The transition rate of an Unruh detector in a general spacetime

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What are "particles" in QFT?

In flat space $\longrightarrow$ $|0\rangle$, $\{|n\rangle_{\vec{k}}\}$

In curved space $\longrightarrow$?

* One possible answer: Fields are fundamental. Particles are just "what particle detectors detect."

* Detector = elementary quantum system w/ energy levels

$\longrightarrow$ couple to field $\longrightarrow$ transitions

Interpret transitions as absorption / emission of particles

Unruh - DeWitt particle detector

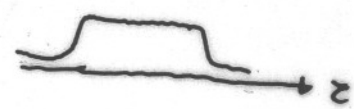
$|1\rangle_d$, $E = \omega$

$|0\rangle_d$, $E = 0$

$$H = H_d + H_\phi + H_{int}$$

$$H_{int} = c\, \mu(\tau)\, \chi(\tau)\, \phi(x(\tau))$$

$\phi(x(\tau))$ → quantum scalar field at $x(\tau)$

$\chi(\tau)$ → switching function 

Q: If at $\tau \to -\infty$ the state is $|\Psi\rangle = |\psi\rangle \otimes |0\rangle_d$,

then what is the probability of a transition to $|\text{any}\rangle \otimes |1\rangle_d$?

A: $$P(\omega) = \underbrace{c^2\, |{}_d\langle 0|\mu(0)|1\rangle_d|^2}_{\text{irrelevant constant}} \underbrace{\int_{-\infty}^{+\infty} d\tau' \int_{-\infty}^{+\infty} d\tau''\, e^{-i\omega(\tau'-\tau'')}\, \chi(\tau')\, \chi(\tau'')}_{= F(\omega) \text{ response function}}$$

$$\times \underbrace{\langle\psi|\, \phi(x(\tau'))\, \phi(x(\tau''))\, |\psi\rangle}_{= W(\tau', \tau'') \text{ Wightman function}}$$

Problem: $F(\omega)$ still depends on $\chi(\tau')$.

Solution (?): make $\chi(\tau') = \theta(\tau'-\tau_0)\,\theta(\tau-\tau')$

then define the transition rate:

$$\dot{F}_\tau(\omega) = 2\,\mathrm{Re}\int_0^{\Delta\tau} ds\, e^{-i\omega s}\, W(\tau,\tau-s)$$

For Rindler trajectory in Minkowski, this is Planckian spectrum. (if $\tau_0 \longrightarrow -\infty$)

<u>However</u>: $W(x,x')$ is actually a distribution

$$W_\varepsilon(x,x') \sim \frac{1}{4\pi^2}\left[\ \frac{1}{\sigma_\varepsilon} + \left[m^2 + \left(\xi - \frac{1}{6}\right)R(x)\right]\ln\sigma_\varepsilon + \cdots\right.$$

$$\sigma_\varepsilon(x,x') = \sigma(x,x') + 2i\varepsilon\left[T(x)-T(x')\right] + \varepsilon^2$$

$\to$ arbitrary time function

and $\varepsilon \to 0$ should be taken only after integration against <u>smooth</u>, compact-supported functions.

Solution: study $F(\omega)$ with smooth χ, and take the $\varepsilon \to 0$ limit under the integral, for a general spacetime, state and trajectory.

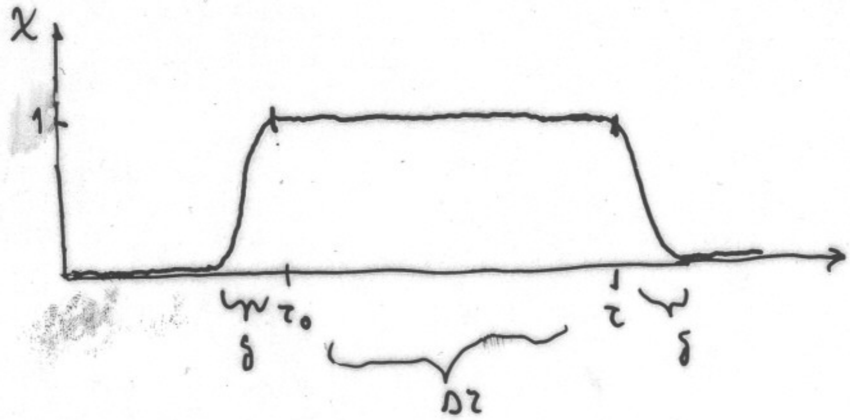
$$\Rightarrow F(\omega) = -\frac{\omega}{4\pi} \int_{-\infty}^{+\infty} du \, [\chi(u)]^2 + \frac{1}{2\pi^2} \int_0^{+\infty} \frac{ds}{s^2} \int_{-\infty}^{+\infty} du \, \chi(u) \left[\chi(u) - \chi(u-s) \right]$$

