PROGRAM FOR THE SEMINAR ON MAPPING CLASS GROUPS AND CLASSIFYING SPACES.

NOÉ BÁRCENAS AND ALEJANDRA TRUJILLO

In this semester, we will continue our study of mapping class groups by learning some basic aspects of Teichmüller Theory. The goal of the Seminar will be to describe the classifying space of proper actions for all mapping class groups.

The seminar will consist of two sections, one dealing with basic aspects from Teichmüller Theory, and a second one which should set up these methods in connection to the construction of classifying spaces for families of subgroups.

Given a closed surface S, the mapping class group $Mod(S)$ is defined as the component set of the space of orientation preserving Homeomorphisms of S. In symbols:

$$Mod(S) = \pi_0(Homeo^+(S))$$

There exist versions for surfaces with boundary, taking into acount data of the boundary components, as well as versions taking into account marked points.

On the other hand, considerations which go back to Riemann himself describe the action of this group on the space of hyperbolic structures, or metrics of the given surface. This space is currently known as the Teichmüller Space.

We will describe three models of it, each one of them being particularly well suited to different purposes. We will be able to prove that the Teichmüller space is $6g-6$ dimensional and by introducing the Fenchel-Nielsen coordinates we will prove that it is diffeomorphic to a $6g-6$-dimensional open disk.

We will use the theory of quasiconformal mappings will let us introduce a metric on it, the Teichmüller metric.

This metric will be the main tool to prove that the action of the mapping class group $Mod(S_g)$ on the Teichmüller space is proper, and further results by Broughton give a cocompact model, thus proving that there is a finite model for $\underline{E}Mod(S_g)$. Finally, well known methods for the construction of classifiying spaces for families together with the Birman exact sequence will provide us the tools for proving the main theorem of this seminar:

Theorem 0.1. *Let $S_g^{n,r}$ be a surface of genus g, n marked points and r boundary components. Then, there exists a finite model for the classifying space for proper actions $\underline{E}Mod(S_g^{n,r})$.*

(i) First part: Basics of Teichmüller Theory
  - First talk. Basic definitions. This talk introducess the Teichmüller space as the space of hyperbolic structures and metrics. Sections 10.1 and 10.2 in [2].
  - Second talk. Here, the algebraic topology and first dimension count will be made. Section 10.3 in [2].
  - Third talk: The pants decomposition and the Fenchel-Nielsen coordinates. Material in [2], Section 10.5 and 10.6, omiting the open surface case.

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- Fourth talk. The speaker in this talk should introduce the theory of quasiconformal mappings. Section 11.2 in [2].
- Fifth talk: Teichmüller metric.The goal of the talk is to introduce the Teichmüller metric, discussed in [2], 12.4 and to prove Wolpert's Lemma, 12.5 in [2].

(ii) Second part: classifying spaces

- Sixth talk: Basic introduction to classifying spaces for families and methods to construct them. Reference:[3].
- Seventh talk: The main result of the talk is to prove that the teichmüller space of a closed surface S_g is a compact model for $\underline{E}Mod(S-g)$ The proof of properness is in section 12.5 of [2] and section 2 of [1]. The existence of a compact model is discussed there.
- Eigth talk: We will finish the proof of the main result, puting all together the previously described methods. Main reference is [4].

References

[1] S. A. Broughton. The equisymmetric stratification of the moduli space and the Krull dimension of mapping class groups. *Topology Appl.*, 37(2):101–113, 1990.

[2] B. Farb and D. Margalit. A primer on mapping class groups. Book Project.

[3] W. Lück. Survey on classifying spaces for families of subgroups. In *Infinite groups: geometric, combinatorial and dynamical aspects*, volume 248 of *Progr. Math.*, pages 269–322. Birkhäuser, Basel, 2005.

[4] G. Mislin. Classifying spaces for proper actions of mapping class groups. *Münster J. Math.*, 3:263–272, 2010.

E-mail address: barcenas@matmor.unam.mx

E-mail address: alexatrn@gmail.com