Real clocks and rods in quantum mechanics.

Rodolfo Gambini

Universidad de la Republica
Uruguay

With Jorge Pullin (Louisiana State) and
Rafael Porto (Carnegie Mellon)

OUTLINE

1) The evolution equation in terms of a real clock variable

2) Limitations to how good a clock or a measuring rod can be

3) Loss of unitarity and loss of entanglement

4) Consequences for the problem of measurement in Quantum Mechanics

The evolution equation in terms of a real clock variable

Given a physical situation of interest described by a (multi-dimensional) phase space *q, p* we start by choosing a ``clock''. By this we mean a physical quantity (more precisely a set of quantities, like when one chooses a clock and a calendar to monitor periods of more than a day) that we will use to keep track of the passage of *time*.

An example of such a variable could be the angular position of the hand of an analog watch. Let us denote it by *T(q,p)*. We then identify some physical variables that we wish to study as a function of time. We shall call them generically *O(q,p).*

We then quantize the system and work in the Heisenberg picture
Notice that we are not in any way modifying quantum mechanics. We assume that the system has a Hamiltonian evolution in terms of an external parameter *t*, which is a classical variable.

$$\mathcal{P}\left(O \in [O_0 - \Delta O, O_0 + \Delta O] | T \in [T_0 - \Delta T, T_0 + \Delta T]\right) = \lim_{\tau \to \infty} \frac{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(P_{O_0}(t) P_{T_0}(t) \rho P_{T_0}(t)\right)}{\int_{-\tau}^{\tau} dt \, \mathrm{Tr}\left(P_{T_0}(t)\rho\right)}$$

The reason for the integrals is that we do not know for what value of the external ideal time *t* the clock will take the value T_0

If one assumes that the clocks and the observed system are independent one can show that the system will be governed by an effective density matrix given by:

$$\rho(T) \equiv \int_{-\infty}^{\infty} dt U_{\rm sys}(t) \rho_{\rm sys} U_{\rm sys}(t)^\dagger \mathcal{P}_t(T)$$

Where *P* the probability density that the measurement of the clock variable takes the value *T* when the ideal time takes the value *t* .

Unitarity is lost since one ends up with a density matrix that is a superposition of density matrices associated with different values of *t.*

If we assume that the “real clock" is behaving semi-classically the probability distribution will be a very picked function concentrated in a neighborhood of the the ideal time t that spreads slowly with time.

$$\frac{\partial \rho(T)}{\partial T} = i[\rho(T), H] + \sigma(T)[H,[H,\rho(T)]].$$

$$\sigma(T) = \partial b(T)/\partial T.$$

Limitations to how good a clock or a rod can be

We have established that when we study quantum mechanics with a physical clock, unitarity is lost, and pure states evolve into mixed states. The effects are more pronounced the worse the clock is.

Which raises the question: is there a fundamental limitation to how good a clock can be? As we don’t have a complete quantum gravity theory this is a contentious point: I will mention three independent arguments leading to an estimate of such a limitation:

A) Salecker and Wigner (1957) and Ng and van Dam (1995)

They noted that the accuracy of a clock is limited by quantum uncertainties

$$\delta t \approx \sqrt{\frac{t}{M}} \qquad (\hbar = c = 1)$$

The amount of mass of the clock cannot be increased indefinitely. if one piles up enough mass in a concentrated region of space one ends up with a black hole.

$$\delta T = T^{1/3} t_p^{2/3}$$

There is a corresponding uncertainty for the measurements of lengths.

B) S.Lloyd and J. Ng Scientific American 291 52 (2004) Giovannetti, Lloyd and Maccone Science 306 1330 (2004)

In order to map out the geometry of spacetime they fill space with clocks exchanging signals with the other clocks and measuring their time of arrival, like the GPS. We can think of this procedure as a special kind of computation. Making use of the Margolus Levitin theorem they prove that the total number of elementary computations per unit volume is bounded by: $n < l^2 / l_{Pl}^2$

And from here they argue that each computation will require a cell of volume

$$\frac{l^3}{l^2 / l_P^2} = l l_P^2$$

And therefore the cells are separated by an average distance

$$\delta l = l^{1/3} l_P^{2/3}$$

Notice that this limitation for the measurement of length and times is also related with the holographic bound if one assumes an entropy per cell of order one. This limits were obtained from heuristic considerations. Is there a derivation from first principles of these bounds? We don't have a complete theory of quantum gravity but…

We have recently established that the kinematical structure of loop quantum gravity in spherical symmetry implies the holographic principle irrespective of the details of the dynamics. It stems from the fact that the elementary volume that any dynamical operator may involve goes as rl_P^2

These limitation on the the time measurements lead to loss of coherence.

$$\rho(T)_{nm} = \rho_{nm}(0) e^{-i\omega_{nm} T} e^{-\omega_{nm}^2 T_{\text{Planck}}^{4/3} T^{2/3}}.$$

Real rods and loss of entanglement.

