf{q}_i \int_{-\infty}^{+\infty} d\mathbf{q}'_i \int_{\mathbf{q}_i}^{\mathbf{q}_f} D\mathbf{q} \int_{\mathbf{q}'_i}^{\mathbf{q}_f} D\mathbf{q}'$$

$$\exp\left(\frac{i}{\hbar}\left\{S_b[\mathbf{q}] + S_{\text{int}}[x, \mathbf{q}] - S_b[\mathbf{q}'] - S_{\text{int}}[x', \mathbf{q}']\right\}\right) \times \rho_b(\mathbf{q}_i, \mathbf{q}'_i, t_i)$$

Influence Action

$$= \exp\left(\frac{i}{\hbar}\delta\mathcal{A}[x, x']\right)$$

Influence functional for a Paramp

$$\mathcal{F}[x,x'] = \exp\Big\{ -\frac{i}{\hbar}\int_{t_i}^{t} ds \int_{t_i}^{s} ds' \Big[F(x(s)) - F(x'(s))\Big]\mu(s,s')\Big[F(x(s')) + F(x'(s'))\Big]$$
$$-\frac{1}{\hbar}\int_{t_i}^{t} ds \int_{t_i}^{s} ds' \Big[F(x(s)) - F(x'(s))\Big]\nu(s,s')\Big[F(x(s')) - F(x'(s'))\Big]\Big\}$$

$$\Sigma(s) = \frac{1}{2}\Big(F(x(s)) + F(x'(s))\Big),$$

$$\Delta(s) = F(x(s)) - F(x'(s)),$$

Dissipation μ and Noise ν Kernels

$$\mathcal{F}[x,x'] = \exp\Big\{ \frac{i}{\hbar}\int_{t_i}^{t} ds\Delta(s)\langle\xi(s - \frac{1}{\hbar^2}\int_{t_i}^{t} ds \int_{t_i}^{s} ds'\Delta(s)\Delta(s')C_2(s,s') \Big\}$$

$$\langle\bar{\xi}(t)\bar{\xi}(t')\rangle = C_2(s,s') \equiv \hbar\nu(s,s')$$

Langevin Equation:

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - 2\frac{\partial F(x)}{\partial x}\int_{t_i}^{t}\mu(t,s)F(x(s))ds = -\frac{\partial F(x)}{\partial x}\bar{\xi}(t)$$

Noise and Dissipation Kernels

Equation of Motion for the amplitude function of a Parametric Oscillator

$b_n = 0$ and $m = 1$ $\quad \kappa_n = m_n(t_i)\omega_n(t_i)$ $\quad \ddot{X}_n + \omega_n^2(t)X_n = 0,$

$$\mu(s,s') = \frac{i}{2}\int_0^\infty d\omega I(\omega,s,s')\Big[X_\omega^*(s)X_\omega(s') - X_\omega(s)X_\omega^*(s')\Big],$$

$$\nu(s,s') = \frac{1}{2}\int_0^\infty d\omega I(\omega,s,s')\coth\left(\frac{\hbar\omega(t_i)}{2k_BT}\right)\Big[\cosh 2r(\omega)\Big[X_\omega^*(s)X_\omega(s') + X_\omega(s)X_\omega^*(s')\Big]$$

$$-\sinh 2r(\omega)\Big[e^{-2i\phi(\omega)}X_\omega^*(s)X_\omega^*(s') + e^{2i\phi(\omega)}X_\omega(s)X_\omega(s')\Big]\Big].$$

Spectral Density Function

$$I(\omega,s,s') = \sum_n \delta(\omega - \omega_n)\frac{c_n(s)c_n(s')}{2\kappa_n}$$

$I(\omega) \sim \omega^n$ $\quad$ n=1: Ohmic, n>1 Supra Ohmic; n<1 Subohmic

Squeezed and Rotation parameters:

$$\hat{\rho}_b(t_i) = \prod_n \hat{S}_n(r(n),\phi(n))\hat{\rho}_{\rm th}\hat{S}_n^\dagger(r(n),\phi(n))$$

e.g., for an initial squeezed thermal bath

Stochastic Equations

Non- Markovian Master Equation:

$$i\hbar\frac{\partial}{\partial t}\hat{\rho}_r(t) = [\hat{H}_{\rm ren},\hat{\rho}] + iD_{pp}[\hat{x},[\hat{x},\hat{\rho}]] + iD_{xx}[\hat{p},[\hat{p},\hat{\rho}]]$$

