The bundle structure in discrete models for gauge theory and its relevance

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— Outline —

Motivation

Spacetime discretizations and topology

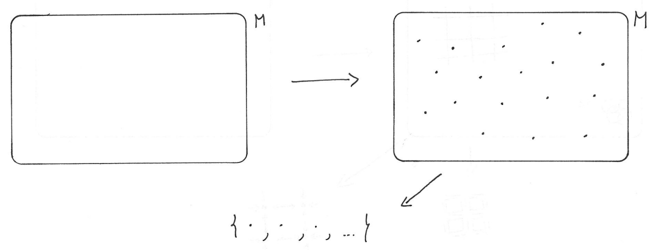
Models for the bundle over a discretized spacetime and their structure

Motivation

- Fields propagate in spacetime $\longrightarrow$
  Histories in FT are local sections of a bundle over spacetime
- Models for FT on a discretization of spacetime
  can keep track of
  spacetime's topology
- Models for gauge FT over a discretization of spacetime
  can retain the bundle's structure,
  but standard LGT does **not** do it

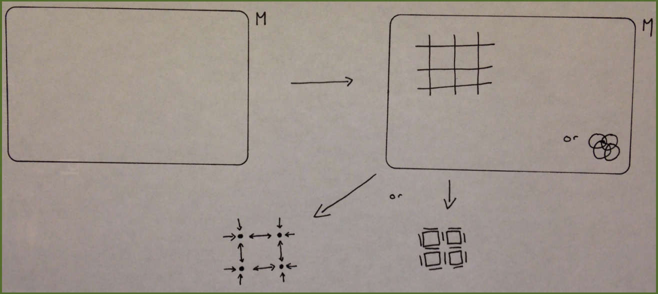
Spacetime discretizations and topology

Spacetime discretization discarding topological information



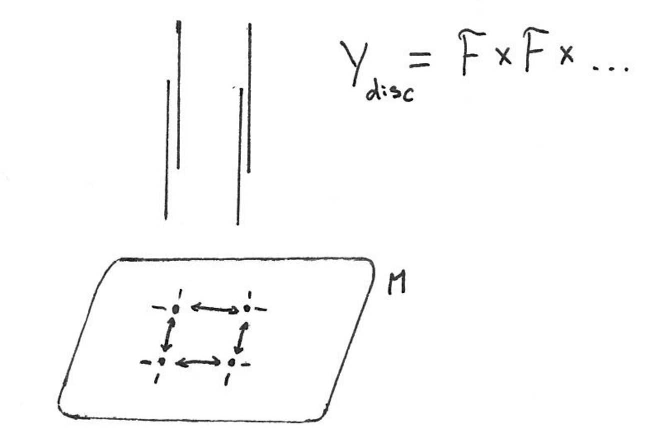
Spacetime discretizations and topology

Discretizations of spacetime which retain the topology



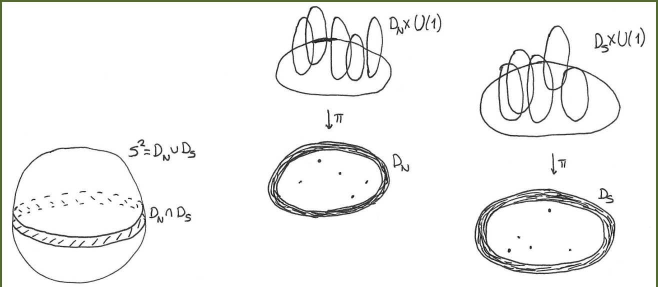
Models for the bundle over a discretized spacetime and their structure

A model for the bundle over a discretized spacetime which does not keep track of the bundle structure



Models for the bundle over a discretized spacetime and their structure

Inequivalent bundles over the same base
and with the same fiber

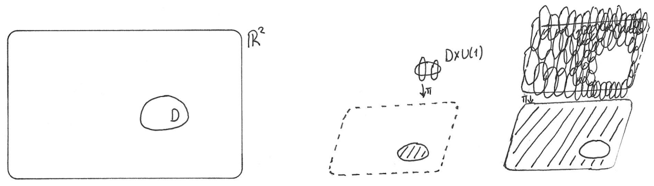


Gluing the trivial bundles on $D_N \cap D_S = S^1 \times [-\epsilon, \epsilon]$

$$
\begin{array}{ccc}
(S^1 \times [-\epsilon,\epsilon] \times U(1))_N & \xrightarrow{g_{NS}} & (S^1 \times [-\epsilon,\epsilon] \times U(1))_S \\
(\theta, \lambda, e^{i\theta_N})_N & \longmapsto & (\theta, \lambda, e^{i\theta_S} = e^{i\theta_{NS}} e^{i\theta_N})_S
\end{array}
$$

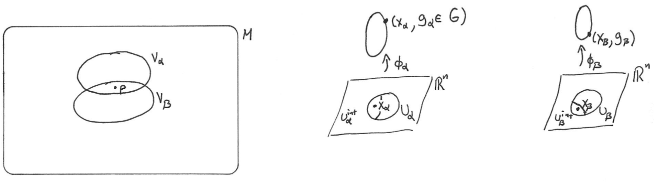
Models for the bundle over a discretized spacetime and their structure

I am interested in local properties of the field,
should I care about this "seemingly global" issue?



