

**PROGRAM FOR THE TRANSCONTINENTAL SEMINAR:
“COARSE GEOMETRY, ASSEMBLY AND EXPANDERS”**

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1. INTRODUCTION

The sixth edition of the Transcontinental Seminar analyzes the relationship between coarse geometry and the Baum-Connes Assembly Map. In the first part of the seminar the basics of operator algebras, K -Theory and K -homology will be discussed, with particular emphasis on their connection to the Baum-Connes conjecture.

The Baum-Connes conjecture [1] predicts an isomorphism of abelian groups

$$\mu_G : K_*^G(\underline{E}G) \longrightarrow K_*^{\text{top}}(C_r^*(G)),$$

where $K_*^G(-)$ denotes equivariant K -homology, a generalized equivariant homology theory. The space $\underline{E}G$ is the classifying space for proper actions, $K_*^{\text{top}}(A)$ denotes *topological* K -theory of the C^* -algebra A , and $C_r^*(G)$ denotes the *reduced* C^* -algebra of the discrete group G .

Given a finite generating set S for a group G , the word-length metric associated to S defines a metric on the Cayley graph $C(G, S)$ that is a quasi-isometry invariant of the group G . This metric space captures the so-called “large scale” geometry of G . The second part of the seminar will introduce the basic methods from coarse geometry and will introduce K -homology associated to a coarse structure.

Coarse geometry associates to a metric space X (or more general structures) a C^* -algebra $C^*(X)$ encoding the large scale geometry of the space X . Methods from C^* -algebra and operator theory, originally developed in index theory for non-compact manifolds, associate to this data an assembly map

$$K_*(X) \rightarrow K_*(C(X)).$$

The statement of the Coarse Baum-Connes conjecture [11] is that this map is an isomorphism. The Baum-Connes conjecture is related to several conjectures, like Novikov’s conjecture on the homotopy invariance of higher signatures, the Kaplansky conjecture, and many others (see [6] for more on this topic). A generalization of the Baum-Connes conjecture allows for more general coefficients. It predicts the existence of an isomorphism

$$A : K_*(\underline{E}G, A) \rightarrow K_*(A \wr_r G)$$

for any G - C^* -algebra A . Counterexamples to this conjecture are known due to the work of Higson-Lafforgue-Skandalis, but it is still studied since it has nice inheritance properties (like passing to subgroups). In the third part of the seminar we will consider a description of the coarse structures that lead to these counterexamples.

The seminar is organized as follows:

- (i) Part 1: Basics of topological K -Theory, K -homology and C^* -algebras.
 - First Talk: General Overview of the Baum Connes conjecture, Coarse Geometry and expanders. This talk should give a general introduction

- to the diverse topics of the seminar. Introduce some Basics on C^* -algebras, including the Gelfand-Naimark Theorem.
- Second Talk: The definition of K -theory for C^* -algebras. K_0 as Murray-von Neumann equivalence classes of projectors in A , $K_1(A)$ as the quotient of the infinite dimensional general linear group in A modulo the identity component. Strengthened Bott periodicity and homotopy invariance as the crucial difference to algebraic K -theory (without proofs). References: [5], chapter 4, [3], [10], Chapter 1 of [5].
 - Third Talk: Classical K -theory and K -homology. This talk should introduce topological K -theory, state the Serre-Swan Theorem relating operator theory to topological K -theory. Also, K -homology should be introduced as an extension group, and as the K -theory of the Paschke dual algebra. Comment on how involved it is to prove the homotopy invariance of K -homology in any of these formulations. Reference: [5], chapter 5. Description of the classical Baum-Connes map.
- (ii) Part 2: Coarse geometry and coarse K -homology.
- First talk. Coarse structures. This talk will introduce the notions of coarse structures, controlled and bounded sets, cones and the C^* algebra of a Coarse space. special interest should be devoted to the relationship between coarse structures associated to Cayley graphs and quasi-isometry. References: [5], chapter 6 pages 142- 152, [8] for the emphasis on groups and coarse structures associated to them.
 - Second talk. K -homology of coarse structures and the coarse Assembly map. References: [5], pages 152- 163. For the coarse assembly map, [5], chapter 12 section 3.
- (iii) Third talk. Scaleable spaces. This talk will deal with sections 4, 5 in [5].
- (iv) Part 3: Coarse structures, expanders and assembly maps.
- Expanders, coarse geometry and failure of surjectivity of the coarse assembly map. This talk will deal with the example in section 6 of [4].
 - Coarse embeddability and finite asymptotic dimension. This talk will give some indication of how these properties are used to prove positive results on the coarse assembly map. References: [13, 7].

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