$$+ 2 \, \mathrm{Re} \int_{-\infty}^{+\infty} du \, \chi(u) \int_0^{+\infty} ds \, \chi(u-s) \left(e^{-i\omega s} W_0(u, u-s) + \frac{1}{4\pi^2 s^2} \right)$$

which is fully equivalent to distributional expression with

$\lim_{\varepsilon \to 0} \int \ldots \, W_\varepsilon(\ldots)$

Next make $\chi(u)$ of the form



and study $F(\omega)$ for $\Delta\tau \gg \delta$, and take the τ-derivative

$$\boxed{\dot{F}_\tau(\omega) = -\frac{\omega}{4\pi} + \frac{1}{2\pi^2 \Delta\tau} + 2\,\mathrm{Re}\int_0^{\Delta\tau} ds \left(e^{-i\omega s}\, W_0(\tau,\tau-s) + \frac{1}{4\pi^2 s^2} \right) + O\left(\delta/\Delta\tau^2\right)}$$

- General expression for any μ, $|\Psi\rangle$ and $x(\tau)$.
- No ε-regulator $\longrightarrow$ suitable for numerics.
- Recovers Unruh effect.
- Simple expression for difference in response:

$$\boxed{\Delta \dot{F}_\tau(\omega) = 2\,\mathrm{Re}\int_0^{\Delta\tau} ds\; e^{-i\omega s} \left(W_0^A(\tau,\tau-s) - W_0^B(\tau,\tau-s) \right)}$$

Useful when W_0^A is the W in a reference situation "A" for which $\dot{F}_\tau$ is known.

Application : Static, Newtonian spacetime

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$h_{\mu\nu} = h_{\mu\nu}(\vec{x}) \quad , \quad \partial_\mu \bar{h}^{\mu\nu} = 0$

$$T_{\mu\nu} = \begin{pmatrix} \rho(\vec{x}) & 0\ 0\ 0 \\ 0 & \\ 0 & \bigcirc \\ 0 & \end{pmatrix}$$

Can use $G_F(x,x')$ instead of $W(x,x')$ because only $\Delta t > 0$ is needed.

$$\left(\Box_x - \xi R(x)\right) G_F(x,x') = -\frac{1}{\sqrt{g(x)}} \delta(x-x')$$

$$G_F = G_F^{(0)} + G_F^{(1)} \quad ; \quad \Box = \Box^{(0)} + \Box^{(1)}$$

$\rightsquigarrow$ solve for $G_F^{(1)}(x,x')$ and use $iG_F^{(1)} = W^{(1)}$ for:

$$\dot{F}_2(\omega) = -\frac{\omega}{2\pi}\Theta(-\omega) + 2\,\mathrm{Re}\int_0^\infty ds\, e^{-i\omega s}\, W^{(1)}\big((t,\vec{x}),(t-s,\vec{x})\big)$$

(detector at rest)

Result for a general source $\rho(\bar{x})$ is:

$$\Delta \dot{F}_{\bar{x}}(\omega) = \frac{G}{\pi}\,\theta(-\omega)\int d^3x'\,\rho(x')\left[\xi\,\frac{\sin(2\omega X)}{X^2} + \frac{\omega\cos(2\omega X)}{X} + 2\omega^2\,\mathrm{Si}(2\omega X) + \pi\omega^2\right]$$

where $X = |\bar{x}-\bar{x}'|$

* No excitations (no "particles present")

* Nontrivial correction to de-excitation rate.

* "Nonlocal" dependence on the source.

E.g. star of constant density, mass M and radius R_0

$$\rightsquigarrow \Delta \dot{F}_r(\omega) \underset{r\to\infty}{\sim} -\left(\xi-\frac{1}{2}\right)\theta(-\omega)\,\frac{3MG}{4\pi r^2}\,\frac{1}{\omega^2 R_0^2}\,\sin(2\omega r)\cos(2\omega R_0)$$

The leading order depends on R_0 as well as M.

Effect is of order $\dfrac{\Delta\dot{F}}{\dot{F}^{(0)}} \sim 10^{-42} \longrightarrow$ unobservable!

Conclussions

* The instantaneous transition rate of an U-DW detector can be defined as:

$$\dot{F}_\tau(\omega) = -\frac{\omega}{4\pi} + \frac{1}{2\pi^2 \Delta\tau} + 2\,\mathrm{Re} \int_0^{\Delta\tau} ds \left(e^{-i\omega s}\, W_0(\tau, \tau - s) + \frac{1}{4\pi^2 s^2} \right)$$

for general spacetime, Hadamard state and trajectory.

* For a detector at rest in a Newtonian static spacetime:

$$\dot{F}_\tau(\omega) = -\frac{\omega}{2\pi}\Theta(-\omega) + G\,\Theta(-\omega) \int d^3x'\, \rho(\bar{x}') \ldots\ldots$$

* Contains the full information about $\rho(\bar{x})$ at all orders in an asymptotic expansion.

* What for a collapsing star...?