Nonlocal dissipation
Nonlocal fluctuations
(Colored noise)

$$+ iD_{xp}[\hat{x},[\hat{p},\hat{\rho}]] + iD_{px}[\hat{p},[\hat{x},\hat{\rho}]] + \Gamma[\hat{x},\{\hat{p},\hat{\rho}\}],$$

$$\hat{H}_{\rm ren} = \frac{\hat{p}^2}{2M(t)} - \frac{B(t)}{4}(\hat{p}\hat{x}+\hat{x}\hat{p}) + \frac{M(t)}{2}\Omega_{\rm ren}(t)\hat{x}^2.$$

Wigner Function:

$$F_W(\Sigma,p,t) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{ip\Delta/\hbar}\left\langle\Sigma-\frac{\Delta}{2}\middle|\hat{\rho}\middle|\Sigma+\frac{\Delta}{2}\right\rangle d\Delta,$$

Fokker-Planck or Wigner Equation: (Non-Markovian)

$$\frac{\partial}{\partial t}F_W(\Sigma,p,t) = \left[-\frac{p}{M(t)}\frac{\partial}{\partial\Sigma} + \frac{1}{2}M(t)\Omega^2_{\rm ren}(t)\,\Sigma\frac{\partial}{\partial p} + \Gamma(t)\frac{\partial}{\partial p}p - 2D_{pp}(t)\frac{\partial^2}{\partial p^2}\right.$$
$$\left. -\hbar D_{xx}(t)\frac{\partial^2}{\partial\Sigma^2} + 2\Big(D_{xp}(t)+D_{px}(t)\Big)\frac{\partial^2}{\partial\Sigma\partial p}\right]F_W(\Sigma,p,t).$$

Closed system of n interacting molecules:

Molecular Dynamics is unitary

Hamiltonian
$$H = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2m} + \frac{1}{2}\sum_{i \neq j} V_{ij}; \qquad V_{ij} = V\left(|\mathbf{x}_i - \mathbf{x}_j|\right) \tag{2.3.1}$$

(We assume no self energies: $V_{ii} = 0$) leads to the Hamilton equations

$$\frac{d\mathbf{x}^i}{dt} = \frac{\partial H}{\partial \mathbf{p}_i} = \frac{\mathbf{p}^i}{m}; \qquad \frac{d\mathbf{p}^i}{dt} = -\frac{\partial H}{\partial \mathbf{x}_i} = -\sum_{i \neq j} \frac{\partial V_{ij}}{\partial \mathbf{x}_i}. \tag{2.3.2}$$

Equivalently we may describe the state of the system through a $6N$- dimensional distribution function $\rho = \rho\left((\mathbf{x}_1, \mathbf{p}_1), ..., (\mathbf{x}_N, \mathbf{p}_N), t\right)$, which satisfies the Liouville equation

$$\frac{\partial \rho}{\partial t} = -\{H, \rho\}, \tag{2.3.3}$$

Expressed in terms of nth order correlation functions: BBGKY hierarchy contains full information of molecular gas

Truncation of hierarchy: Keeping only the one particle distribution function, Imposing the molecular chaos assumption

➔ Boltzmann equation with source (collision integral) given by the two particle correlation function (initially factorizable) describes an **effectively open system**

Nonlocality residing in the higher order correlation functions manifest as *nonlocal dissipation* and *colored noise*

Dissipation in Boltzmann Eqn:

$$\frac{\partial f_1}{\partial t}(\mathbf{x}_1, \mathbf{p}_1) = -\frac{\mathbf{p}_1}{m}\frac{\partial f_1}{\partial \mathbf{x}_1} + \frac{\partial}{\partial \mathbf{p}_1}\int d\mathbf{x}_2 d\mathbf{p}_2 \left[\frac{\partial}{\partial \mathbf{x}_1} V(|\mathbf{x}_1 - \mathbf{x}_2|)\right] f_2((\mathbf{x}_1, \mathbf{p}_1), (\mathbf{x}_2, \mathbf{p}_2)). \qquad (2.3.12)$$

