Approaches to Quantum Gravity – A conceptual overview

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A quantum theory of gravity?

We have two hugely successful theories of fundamental physics:

- General Relativity (gravity, structure of the universe)
- The Standard Model of Particle Physics (electromagnetic and nuclear interactions, structure of matter)

Obviously, we would like to have one theory encompassing them both. The key obstacle is that we have no quantum theory of gravity. So how do we get one?

Spacetime in general relativity

General relativity has taught us that spacetime is a four dimensional manifold whose properties are described through a *Lorentzian metric* which interacts with matter fields as described in the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- spacetime: $g_{\mu\nu}$ metric, R scalar curvature, $R_{\mu\nu}$ Ricci curvature
- matter fields: $T_{\mu\nu}$ energy-momentum tensor
- constants: c speed of light, G Newton's constant, Λ cosmological constant

G and Λ are small and can often be neglected. Spacetime is then approximately *Minkowski space* (if matter density is low).

Different starting points: Primary approaches

- Usually quantum theories are constructed from classical ones through a process of *quantization*.
- *Primary* approaches to quantum gravity consists in trying to apply a quantization procedure to classical general relativity.
- Advantage: One might expect to reproduce general relativity as the classical limit.
- Disadvantage: Other interactions must be treated separately.
- Examples:
  - Perturbative Quantum Gravity
  - Quantum Geometrodynamics
  - Loop Quantum Gravity
  - Dynamical Triangulations

Example: perturbative quantum gravity

- Einstein-Hilbert action of general relativity:

$$S[g] = \frac{c^4}{16\pi G} \int \mathrm{d}^4x \sqrt{-g}(R - 2\Lambda)$$

- know how to quantize field theories on Minkowski space

$\rightarrow$ decompose metric $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G} h_{\mu\nu}$ with $\eta_{\mu\nu}$ Minkowski metric

- use path integral over *graviton field h* to calculate *n*-point functions

$$\int \mathcal{D}h \, h_{\mu_1\nu_1}(x_1) \cdots h_{\mu_n\nu_n}(x_n) \exp(iS[\eta + \sqrt{G}h])$$

- obtain perturbative expansion in coupling $\sqrt{G}$
- scattering cross sections of gravitons can be extracted from *n*-point functions

Problem: perturbative quantum gravity is non-renormalizable, i.e. is non-predicitive due to infinitely many adjustable parameters

Example: effective quantum gravity

- But: perturbative quantum theory is predictive at low energies (*effective field theory*)
- Example result: Newton potential with correction

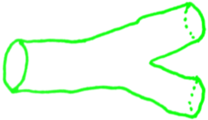
$$V(r) = -\frac{Gm_1 m_2}{r}\left(1 + \frac{3G(m_1 + m_2)}{2rc^2} + \frac{41G\hbar}{10\pi r^2 c^3} + \cdots\right)$$

Different starting points: Secondary approaches

- *Secondary* approaches to quantum gravity consist either in quantizing some other classical theory or in building a quantum theory from scratch via other means.
- Advantage: One might hope to directly obtain a unified theory (unifying all forces) or a new framework for quantum theory.
- Disadvantage: Starting point speculative, physical motivation might be weak
- Examples:
  - String Theory
  - Causal Sets

Example: String Theory (I)

The basic idea of string theory is to replace the point-like excitations of quantum field theory with string like excitations.



The strings are modeled as minimal surfaces in a background spacetime. Suppose a cylinder $R \times S^1$ is embedded into $\mathbb{R}^n$ with metric $\eta_{\mu\nu}$ via $(\tau, \sigma) \mapsto X(\tau, \sigma)$. The Nambu-Goto action describing the surface area is to be minimized:

$$S = -\int d\tau d\sigma \sqrt{-\det(\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu})}$$

This classical theory is then quantized. The (Euclidean version) of perturbative String Theory is then given by summing the resulting amplitudes for all toplogigcal classes of such surfaces.

Example: String Theory (II)

- the theory contains spin-2 particles that might be interpreted as *gravitons*

$\longrightarrow$ hope: includes quantum gravity

- the theory also produces many other particles

$\longrightarrow$ hope: may provide a *unified theory*

- but: theory is inconsistent in $3+1$ spacetime dimensions
- using *supersymmetry* theory can be defined in $9+1$ spacetime dimensions
- try to save theory by postulating that spacetime has 6 extra compactified dimensions that are so small that they have not been observed so far
- arising complications might render theory unpredictive (*landscape*)
- progress in development of theory (dualities, AdS/CFT), but still no underlying principle to say what string theory really is

Different methods

The main approaches to quantum theory are traditionally classified into "covariant" and canonical approaches. This is a distinction based on technical aspects of the approaches.

