BACKREACTIONS IN COSMOLOGICAL PERTURBATION THEORY

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BASED ON:

- "A PERTURBATIVE APPROACH TO DIRAC OBSERVABLES AND THEIR SPACE-TIME ALGEBRA"
  B. DITTRICH, JT gr-qc/0610060 (CQG 2007)

- "GAUGE INVARIANT PERTURBATIONS AROUND SYMMETRY REDUCED SECTORS OF GENERAL RELATIVITY: APPLICATIONS TO COSMOLOGY"
  B. DITTRICH, JT gr-qc/0702093
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OVERVIEW

1) COSMOLOGICAL PERTURBATION THEORY: STANDARD TREATMENT

2) A NEW HAMILTONIAN FRAMEWORK

3) EXAMPLE: BIANCHI-I

4) CONCLUSIONS

COSMOLOGICAL PERTURBATION THEORY: STANDARD TREATMENT

[See e.g. Brandenberger, Feldmann, Mukhanov: Phys. Rep. 215, 1992]

FRW-solution (for vanishing spatial curvature):

$$ds^2 = a^2(\eta)\,(d\eta^2 - d\vec{x}^{\,2})$$

$$\Big\downarrow \text{ put in Einstein's field equations + desired matter contribution } T^{\mu}{}_{\nu} = \mathrm{diag}(\rho, -p, -p, -p)$$

Friedman-equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad ; \quad \dot{\rho} = -3H(\rho + p)$$

Friedman-eq. describe dynamics of an ENTIRELY HOMOGENEOUS and ISOTROPIC universe.

- How to deal with an INHOMOGENEOUS and ANISOTROPIC universe?

$$g_{\mu\nu} = \underset{\substack{\uparrow \\ \text{FRW}}}{\overset{(0)}{g}_{\mu\nu}} + \underset{\substack{\uparrow \\ \text{10 components}}}{\delta g_{\mu\nu}}$$

- characterize 10 entries of $\delta g_{\mu\nu}$ according to their behaviour under SO(3):

4 scalar modes

4 vector modes

2 tensor modes

- What are the physical, i.e. gauge independent degrees of freedom?

COSMOLOGICAL PERTURBATION THEORY: STANDARD TREATMENT

LINEAR PERTURBATION THEORY:

- what is small? $\quad |\delta g_{\mu\nu}| \ll |g_{\mu\nu}|$, etc.
- 10 components of $\delta g_{\mu\nu}$ $\not\Leftrightarrow$ 2 independent dof's of gravity

  $\Rightarrow$ Have to find the physical degrees of freedom in linear perturbation theory.

- in linear order:
  - modes do NOT couple
  - can concentrate on SCALAR modes
  - gauge invariant quantities are known: "BARDEEN POTENTIALS" (Bardeen 1980)

    Bardeen potentials are invariant under SMALL gauge trafo's (linearized constraints in a Hamiltonian language)
  - or: gauge-fixing, e.g. "longitudinal gauge"

- insert BP's + your choice of matter (e.g. scalar field) into Einstein's field eq.

  $\Rightarrow$ keep only terms linear in $\delta g_{\mu\nu}$, $\delta T^{\mu}{}_{\nu}$

  $\Rightarrow$ system of DE's

  $\Rightarrow$ Dynamical evolution of matter- and gravitational perturbations on a FRW-background.

  $\rightarrow$ most important tool to study physics of the early universe by observing the CMB.

COSMOLOGICAL PERTURBATION THEORY: STANDARD TREATMENT

PROBLEMS with generalization to HIGHER ORDER:

- modes couple
  - ⇒ · problematic to extract gauge-inv. observables already to 2nd order
  - ⇒ difficult to distinguish PHYSICAL PERTURBATIONS from mere COORDINATE ARTEFACTS
  - ⇒ cannot trust dynamical evolution

- backreactions
  - split into BACKGROUND and PERTURBATIONS gets ambigous in higher order
  - background-dynamics gets influenced by perturbations starting at 2nd order
    (i.e FRW is only a 0th order approx. to the dynamical behaviour of homogeneous and isotropic quantities)

- messy calculations!

A NEW HAMILTONIAN FRAMEWORK:

- based on RELATIONAL OBSERVABLES [Rovelli 90's, Dittrich '04]

- "coordinates" are not put in by hand from the exterior but are defined in a purely dynamical way
  (-> use physical fields to define what space and time are)

- "background" emerges through an averaging procedure over dynamical dof's on the spatial hypersurface

- symmetry reduced sector of homogeneous and isotropic fields ("background") stays entirely dynamical
  -> important for consistency in higher orders

- "Feynman-like" interpretation in terms of
  FREE PROPAGATION AND INTERACTIONS

  ↑ linear evolution of perturbations on FRW-background

  ↑ couplings between different modes

A NEW HAMILTONIAN FRAMEWORK:

- start from relational observables of full GR: [Dittrich '04]

$$F_{[A(\sigma), T^K(\sigma)]}(\tau^K) = \sum_{n=0}^{\infty} \frac{1}{n!}\{\ldots\{A, \tilde{C}_{K_1}\}, \ldots \tilde{C}_{K_n}\} \times (\tau^{K_1} - T^{K_1}) \cdot \ldots \cdot (\tau^{K_n} - T^{K_n})$$

$$\tilde{C}_K = (A^{-1})_K^{\ j} C_j \qquad A^K_{\ j} = \{T^K, C_j\}$$

$T^K \hat{=}$ "clock-function" (dynamical definition of a coordinate system)

- F is a Dirac-observable of full GR, i.e. weakly commutes with all the constraints of GR.