To obtain the dynamics for f_1 we need the dynamics for f_2. This is obtained in an analogous way

$$\frac{\partial f_2}{\partial t} = -N(N-1)\int \prod_{j=3}^{N} d\mathbf{x}_j d\mathbf{p}_j \ \{H, \rho\} \qquad (2.3.13)$$

Nonlocal Dissipation

Fluctuations? Yes. Nonlocal

$$\frac{\partial f}{\partial t} + \frac{\mathrm{p}}{\omega_p}\nabla f = \frac{1}{\omega_p} I_{col} + j(X, \mathrm{p}) \qquad (2.3.52)$$

The Γ matrix has an asymmetric part (coming from the spatial gradients term) and a symmetric part (coming from the linearization of the collision integral). Only the latter contributes to the noise auto-correlation, and so we obtain

$$\langle j(X, \mathbf{p})\, j(Y, \mathbf{q})\rangle = -\left\{ \frac{1}{\omega_p} \frac{\delta I_{col}(X, \mathbf{p})}{\delta F(Y, \mathbf{q})} + \frac{1}{\omega_q} \frac{\delta I_{col}(Y, \mathbf{q})}{\delta F(X, \mathbf{p})} \right\} \qquad (2.3.53)$$

Thermodynamic Force

$$F(x, \mathbf{p}) = -\frac{\delta S}{\delta(\delta f)} = \frac{1}{(2\pi)^3} \frac{\delta f(x, \mathbf{p})}{[1 + f_{eq}(p)]\, f_{eq}(p)}$$

Entropy current

in terms of 1-particle distribution function:

$$S^\mu(x) = \int Dp\, p^\mu \left\{ [1 + f(p)] \ln [1 + f(p)] - f(p) \ln f(p) \right\}.$$

Boltzmann-Langevin Equation

Nonlocality in time: Memory effects and Stochasticity – colored noise

occur naturally in open systems
and effectively open systems

Nonlocality and Stochasticity in Quantum Gravity

Ought to be able to demonstrate in various approaches in QG:

- string interactions (duality?)
- Simplicial (can nonlocal simplices constitute quasilocal neighborhoods in macroscopic spacetime with manifold structure)
- Loop (how does intertwining operators generate weave states)
- Spin-nets
- Causal sets

I don't understand enough about nonlocality in quantum gravity.

Will talk about something I understand, starting from semiclassical gravity

Semiclassical Gravity

Schematically:

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa\langle\hat{T}_{\mu\nu}\rangle_q$$

$\tilde{G}_{\mu\nu}$ is the Einstein tensor (plus covariant terms associated with the renormalization of the quantum field – details in a later slide)

$\kappa = 8\pi G_N$ and G_N is Newton's constant

Free massive scalar field $(\Box - m^2 - \xi R)\hat{\phi} = 0.$

$\hat{T}_{\mu\nu}$ is the stress-energy tensor operator

$\langle\,\rangle_q$ denotes the expectation value

Semiclassical Einstein Equation I

$$G_{ab}[g] + \Lambda g_{ab} - 2(\alpha A_{ab} + \beta B_{ab})[g] = 8\pi G \langle \hat{T}^R_{ab}[g] \rangle,$$

- Matter: Quantum Scalar Field

$$S_m[g,\phi] = -\frac{1}{2}\int \mathrm{d}^4x \sqrt{-g}\,[g^{ab}\nabla_a\phi\nabla_b\phi + (m^2 + \xi R)\phi^2],$$

$$T^{ab}[g,\phi] = \nabla^a\phi\nabla^b\phi - \tfrac{1}{2}g^{ab}(\nabla^c\phi\nabla_c\phi + m^2\phi^2) + \xi(g^{ab}\Box - \nabla^a\nabla^b + G^{ab})\phi^2,$$