“Covariant” approaches

“Covariant” approaches use the *Feynman path integral* to quantize a classical theory. They treat spacetime as a whole, i.e., don’t perform a split into space and time, hence the name. However, they are usually based on a background metric and *not* generally covariant, so the name is slightly misleading.
Examples:

- Perturbative/effective quantum gravity
- Supergravity
- String Theory

Canonical approaches

Canonical approaches are based on *canonical quantization*. To describe the phase space spacetime is usually foliated into Cauchy hypersurfaces. Independence of the foliation then has to be demonstrated.

Examples:

- Quantum Geometrodynamics
- Loop Quantum Gravity

Dirac quantization of GR

canonical description

Start with a Hamiltonian formulation of general relativity:

- Assume global hyperbolicity of spacetime $\mathbb{R} \times M$ and foliate into Cauchy hypersurfaces
- Identify the space of solutions (phase space) with initial data on hypersurfaces:
    - 3-metric q_{ij} intrinsic to hypersurface ("positions")
    - extrinsic curvature K^{ij} describing embedding
- define $p^{ij} = \frac{\sqrt{q}}{16\pi G}(K^{ij} - q^{ij}K)$ ("momenta")
- then q_{ij} and p^{ij} are *conjugate* variables
- Poisson brackets are: $\{q_{ij}(x), p^{kl}(x')\} = \frac{1}{2}(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k)\delta(x - x')$

Dirac quantization of GR
constraints

- diffeomorphisms are gauge symmetries

$\longrightarrow$ need to quotient phase space by symmetries to get physical *reduced* phase space

- generate symmetries through *constraints*: Hamiltonian constraints $H(x)$ and 3-diffeomorphism constraints $H^i(x)$
- these arise from hypersurface deformations

Now "quantize" the theory:

- *kinematical* Hilbert space $\mathcal{H}_{\text{kin}}$ is a space of functions on the space of "positions"
- promote observables O on phase space to operators $\hat{O}$ on $\mathcal{H}_{\text{kin}}$ such that $[\hat{O}_1, \hat{O}_2] = i\hbar\{O_1, O_2\}$
- implement constraints: physical Hilbert space $\mathcal{H}_{\text{phys}} \subset \mathcal{H}_{\text{kin}}$ is kernel of constraints $\hat{H}(x)$ and $\hat{H}^i(x)$
- need gauge invariant observables to commute with constraints

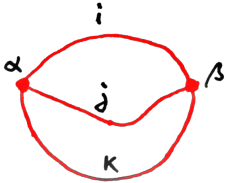
Problems:

- all steps are very difficult, few has been accomplished
- even if a solution is found, what is the interpretation of $\mathcal{H}_{\text{phys}}$ and the operators acting on it?

Dirac quantization of GR
LQG

- think of GR as a gauge theory rather than a theory of the metric
- use self-dual formulation and gauge fix to reduce gauge group from $SO(3,1)$ to $SU(2)$
- "position" variables are connections A_a^i
- conjugate "momentum" variables are 3-frame fields E_i^a
- Poisson brackets: $\{E_a^i(x), A_j^b(x')\} = -8\pi G \delta_a^b \delta_j^i \delta(x - x')$
- constraints take a particularly simple form

- $\mathcal{H}_{\text{kin}}$ successfully constructed, using spin network functions
- $\mathcal{H}_{\text{kin}}$ is non-separable
- 3-diffeormorphism constraint successfully implemented, $\mathcal{H}_{\text{diff}} \subset \mathcal{H}_{\text{kin}}$ is separable
- Hamiltonian constraint still elusive. . .



Classical versus quantum physics

- Classical and quantum physics have very different ways to describe reality.
- Spacetime appears in very different ways in the two frameworks.
- Elements from each of the two types of descriptions contradict assumptions of the other.

This is a core problem for any approach to quantum gravity

Reality in classical physics

Reality in classical physics is objective and deterministic.