Models for the bundle over a discretized spacetime and their structure

Fields in a sigma model in the continuum know how to glue neighboring parts of the bundle



$$\phi \equiv \{\phi_\alpha\}_{\alpha\in I} \quad \text{with} \quad g_\beta(x_\beta) = t_{\alpha\beta}(x_\alpha) g_\alpha(x_\alpha)$$

The transition functions can be retrieved from the field

$$\phi \equiv \{\phi_\alpha\}_{\alpha\in I} \quad \longleftrightarrow \quad t_{\alpha\beta} : U_\alpha^{\text{int}} \to G$$

Models for the bundle over a discretized spacetime and their structure

Can $t_{\alpha\beta} : U_\alpha^{\mathsf{int}} \to G$ be recovered from a field in the lattice?

No.
Only the evauation of $t_{\alpha\beta}$ on a discrete subset of U_α^{int}
is available in the lattice.

Can essential gluing information be stored in the lattice?

Yes.
If different $t_{\alpha\beta} : U_\alpha^{\mathsf{int}} \to G$ lead to inequivalent bundles,
the different *equivalence classes* are labeled by
local data that can be declared as part of the field.

Models for the bundle over a discretized spacetime and their structure

The origin of this lattice data in the continuum is the homotopy types of a collection of the maps:

$$t_{\alpha\beta}|_{F(\alpha\beta)} : F(\alpha\beta) \subset U_\alpha^{\text{int}} \to G$$

with $t_{\alpha\beta}$ fixed on the boundary of the $n-1$ face $F(\alpha\beta) \subset U_{\alpha\beta} = U_\alpha^{\text{int}}$.

$$t_{\alpha\beta\gamma}|_{F(\alpha\beta\gamma)} : F(\alpha\beta\gamma) \subset U_{\alpha\beta\gamma} \to G$$

with $t_{\alpha\beta\gamma}$ fixed on the boundary of the $n-2$ face $F(\alpha\beta\gamma) \subset U_{\alpha\beta\gamma}$

...

where the boundary of a $n-(n-1)$ face are two vertices in which the lattice data may provide the evaluation of $t_{\alpha\beta\ldots}$

Models for the bundle over a discretized spacetime and their structure

Gauge fields in the continuum contain gluing information

Consider a cellular decomposition of spacetime.
The bundle Y is divided in portions over the cells.
The bdl over each n cell is glued to bdls over its bdary faces.
Each bundle is trivial, but the gluing may not be so.

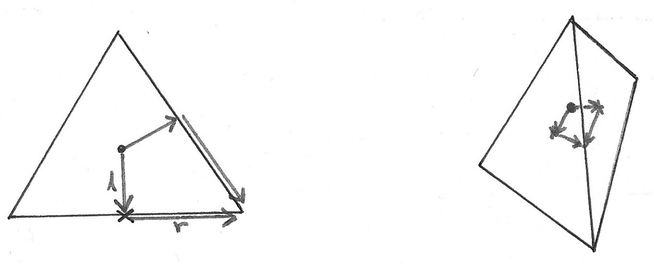
$$[(g \in G, p \in \mathsf{Path}_{\nu,x})] \in Y|_\nu \quad , \quad g_1 \mathrm{PT}^\nu(p_1) = g_2 \mathrm{PT}^\nu(p_2)$$

where the starting point of p is $C\nu$ and the end point is $x \in \nu$.

Gluing of $Y|_\nu$ with $Y|_\tau$ for $\tau \subset \nu$
$[g_1, p_1] \in Y_\nu|_x$ is equivalent to $[g_2, p_2] \in Y_\tau|_x$ if
$g_1 \mathrm{PT}^\nu(p_2^{-1} \circ p_1) = g_2$.

Models for the bundle over a discretized spacetime and their structure

Storing bundle gluing information in LGT



$$\nu \quad \xrightarrow{\tilde{A}} \quad (\nu, \{h_l \in G\}_{l \subset \nu}; \{\tau^{n-1}, \{k_{r_{n-1}} \in G\}_{r \subset \tau^{n-1}}\}_{\tau^{n-1} \subset (\partial\nu)^{n-1}};$$
$$...; \{\tau^0\}_{\tau^0 \subset (\partial\nu)^0}; W = \{ \text{ h. type of gluing } \nu \text{ and } \tau \text{ loc. triv. } \}_{\tau \subset \partial\nu})$$

Interpretational advantages of keeping the bundle

- A sharper understanding of the relation between LGT and continuum gauge theory
- An honest continuum classical limit of LGT may recover classical gauge theory!
- The path integral in LGT could be restricted to sum only over gauge fields of a given bundle
- ...

Ignoring gluing data may have consequences!

- 2d gravity is NOT trivial (if gluing data is ignored)
  $SO(2)$ gauge theory in 2d and $\pi_1(SO(2)) = \mathbb{Z}$.
- In 4d it is relevant that $\pi_3(SU(n)) = \mathbb{Z} \quad n \geq 2$
- Compact QED gets "decompactified"

Thank you for your attention!

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