Semiclassical Einstein Equation II

- Spacetime: Dynamics from a semiclassical action

$$S_g[g] = \frac{1}{8\pi G}\int d^4x\sqrt{-g}\left[\frac{1}{2}R - \Lambda + \alpha C_{abcd}C^{abcd} + \beta R^2\right],$$

$$\begin{aligned}A^{ab} &= \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g_{ab}}\int d^4\sqrt{-g}\,C_{cdef}C^{cdef} \\ &= \tfrac{1}{2}g^{ab}C_{cdef}C^{cdef} - 2R^{acde}R^{b}{}_{cde} + 4R^{ac}R_{c}{}^{b} - \tfrac{2}{3}RR^{ab} \\ &\quad - 2\,\Box R^{ab} + \tfrac{2}{3}\nabla^a\nabla^b R + \tfrac{1}{3}g^{ab}\,\Box R,\end{aligned}$$

$$\begin{aligned}B^{ab} &= \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g_{ab}}\int d^4\sqrt{-g}\,R^2 \\ &= \tfrac{1}{2}g^{ab}R^2 - 2RR^{ab} + 2\nabla^a\nabla^b R - 2g^{ab}\,\Box R,\end{aligned}$$

Back-reaction of quantum field processes (e.g., particle creation) engenders **nonlocal dissipation** in the spacetime dynamics:

Anisotropy Damping: Bianchi I + conformal scalar field (Hartle-Hu 79, Calzetta-Hu 86)

$$\frac{\delta}{\delta\beta^{+}_{ij}}(S_g+\Gamma_f)\big|_{\beta^{\pm}=\beta^{-}=\beta} = -2\kappa\frac{d}{d\eta}(a^2\beta'_{ij})$$

$$+\frac{1}{30(4\pi)^2}\frac{d^2}{d\eta^2}[\beta''_{ij}\ln(\mu a)] + \frac{1}{90(4\pi)^2}\frac{d}{d\eta}[((\frac{a'}{a})^2+\frac{a''}{a})\beta'_{ij}]$$

$$-\frac{1}{30(4\pi)^2}\int_{-\infty}^{\infty}d\eta'\beta_{ij}(\eta')R_4(\eta-\eta') - \frac{1}{30(4\pi)^2}\int_{-\infty}^{\infty}d\eta'\beta_{ij}(\eta')iI_4(\eta-\eta')$$

$$= -J_{ij}(\eta)$$

Dissipation kernel R is nonlocal: contains effect of particles created in the whole history

Another angle towards nonlocality: **Correlations** related to **Fluctuations**

- **Strongly correlated (mesoscopic) systems**.

  Plenty of condensed matter physics models --

  - Ising model (2D) [Wan 06], Quantum Graphity [Konopka, FM, Smolin 06]

  - cluster states (built-in correlation in system)

  - star states (equal mutual interaction)
- Start from microscopic entities, examine the **conditions for the hydrodynamic / thermodynamic states to emerge**.

  If no such conditions, *why*?
- **Correlation** functions measure **nonlocality** and can be related to **fluctuations** [Fluctuation-dissipation theorem can be phrased In terms of correlations]
- Can add interaction with environment to examine

  Decoherence and Q → C transition. An important factor in quantum information processing with many qubits.

  E.g., Decoherence of strongly correlated spin systems

Stochastic Gravity

Schematically:

$$\tilde{G}_{\mu\nu}(g_{\alpha\beta}) = \kappa \left(T^{\mathrm{c}}_{\mu\nu} + T^{\mathrm{qs}}_{\mu\nu} \right)$$

$T^{c}_{\mu\nu}$ is due to classical matter or fields

$$T^{\mathrm{qs}}_{\mu\nu} \equiv \langle \hat{T}_{\mu\nu} \rangle_{\mathrm{q}} + T^{\mathrm{s}}_{\mu\nu}$$

$T^{\mathrm{qs}}_{\mu\nu}$ is a new stochastic term related to the quantum fluctuations of $T_{\mu\nu}$