- The description of a system involves a space of *motions*. The elements of this space correspond directly to the “possible worlds” or “states” of the system.
- These worlds are objective entities. A world is either realized or not, independent of any observation process to establish this.
- Any property of a world is in principle observable. Any function on the space of motions can serve as observable.
- Measurements have the goal of determining which of the worlds (or class of worlds) is realized.
- Given such a determination allows predictions about properties of the world with certainty.
- A spacetime is part of each world.
- Reality is *local* in the sense that local perturbations do not have instantaneous effects at a distance.

Reality in quantum physics

- The description of a system involves a space of *states*. This is a Hilbert space.
- A state encodes information about potential properties of the system.
- Only probabilistic predictions about the outcome of a measurement can be made using a state.
- Observation modifies a state – The observer and the system are inevitably coupled.
- The observer must be *external* to the system and is subject to a *classical* description.
- Assuming that a state is an image of the reality of the system leads to the conclusion that this reality is *non-local*. (collapse of the wavefunction, Copenhagen interpretation)

Spacetime in quantum physics

- Spacetime is a fixed entity (usually Minkowski space).
- All states associated to a system relate to the same spacetime.

This is justified by the following requirements.

- Consistency demands that probabilities are conserved *in time*. For this, an external notion of time (independent of the state) is essential.
- In QM: Time is essential in determining how consecutive measurements influence each other.
- In QFT: Spacetime is essential in determining how measurements influence each other, given a causal relation between them.

Different strategies

background metric approach:

- split spacetime into a fixed metric plus a perturbation: $g_{\mu\nu} = \eta_{\mu\nu} + f_{\mu\nu}$
- fixed metric $\eta_{\mu\nu}$ is basis for interpretation of quantum theory.
- perturbation $f_{\mu\nu}$ is dynamical and quantized.

"no spacetime" approach:

- do away with the quantum theoretical postulates relating to external spacetime
- allow different states to encode different spacetimes
- hope to recover consistency conditions (approximately) in suitable regimes

extended quantum theory:

- weaken the notion of background spacetime to that of differentiable or topological manifold
- extend quantum theoretical postulates on spacetime to this case

The background approach

Advantages:

- Powerful methods and accumulated expertise on quantum (field) theory can be fully used
- The connection to and comparison with known quantum physics (standard model) is easy

Problems:

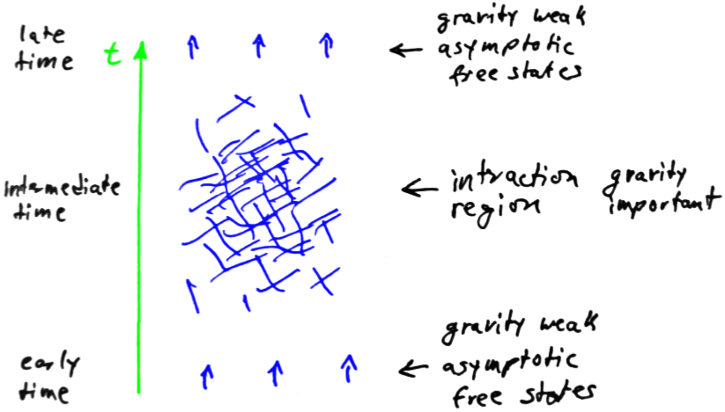
- Description only approximate, for small deviations from background
- When trying to construct “full” theory by piecing together backgrounds the old problems return and standard quantum theory must be abandoned

Examples:

- Perturbative/effective quantum gravity
- Supergravity
- String theory

Example: S-matrix (I)

- decompose metric $g_{\mu\nu} = \eta_{\mu\nu} + f_{\mu\nu}$, $\eta_{\mu\nu}$ Minkowski metric
- assume spacetime asymptotically flat, $f_{\mu\nu} \to 0$ for $t \to {}^{+\infty}_{-\infty}$



Example: S-matrix (II)

- gravitational perturbations approximately linear at early and at late times,

$$\Box f_{\mu\nu} = -16\pi G\,(T_{\mu\nu} - 1/2\eta_{\mu\nu}T)$$

$\longrightarrow$ associated particles (*gravitons*) are free

- S-matrix $\langle q_1, \ldots, q_n | S | p_1, \ldots, p_m \rangle$ describes scattering amplitude of free initial gravitons $p_1, \ldots, p_m$ to free final gravitons $q_1, \ldots, q_n$
- perturbative quantum field theory yields S-matrix