- split phase-space of GR into HOMOGENEOUS, ISOTROPIC SECTOR and EVERYTHING ELSE
  - can be done by using a PROJECTION OPERATOR acting on a canonical basis:

$$\boxed{\mathcal{P} := \frac{1}{3}\int_{\Sigma} d\sigma \, tr(\ldots)}$$

  - work with Ashtekar's variables:

$$A_a^{\ j}(\sigma) = \underbrace{A \beta \delta_a^j}_{\mathcal{P}} + \underbrace{a_a^{\ b}(\sigma) \beta \delta_b^j}_{(id - \mathcal{P})}$$

$$E^a_{\ j}(\sigma) = E \beta^{-1} \delta^a_j + e^a_{\ b}(\sigma) \beta^{-1} \delta^b_j$$

$\Rightarrow$ symplectic structure

$$\{A, E\} = \frac{\kappa}{3} \qquad \{a_a^{\ b}(\sigma), e^c_{\ d}(\sigma')\} = \kappa\left(\delta_a^c \delta_d^b - \frac{1}{3}\delta_a^b \delta_d^c\right)$$

- INSERT this split into F and sort by powers of fluctuations
  $\Rightarrow$ APPROXIMATE DIRAC OBSERVABLES

A NEW HAMILTONIAN FRAMEWORK:

linear perturbation theory:

- calculate $^{[1]}F_{[f,T^K]}(\hat{\tau}^K)$ for a 1st order quantity f

  (i.e. keep only linear terms in complete observable)

- can reproduce standard results of linear pert. theory using a specific choice of clock-variables:
  $\rightarrow$ "longitudinal clocks"

BACKREACTION EFFECTS:

- $^{[0]}F_{[f,T^K]}(\hat{\tau}^K) =: \alpha_{free}^{t-T^0(0)}(f)$ coincides with usual FRW-dynamics for a 0th order variable f

- calculate $^{[2]}F_{[f,T^K]}(\hat{\tau}^K)$ for f 0th order:

$$^{[2]}F_{[f,T^K]}(\hat{\tau}^K) = \alpha_{free}^{t-T^0(0)}(f) + \int_0^{t-T^0(0)} ds\; \alpha_{free}^{t-T^0(0)}\Big[\underbrace{\{\alpha_{free}^{s}(f),\, {}^{(2)}C\}}\Big]$$

\+ gauge inv. ext. of these terms

coupling of homogeneous and inhomogeneous modes: INTERACTION TERMS

EXAMPLE: BIANCHI-I MODEL

- homogeneous but **ANISOTROPIC** cosmology, can be solved exactly
- $A_a^{\;j} = \beta \,\mathrm{diag}(A_1, A_2, A_3) \qquad E^a_{\;j} = \beta^{-1}\mathrm{diag}(E_1, E_2, E_3)$

  $\phi(x) = \phi \qquad\qquad \pi(x) = \pi$

  Hamiltonian constraint:

  $$C = \frac{2\beta^2}{\kappa}\left(E_1A_1E_2A_2 + E_1A_1E_3A_3 + E_2A_2E_3A_3\right) + \frac{1}{2\kappa}\pi^2$$

- treat Bianchi-I as anisotropic perturbation around FRW:

  $A := \frac{1}{3}\sum_i A_i \qquad\qquad E := \frac{1}{3}\sum_i E_i$

  $a_i := A_i - A \qquad\qquad e_i := E_i - E$

- consider complete observable associated to E and the clock $T = \frac{\phi}{\pi}$

  exact solution:

$$F_{[E,T]}(\tau) = \frac{1}{3}\sum_i E_i \exp\left[2\beta^2 A_i E_i\left(\tau - \frac{\phi}{\pi}\right)\right]\exp\left[-2\beta^2\sum_j A_j E_j\left(\tau - \frac{\phi}{\pi}\right)\right]$$

