Program for the Block Seminar with topic "Introduction to Controlled K-Theory"

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Abstract

In this seminar we will discuss selected topics of controlled algebra. After recalling some basic concepts from algebraic K-theory, we will prove in detail the topological invariance of Whitehead torsion, following Ranicki and Yamasaki [RY95].

Controlled methods play a relevant role in geometric topology. The introduction of controlled topology goes back to the foundational work of Connell-Hollingsworth [CH69] and the crucial developments by Quinn [Qui79] and Chapman [Cha80].

Controlled topology uses K-theoretic invariants of modules or chain complexes with a reference map to a metric space, which takes care of the " size" of performed operations. Under a good "control" of the size of performed operations, one can actually obtain results about classical K-theoretic invariants.

The seminar will be concerned with one of the first applications of these techniques: the proof of the topological invariance of Whitehead torsion, going along the path by given by Ranicki and Yamasaki [RY95]. Several approaches to this proof are available, among them these by Quinn[Qui79] and Pedersen [Ped00].

Further lectures on the topic include [FHP94], where a survey of controlled surgery theory is given, as well as [Ros10], where the connections with the isomorphisms conjectures of Farrell-Jones are stated.

The seminar will combine the use of videoconference technologies and traditional talks. In the first series consisting of six lectures, the basic notions of controlled algebra will be introduced. This part of the seminar will take place weekly between April 29th and June 3rd, on Friday afternoon and will include the use of videoconference technologies. The second part of the seminar will consist of two sessions on Friday, June 10th and Friday, June 17th.

The seminar is part of a collaboration with the National University of Mexico (UNAM), started by Dr. Noe Bárcenas and Dr. Daniel Juan-Pineda and will include the visit of a number of students of the UNAM Graduate School Program in Pure Mathematics. The activities of the Transcontinental Seminar are supported by the UNAM Graduate School in Mathematics and the Mexican Talent Network in Germany.

- Introduction to the projective class group K_0. In this talk, the speaker shall introduce the projective class group and Wall's finiteness obstruction. Pages 1-7 in [Ros94] for the basic definitions, which at least should be illustrated with the examples provided by theorem 1.3.1 , page 11 in [Ros94]. See [Ros94] 41-58 for the Finiteness obstruction.

- Whitehead torsion and simple homotopy theory. In this talk, Whitehead groups and whitehead Torsion will be introduced. For the definition of whitehead groups, the reference is [KL05], pages 35 to 39 (without the discussion of the Bass-Heller-Swan Decomposition). For whitehead Torsion, chapter 6, page 43 to 53 in [KL05].

- Nil Groups and the Bass-Heller Swan decomposition. The speaker should explain how the two copies of Nil groups lie inside $K_1(R(t, t^{-1})$ and how does the Whitehead torsion decomposes into the different factors, this is needed for the third lecture in the second part. References [Ros94], pages 132 to 153.

- Geometric modules and controlled algebra. The speaker should introduce the notion of geometric modules and geometric chain complexes, as well as the notion of a projective object in these categories. Reference [RY95], section 1.

- Controlled Finiteness obstruction. The speaker should introduce the notion of controlled K-theory Groups and controlled chain equivalence of controlled projective modules. Reference [RY95], section 3.

- Controlled Whitehead Torsion. In this talk, the notion of controlled relative Whitehead groups will be introduced, as well as the notions of n-stable, ϵ simple equivalences, and ϵ-n stable equivalences, which will be used to give the controlled relative whitehead groups an abelian group structure, proposition 4.1 in [RY95], and to prove the composition formula, proposition 4.8 in [RY95].

Second session

- Relative K-theory. After recalling briefly the notion of controlled relative whitehead groups, the speaker shall discuss the notion of an ϵ-domination, and prove the "stable exact sequence theorem ", Theorem 5.3 in [RY95].

- Excision and controlled Mayer-Vietoris Sequences. The speaker should state the Controlled Mayer-Vietoris sequence in theorem 6.2 in [RY95].

- Controlled Whitehead Group of $M \times S^1$. In this talk, the goal is to prove a controlled version of the Bass-Heller-Swan decomposition, [RY95], section 7.

- Proof of the "Eventually Vietoris Theorem". The speaker should define negative controlled K-theory, and to prove Theorem 8.1 in [RY95], section 8, and its corollaries, particularly 8.3. For the definition of negative controlled K-theory, see the appendix in the same reference.

- Proof of the topological invariance of the Whitehead Torsion. After introducing controlled Whitehead torsion using the theory of transverse CW-complexes, this talk is aimed to finish the proof of the topological invariance of whitehead torsion, Theorem 10.1 in [RY95].

References

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[FHP94] Steven C. Ferry, Ian Hambleton, and Erik K. Pedersen. A survey of bounded surgery theory and applications. In *Algebraic topology and its applications*, volume 27 of *Math. Sci. Res. Inst. Publ.*, pages 57–80. Springer, New York, 1994.

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[Qui79] Frank Quinn. Ends of maps. I. *Ann. of Math. (2)*, 110(2):275–331, 1979.

[Ros94] Jonathan Rosenberg. *Algebraic K-theory and its applications*, volume 147 of *Graduate Texts in Mathematics.* Springer-Verlag, New York, 1994.

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[RY95] Andrew Ranicki and Masayuki Yamasaki. Controlled K-theory. *Topology Appl.*, 61(1):1–59, 1995.