PROGRAM FOR THE TRANSCONTINENTAL SEMINAR "BASICS ON BRAID AND MAPPING CLASS GROUPS"

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In the eighth edition of the seminar we will study the basics of the theory of surface mapping class groups and surface braid groups, focusing in the relationship existent between them. Mapping class groups were first introduced by Dehn and have atracted in the last years the attention of geometric group theorists, topologists and algebraic Geometers. On the other hand, Braid groups were first introduced by Artin and have became over the years the axis of a close interaction of topology, group theory, the theory of configuration spaces and even theoretical physics.

Given a closed surface S, the mapping class group $Mod(S)$ is defined to be the component set of the space of orientation preserving Homeomorphisms of S. In symbols:

$$Mod(S) = \pi_0(Homeo^+(S))$$

There exist versions for surfaces with boundary, taking into acount data of the boundary components, as well as versions taking into account marked points.

On the other hand, Braid groups might be defined as the fundamental group of a so-called configuration space. Given a surface S, denote by $F_{0,n}(S)$ the subspace of the n-fold cartesian product of S defined by

$$F_{0,n}(S) = \{(x_1, \ldots, x_n) \in \prod_{i=1}^{n} S \mid x_i \neq x_j \; i \neq j\}$$

The pure braid group on n strings of the surface S is defined to be the fundamental group of this space:

$$PB_n(S) = \pi_1(F_{0,n}(S))$$

There exists a group $B_n(S)$, called the braid group of n strings on the surface S, which fits into an exact sequence

$$1 \to PB_n(S) \to B_n(S) \to \Sigma_n \to 1$$

Where Σ_n denotes the symmetric group.

The goal of the seminar is to give a proof of the following two results:

- Birman Exact Sequence. Let S be a surface without marked points and $\pi_i(Homeo^+(S, \partial S)) = 1$. Denote by $Mod(S^*)$ the component set of the space of orientation preserving homeomorphisms of S which preserve n marked points. Then, there exists an exact sequence

$$1 \to PB_n(S) \to Mod(S^*) \to Mod(S) \to 1$$

- Birman-Hilden Theorem. Given a surface S_g^1 of genus g and one boundary component, there exists an element i of order two of the group of orientation preserving homeomorphisms of S_g^1, with centralizer denoted by $SHomeo^+(S_g^1)$. The symmetric mapping class group $SMod(S_g^1)$ is defined as the group obtained by dividing out the isotopy relation, $SHomeo^+(S_g^1)$/isotopy. Then one has

$$SMod(S_g^1) \cong B_n(S_G^1)$$

The seminar will be roughly organized in three blocks: one devoted to basic definitions of Mapping Class Groups, another one dealing with Braid groups, and the last part will deal with the proof of the Birman Exact sequence and the Birman-Hilden Theorem.

Part 1. MAPPING CLASS GROUPS

Talk 1. Definitions of the Mapping Class Groups and Examples. (See Section 1.4 and 2.1 of [2]). For the examples see sections 2.1 and 2.2 of [2]), specially dealing with the Alexander Method, stating a relation to $Sl_2(Z)$.

Talk 2. Basic properties of Dehn Twists (Sections 3.1, 3.3 and 3.4 of [2]). The purpose of this talk is to give a geometric relevant set of generators, so called Dehn twists, as well as to introduce some of their simplest properties.

Talk 3. Presentation of the Mapping Class Group Part 1 (See sections 3.3, 3.5 and 4.4.1 of [2]) This talk introduces the following relations between Dehn twists:
-The disjointness relation (Fact 3.9 of [2])
-The braid relation

Talk 4. Presentation of the Mapping Class Group Part2. The goal of this talk is to finish the proof of the Humphries generator theorem, giving a presentation for the Mapping Class Group. This talk should introduce the last relations between Dehn twists:the chain relation. For Humpries generators theorem See sections 4.4.2, 4.4.3 of [2] and [3].

Part 2. BRAID GROUPS

Talk 5. Geometric and topological definition of surface braid Groups (See Ch. 1 of [1]). This talk will give the basic definitions.

Talk 6. Examples: the plane and the sphere. (See Ch.1 of [1])

Part 3. THE BIRMAN EXACT SEQUENCE AND THE BIRMAN-HILDEN THEOREM .

Talks 7-8. Relationship between the Mapping Class Group and the Braid Group. These talks will deal with the proof of the Birman exact sequence theorem and the Birman-Hilden Theorem.

References

[1] *Joan S. Birman*, "Braids Links and Mapping Class Groups", ANNALS OF MATHEMATICS STUDIES PRINCETON UNIVERSITY.

[2] *Benson Farb and Dan Margalit*,"A primer on mapping class groups", PRINCETON UNIVERSITY.

[3] *Stephen P. Humpries*, "Generators for the Mapping Class Groups"

[4] *Nikolai V. Ivanov*, "Mapping Class Groups"

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