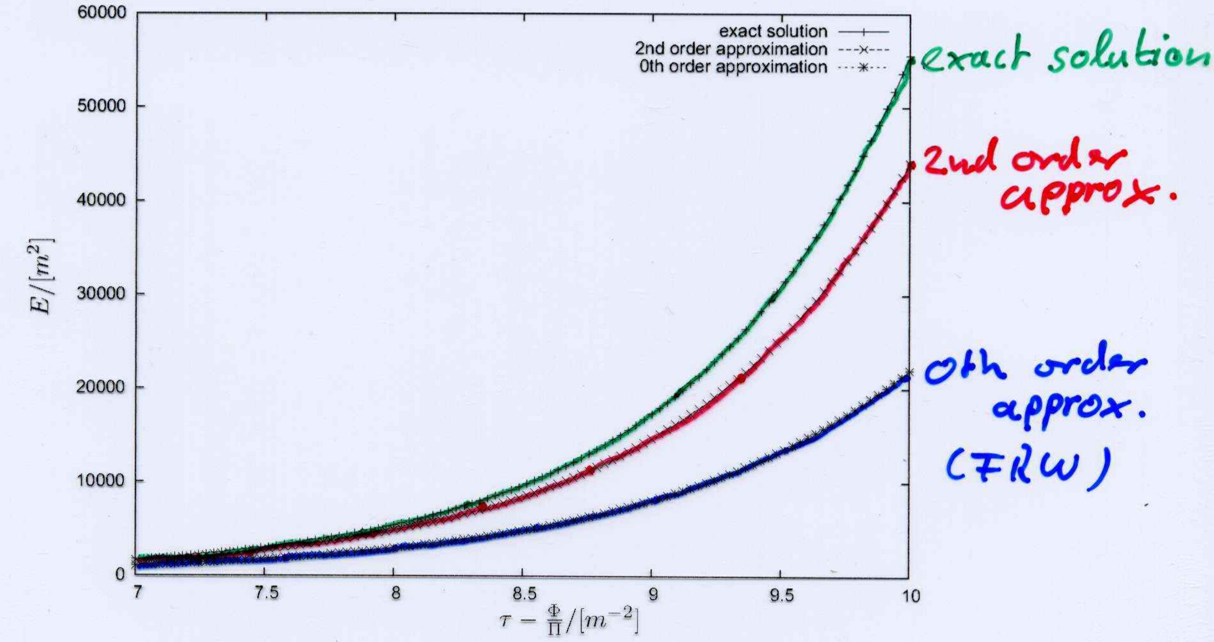
  0th order solution: (FRW)

$${}^{[0]}F_{[E,T]}(\tau) = E\cdot\exp\left[-\omega\left(\tau - \frac{\phi}{\pi}\right)\right] \qquad \omega = 4\beta^2 AE$$

  2nd order solution:

$${}^{[2]}F_{[E,T]}(\tau) = \exp\left[-\omega\left(\tau - \frac{\phi}{\pi}\right)\right]\cdot$$
$$\cdot\Big[E + \frac{2}{3}\beta^2 A\left(\tau - \frac{\phi}{\pi}\right)\sum_i e_i e_i - \frac{2}{3}\beta^2 E\left(\tau - \frac{\phi}{\pi}\right)\sum_i a_i e_i$$
$$+ \frac{\beta^2}{6}\omega A\left(\tau - \frac{\phi}{\pi}\right)^2\sum_i e_i e_i + \frac{\beta^2}{6}\omega\frac{E^2}{A}\left(\tau - \frac{\phi}{\pi}\right)^2\sum_i a_i a_i$$
$$+ \frac{\beta^2}{3}\omega E\left(\tau - \frac{\phi}{\pi}\right)^2\sum_i a_i e_i\Big]$$

EXAMPLE : BIANCHI-I MODEL



initial conditions: $A = 1 \quad E = 1\,m^2$

$a_1 = a_2 = 0.1 \quad a_3 = -0.2$

$e_1 = e_2 = 0.1\,m^2 \quad e_3 = -0.2\,m^2$

Π determined by constraint

- FRW is NOT A GOOD APPROXIMATION to the true dynamical behaviour of homogeneous and isotropic quantities for all times

- BIANCHI-I is to simple to draw any conclusions, would be interesting to calculate backreaction effects in a physically more realistic model (→ DARK ENERGY ?)

CONCLUSIONS

- A NEW FRAMEWORK FOR COSMOLOGICAL PERTURBA-TION THEORY
  - BASED ON RELATIONAL OBSERVABLES OF FULLGR
    → ISSUE OF GAUGE INVARIANCE CAN BE KEPT UNDER CONTROL
  - NO FIXED BACKGROUND STRUCTURE
    → "BACKGROUND" EMERGES THROUGH AVERAGING PROCEDURE OVER DYNAMICAL DEGREES OF FREEDOM
    → "BACKGROUND" STAYS DYNAMICAL
      → IMPORTANT FOR CONSISTENCY
  - CAN CALCULATE BACKREACTION EFFECTS
  - BIANCHI-I CONSIDERATIONS SHOW THAT FRW DOES NOT HAVE TO BE A GOOD APPROXIMATION FOR ALL SITUATIONS
    → WOULD BE INTERESTING TO CONSIDER PHYSICALLY MORE REALISTIC MODELS
    (→ DARK ENERGY ???)