Example: String Theory and AdS/CFT

- also based on S-matrix picture
- perturbative String Theory: if consistent, then should provide predictive theory that agrees with perturbative quantum gravity in its range of validity

new picture: AdS/CFT correspondence

- spacetime is asymptotically 5-dimensional Anti-de-Sitter space (has constant negative curvature) times 5-sphere
- conjecture: obtain *non-perturbative* S-matrix

$\longrightarrow$ spacetime in interior is "genuinely quantum", not expanded about a background

- background still present at "boundary"

Problem: unphysical geometry

The “no spacetime” approach

Advantages:

- dynamical nature of spacetime metric can be fully taken into account
- even non-metric or non-topological notions of spacetime could be implemented

Problems:

- missing elements such as probability conservation in time and description of consecutive measurements
- consistency and predictivity must be ensured by alternative means
- difficulty to compare with and recover standard physics (e.g. standard model of particle physics)
- the quantum cosmology problem can occur

Examples:

- Quantum Geometrodynamics
- Loop Quantum Gravity
- Dynamical Triangulations

Example: Relational observables

- In generic "no spacetime" approaches the objects of the quantum theory are only a Hilbert space of states $\mathcal{H}$ and an algebra $\mathcal{A}$ of operators on it. The space of operators is generated by certain classical observables.
- The relational view on quantum theory suggests that the operational role of external spacetime should be recovered from relations between observables.
- Look for observables that play a role similar to a clock.
- Typically, a question posed in this context could take a form like: What is the value of observable X if observable Y has the value y_0?

Problem: a lot is still missing to ensure a coherent interpretation

Example: Closed FLRW model

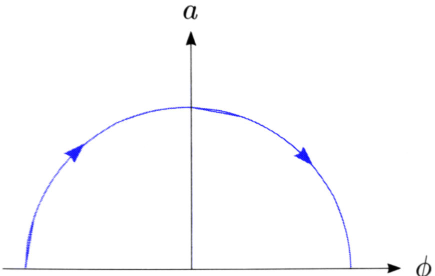
- describe a closed FLRW universe with a massive scalar field ϕ
- metric: $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\Omega_3^2$
- scalar field ϕ homogeneous (only dependent on time)
- Einstein equations take the form:

$$\dot{a}^2 = -1 + a^2\left(\dot{\phi}^2 + \frac{\Lambda}{3} + m^2\phi^2\right)$$

$$\ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} + m^2\phi = 0$$

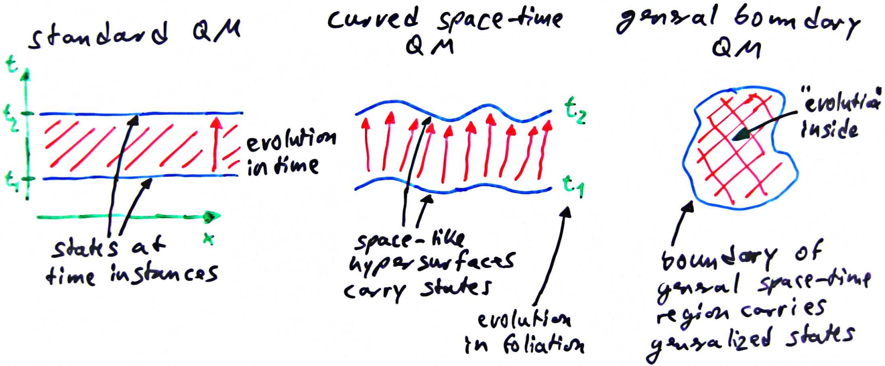
For $m = 0$ solution is

$$\phi(a) = \pm\frac{1}{2}\operatorname{arccosh}\frac{\mathcal{K}}{a^2}$$



- Time t is arbitrary parameter than can be eliminated.
- At early and late times a can play role of *clock variable*.

Extending quantum theory
The General Boundary Formulation (I)



Extending quantum theory

The General Boundary Formulation (II)

TQFT type axioms:

- a state space $\mathcal{H}_\Sigma$ for each hypersurface Σ (not necessarily spacelike)
- an amplitude function ρ_M for each spacetime region M, evaluated on states on the boundary

Extended probability interpretation:
$\mathcal{A} \subset \mathcal{S} \subset \mathcal{H}$ subspaces

$$\boxed{P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_\mathcal{S} \circ P_\mathcal{A}|^2}{|\rho_M \circ P_\mathcal{S}|^2}}$$

Find: probability conservation in spacetime

- perturbative QFT fits into this framework
- 3-dimensional quantum gravity fits
- 4-dimensional quantum gravity not attempted yet...