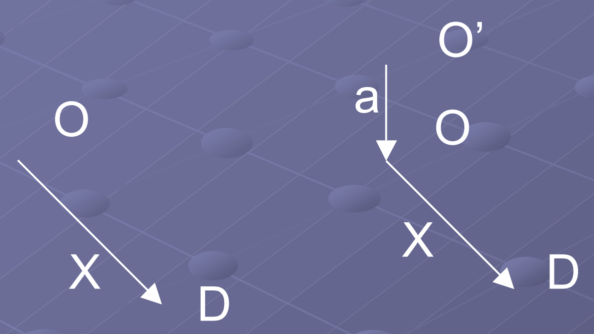
In field theory both time and spatial coordinates are ideal elements. Let us consider non relativistic electrons and let us suppose that we want to compute the probability of finding one electron with certain spin in certain region whose position determined by real rods is given by: $\hat{X}^i(x^j)$

$$P(\varepsilon_a \mid X^j) = \int \prod_i dx_1^i \mathcal{P}_{x_1^j}(X^j)\,\mathrm{Tr}(\hat{P}^z_{x_1^j}(\varepsilon_a)\,\rho)$$

Where P is the probability density that the measurement X^j occurs at x_1^i

Due to the limitations in the measurements of lengths P is *not* a Dirac delta. One may consider that P is a Gaussian whose spread grows with the distance between the origin of the rods and the detector.

$$\delta l = l_P^{2/3}\, l(X)^{1/3}$$



$$P(\varepsilon_a \mid X^j) \neq P(\varepsilon_a \mid X^j + a^j)$$

This effect induces entanglement loss when the system is composed by widely separated portions.

Implications for the measurement problem of quantum mechanics.

The measurement problem in quantum mechanics is related to the fact that in ordinary quantum mechanics the measurement apparatus is assumed to be always in an eigenstate after a measurement has been performed.
The usual explanation for this is that there exists interaction with the environment.

Decoherence then quickly damps superpositions between the localized preferred states when only the system is considered. This is taken as an explanation of the appearance to a local observer of a ``classical'' world of determinate, ``objective'' (robust) properties.

The main problem with such a point of view is how is one to interpret the local suppression of interference in spite of the fact that the total state describing the system-environment combination retains full coherence. One may raise the question whether retention of the full coherence could ever lead to empirical conflicts with the ascription of definite values to macroscopic systems

The usual point of view is that it would be very difficult to reconstruct the off diagonal elements of the density matrix in practical circumstances. However, at least as a matter of principle, one could indeed reconstruct such terms
The evolution of the whole system remains unitary and the coherence of the measurement device will eventually reappear (revivals) .

The fundamental decoherence induced by real clocks suppresses exponentially the off diagonal terms of the density matrix. Revivals of these terms cannot occur no matter how long one waits.

W. Zurek, Phys. Rev. **D26**, 1862 (1982).

S+A: $\{|+>, |->\}.$

E: N two level "atoms" $\{|+>_k, |->_k\}.$

The total Hamiltonian of S+A+E is

$$H_{\rm int} = \hbar \sum_k \left(g_k \sigma_z \otimes \sigma_z^k \otimes \prod_{j \neq k} I_j \right) .$$

If the initial state $|\Psi(0)>$ of the system plus the measurement device is in a quantum superposition, due to the coupling with the environment, the reduced density matrix obtained by taking the trace over the environment degrees of freedom is

$$\rho_c(t) = |a|^2 |+><+| + |b|^2 |-><-| + z(t)ab^* |+><-| + z^*(t)a^*b|-><+|,$$

If *z(t)* vanishes the reduced density matrix is a “proper mixture” representing several outcomes with its corresponding probabilities.

But *z(t)* is a multiperiodic function that will retake the initial value for sufficiently large times. (Poincare Recurrence)

Although this time is usually large, perhaps exceeding the age of the universe, at least in principle it implies that the measurement process does not correspond to a change from a pure to a mixed state in a fundamental way.

If one redoes the derivation using the effective equation we derived for quantum mechanics with real clocks one gets:

$$z'(t) = z(t)\prod_k \exp(-(2g_k)^2 t_P^{4/3} t^{2/3})$$

If one includes real clocks in quantum mechanics revivals are avoided and the pure states resulting from environment decoherence appear to be experimentally undistinguishable from mixed states.

Other procedures for distinguishing between pure and mixed states of the complete system including environment have been proposed.
By analyzing these proposals we were led to conjecture that *when real rods and clocks are taken into account the transition from the pure states resulting from environment decoherence to mixed states seem to be totally unobservable, not only "for all practical purposes" as is usually claimed but because of reasons of principle related with the fundamental structure of spacetime.*

Of course, even if the measuring device is after the measurement in a "proper mixture", problems still persist with the interpretation of quantum mechanics.
We need to explain why we only observe one alternative and not the superposition characteristic of the quantum systems.