Noise Kernel

A physical observable that describes these fluctuations to the lowest order is the noise kernel which is the vacuum expectation value of the two-point correlation function of the stress-energy operator

$$N_{abcd}[g;x,y) = \frac{1}{2}\langle\{\hat{t}_{ab}[g;x),\hat{t}_{cd}[g;y)\}\rangle,$$

$$\hat{t}_{ab}[g;x) \equiv \hat{T}_{ab}[g;x) - \langle\hat{T}_{ab}[g;x)\rangle.$$

Noise associated with the fluctuations of a quantum field

- The noise kernel is real and positive semi-definite as a consequence of stress energy tensor being self-adjoint

the ultraviolet behaviour of $\langle \hat{T}_{ab}(x)\hat{T}_{cd}(y)\rangle$ is the same as that of $\langle \hat{T}_{ab}(x)\rangle\langle \hat{T}_{cd}(y)\rangle$,

- Classical Gaussian stochastic tensor field:

$$\langle \xi_{ab}[g;x)\rangle_s = 0, \qquad \langle \xi_{ab}[g;x)\xi_{cd}[g;y)\rangle_s = N_{abcd}[g;x,y),$$

$\langle \cdots \rangle_s$

denotes statistical average wrt this noise distribution

Classical Stochastic Field assoc.with a Quantum Field

- Stochastic tensor is covariantly conserved in the background spacetime (which is a solution of the semiclassical Einstein equation).

$$\nabla^a \xi_{ab}[g; x) = 0.$$

- For a conformal fie ξ_{ab} is traceless:

$$g^{ab}\xi_{ab}[g; x) = 0;$$

Thus there is no stochastic correction to the trace anomaly

Einstein-Langevin Equation

- Consider a weak gravitational perturbation h off a background g $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu},$ The ELE is given by (The ELE is Gauge invariant)

$$G_{ab}[g+h] + \Lambda(g_{ab} + h_{ab}) - 2(\alpha A_{ab} + \beta B_{ab})[g+h]$$
$$= 8\pi G\left(\left\langle \hat{T}^R_{ab}[g+h]\right\rangle + \xi_{ab}[g]\right).$$

- Nonlocal dissipation and colored noise

Nonlocality manifests with **stochasticity**

because this is an open system

Correlations: Kinetic Theory Approach to the Microscopic Spacetime Structure

GR is a theory of macrostructure of spacetime. QG is a theory of microstructure of ST

Quantizing macroscopic variables of ST unlikely to produce microstructure or QG.

More challenging and urgent task: find the micro-variables.

A ladder from hydrodynamics to microdynamics: First two rungs above classical GR, semiclassical (mean field theory) and stochastic (including fluctuations) gravity.

"***A Kinetic Theory Approach to Quantum Gravity***"
Int. J. Theor. Phys. **41** (2002) 2111-2138 [gr-qc/0204069]

Fluctuation as a useful probe into universality of microscopic structure

I. Wave propagation in a stochastic spacetime

B. L. Hu and K. Shiokawa, Phys. Rev. D **57**, 3474 (1998)

II. Universal `conductance' fluctuations as a signature of microscopic structure

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Mesoscopic fluctuations in stochastic spacetime

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Mesoscopic effects associated with wave propagation in spacetime with metric stochasticity are studied. We show that the scalar and spinor waves in a stochastic spacetime behave similarly to the electrons in a disordered system. Viewing this as the quantum transport problem, mesoscopic fluctuations in such a spacetime are discussed. The conductance and its fluctuations are expressed in terms of a nonlinear sigma model in the closed time path formalism. We show that the conductance fluctuations are universal, independent of the volume of the stochastic region and the amount of stochasticity.

Summary and Tasks

1. **Nonlocality at one level, locality at another:** *How connected?*

2. **Nonlocality and Stochasticity in Statistical Mechanics**
   - Open (Langevin) and Effectively open (Boltzmann) systems
   - Coarse-graining → Noise; Backreaction → Dissipation

3. **Nonlocality and Stochasticity in Quantum Gravity:** examples from **Semiclassical** and **Stochastic gravity**: *non-Markovian dynamics*
   Nonlocality in time (Memory). Nonlocal dissipation and colored noise

4. **Correlation: another angle towards nonlocality and stochasticity**
   Strongly interacting and correlated systems: Condensed matter models

5. Kinetic Theory Approach to QG – **Correlation Hierarchy**
   Mesoscopic spacetime physics: Universal `conductance' fluctuations

6. **Tasks**: understand how these issues play out in known physical models, then apply to spacetime, incorporating its special features: Time / Diff inv.

