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ORIGINAL ARTICLE

Classification of rational 1-forms on the Riemann sphere up to PSL(2, $\mathbb{C}$)

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Abstract

We study the family $\Omega^1(-1^s)$ of rational 1-forms on the Riemann sphere, having exactly $-s \leq -2$ simple poles. Three equivalent $(2s-1)$-dimensional complex atlases on $\Omega^1(-1^s)$, using coefficients, zeros–poles and residues–poles of the 1-forms, are recognized. A rational 1-form is called isochronous when all their residues are purely imaginary. We prove that the subfamily $\mathcal{RI}\Omega^1(-1^s)$ of isochronous 1-forms is a $(3s-1)$-dimensional real analytic submanifold in the complex manifold $\Omega^1(-1^s)$. The complex Lie group PSL(2, $\mathbb{C}$) acts holomorphically on $\Omega^1(-1^s)$. For $s \geq 3$, the PSL(2, $\mathbb{C}$)-action is proper on $\Omega^1(-1^s)$ and $\mathcal{RI}\Omega^1(-1^s)$. Therefore, the quotients $\Omega^1(-1^s)/\text{PSL}(2, \mathbb{C})$ and $\mathcal{RI}\Omega^1(-1^s)/\text{PSL}(2, \mathbb{C})$ admit a stratification by orbit types. Realizations for the quotients $\Omega^1(-1^s)/\text{PSL}(2, \mathbb{C})$ and $\mathcal{RI}\Omega^1(-1^s)/\text{PSL}(2, \mathbb{C})$ are given, using an explicit set of PSL(2, $\mathbb{C}$)-invariant functions.

Keywords Rational 1-forms · Isochronous centers · Proper PSL(2, $\mathbb{C}$) 1-action · Principal PSL(2, $\mathbb{C}$)-bundle · Stratified space by orbit types

Mathematics Subject Classification Primary 32M05 · Secondary 30F30, 58D19

1 Introduction

A compact Riemann surface M_g of genus $g \geq 0$ is determined by its space of holomorphic 1-forms. For $g \geq 1$, the Jacobian variety $J(M_g)$ is defined using the vector space of holomorphic 1-forms. Very roughly speaking, Torelli's theorem says that we can recover M_g from $J(M_g)$; see [12, p. 359]. Moreover, for $g \geq 2$, a classical result of Hurwitz asserts that the automorphism group $\text{Aut}(M_g)$ is finite; see [7, Ch.

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