Stochastic Gravity **in relation to** ***Quantum*** **and** ***Semiclassical***

(w. Enric Verdaguer, Peyresq 98?)

As an example, let's consider

gravitational perturbations $h_{\mu\nu}$ in a FLRW universe with background metric $g_{\mu\nu}$

The Semiclassical Einstein Equation is

$$\Box h = \langle \hat{T} \rangle$$

where < > denotes the quantum vacuum expectation value

With solutions

$$h = \int G \langle \hat{T} \rangle, \qquad h_1 h_2 = \int \int G_1 G_2 \langle \hat{T} \rangle \langle \hat{T} \rangle$$

The Quantum (Heisenberg) Equation is

$$\Box \hat{h} = \hat{T}$$

With solutions

$$\hat{h} = \int G \hat{T}, \qquad \langle \hat{h}_1 \hat{h}_2 \rangle = \int \int G_1 G_2 \langle \hat{T} \hat{T} \rangle_{\hat{h},\hat{\phi}}$$

Where the average is taken with respect to the quantum fluctuations of both the gravitational and matter fields

For **stochastic gravity**, the Einstein Langevin equation is of he form

$$\Box h = \langle \hat{T} \rangle + \tau$$

With solutions

$$h = \int G\langle \hat{T} \rangle + \int G\tau, \qquad h_1 h_2 = \int\int G_1 G_2 [\langle \hat{T} \rangle \langle \hat{T} \rangle + (\langle \hat{T} \rangle \tau + \tau \langle \hat{T} \rangle) + \tau\tau]$$

We now take the noise average $\langle \rangle_\xi$

Recall $$\hat{t}_{\mu\nu}(x) \equiv \hat{T}_{\mu\nu}(x) - \langle \hat{T}_{\mu\nu}(x) \rangle \hat{I}$$

Hence $$\langle \tau \rangle_\xi = 0, \qquad \langle \tau_1 \tau_2 \rangle_\xi \equiv \langle \hat{T}_1 \hat{T}_2 \rangle - \langle \hat{T}_1 \rangle \langle \hat{T}_2 \rangle$$

We get, $$\langle h_1 h_2 \rangle_\xi = \int\int G_1 G_2 \langle \hat{T}\hat{T} \rangle_{\hat{\phi}}$$

Note this **has the same form as in quantum** gravity except that the Average is taken with respect to matter field fluctuations only.

Semiclassical Gravity includes only the **mean value** of the Stress-Energy Tensor of the matter field

Stochastic Gravity includes the two point function of **T**mn in the **Einstein-Langevin equation**

It is **the lowest order in the hierarchy of correlation functions**. The full hierarchy gives full information about the matter field.

At each level of the hierarchy there is a linkage with the gravity sector. The lowest level is the **Einstein equation** relating the **T**mn itself to the Einstein tensor **G**mn

Quantum Fluctuations :: Quantum Correlation :: Quantum Coherence

Thus **stochastic gravity recovers partial quantum coherence in the gravity sector via the metric fluctuations** induced by matter fields

There may exist a <u>meso</u> regime for spacetime structure and dynamics described by a hierarchy of <u>mesoscopic</u> spacetime dynamical equations

similar to the BBGKY hierarchy of which Einstein eqn only describes the lowest rung of macro structure, like the Boltzmann eqn.

This is the underlying idea in the
'Kinetic Theory Approach to Quantum Gravity"

stochastic gravity

level 3: $G_{\mu\nu}(g_{\alpha\beta}) = \kappa \langle T_{\mu\nu}(\phi)\rangle + \xi_{\mu\nu}$

induces $\leftarrow$ $\langle T_{\mu\nu}(x)\, T_{\rho\sigma}(y)\rangle$

$G_{\mu\nu}(x)\, G_{\rho\sigma}(y)$

...

use the n-pt fcn of the Einstein Tensor as ladder to climb up the hierarchy.