Gauge-invariant coherent states for Loop Quantum Gravity

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- Kinematical Hilbert space $\mathcal{H}_{kin} \; = \; L^2(\overline{\mathcal{A}}, \, d\mu_{AL})$.
- Gauss-, Diff-, Hamilton- (or Master-) constraint: $\hat{G}_I$, $\hat{D}_a$, $\hat{H}$ ($\hat{M}$)
- $\longrightarrow$ Difficult to solve! Approximations?
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Complexifier Coherent states

- Good way to deal with these issues: Coherent states
- Candidates: Complexifier coherent states Thiemann, Winkler, hep-th/0005233, 0005237, 0005234
- $\psi_{(A_0,\,E_0)}(A) \;:=\; \left(e^{-\hat{C}}\delta(A,A_0)\right)\Big|_{A_0\to A_0+iE_0}$
- Simplest example: state $\psi^t_{\vec{g}} \;\in\; \mathcal{H}_\gamma$ associated to a graph γ.
- Labeled by $\vec{g}=(g_1,\ldots g_E)\in G^{\mathbb{C}}$: complexified holonomies along edges $e_1,\ldots e_E$ of γ, t: semiclassicality scale.

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Properties:

- $G = SU(2)$:

$$\psi^t_{\vec{g}}(\vec{h}) = \prod_{k=1}^{E} \sum_{j_k \in \frac{1}{2}\mathbb{N}} e^{-j_k(j_k+1)\frac{t}{2}}(2j_k+1)\,\mathrm{tr}_{j_k}\left(g_k\, h_k^{-1}\right)$$

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- With this one can show: i.e. Master constraint correctly implemented on $\mathcal{H}_{kin}$. Good tool for approximations Giesel, Thiemann, gr-qc/0607099, gr-qc/0607100, gr-qc/0607101

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Properties:

- But: complexifier coherent states do not satisfy the constraints (only approximately)
- $\longrightarrow$ purely kinematical!
- Desirable: Coherent states that satisfy the constraints $\hat{G}_I$, $\hat{D}_a$, $\hat{H}$, in order to address dynamical questions.
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The Gauss gauge group

- Given graph γ with E edges and V vertices
- $\Rightarrow$ $\hat{G}_I$ act as gauge group G^V on $\mathcal{H}_\gamma \simeq L^2(G^E)$:

$$\alpha_{k_1,\ldots k_V}\psi(h_1,\ldots h_E) \;=\; \psi\big(k_{b(e_1)}\, h_1\, k_{f(e_1)}^{-1}, \ldots, k_{b(e_E)}\, h_E\, k_{f(e_E)}^{-1}\big)$$

- G compact

$$\Rightarrow \quad \mathcal{P} \;:=\int_{G^V} d\mu_H^{\otimes V}(\vec{k})\,\alpha_{k_1,\ldots k_V}$$

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$$\Psi^t_{[\vec{g}]} = \Psi^t_{[\vec{g}']} \quad \Leftrightarrow \quad \vec{g} = \alpha_{\vec{k}}\vec{g}' \qquad \text{for } \vec{k} \in (G^{\mathbb{C}})^V$$

$$\text{i. e. } g_k = k_{b(e_k)} g'_k k^{-1}_{f(e_k)}$$

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Gauge-invariant coherent states for $G = U(1)$

The case of $G = U(1)$:

- Complexifier coherent states on a graph γ:

$$\psi^t_{\vec{z}}(\vec{\phi}) \;=\; \prod_{k=1}^{E} \sqrt{\frac{2\pi}{t}} \sum_{n_k\in\mathbb{Z}} e^{-\frac{(z_k-\phi_k-2\pi n_k)^2}{2t}}$$

$e^{i\phi_k} \in U(1)$, $e^{iz_k} \in U(1)^{\mathbb{C}}$.

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with

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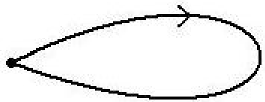
The 1−flower graph:

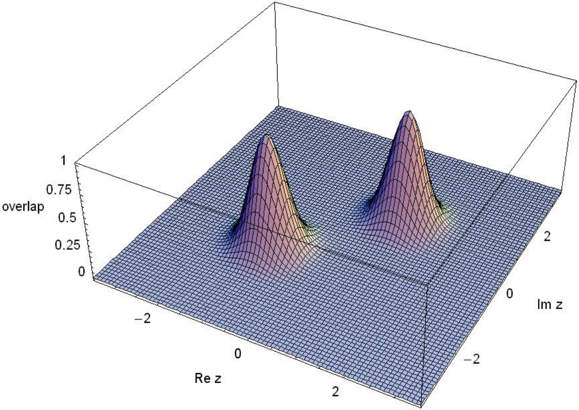
Simplest graph: 1−flower:



$$\Psi^t_{[g]}([h]) \;=\; \Psi^t_{\mathrm{tr}(g)}(\mathrm{tr}(h)) \;=\; \sum_{j\in\frac{1}{2}\mathbb{N}} e^{-j(j+1)\frac{t}{2}}\mathrm{tr}_j(g)\,\mathrm{tr}_j(h)$$

$$\frac{\left|\langle\Psi^t_{\cos w}|\Psi^t_{\cos z}\rangle\right|^2}{\left\|\Psi^t_{\cos w}\right\|^2\left\|\Psi^t_{\cos z}\right\|^2} \;=\; \frac{\left|\sinh\frac{\bar{w}z}{t}\right|^2}{\sinh^2\frac{|z|^2}{t}\;\sinh^2\frac{|w|^2}{t}}\left(1+O(t^\infty)\right)$$

The 1−flower graph



$$\text{overlap} = \frac{\left|\left\langle \Psi^t_{\cos w} \middle| \Psi^t_{\cos z}\right\rangle\right|^2}{\left\|\Psi^t_{\cos w}\right\|^2 \left\|\Psi^t_{\cos z}\right\|^2} \qquad \text{with } w = 1+i,\ t = 0.25.$$

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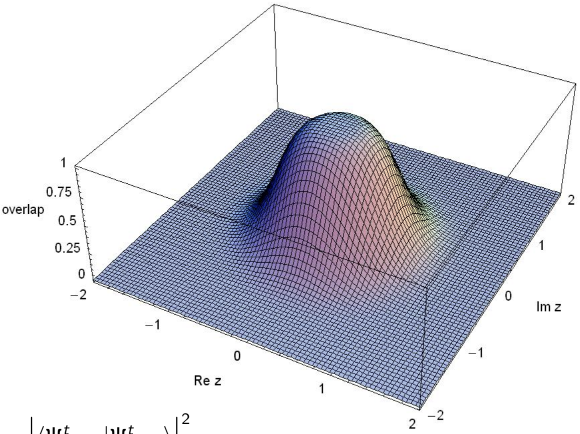

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Other graphs

- Unfortunately, more complicated graphs lead to quite difficult expressions for the overlap.
- $\longrightarrow$ Employ numerical investigations
- Done for 2-flower, sunset graph, tetrahedron. Qualitative results always the same!

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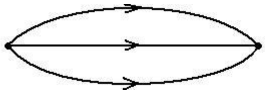
The sunset graph

- Example: the sunset graph



- Gauge-invariant phase space: $[g_1, g_2, g_3] \in SL(2,\mathbb{C})^3 \,/\, SL(2,\mathbb{C})^2$
- Gauge-fixing: three complex parameters $(z_2,\, z_3,\, \theta)$

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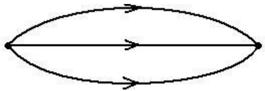

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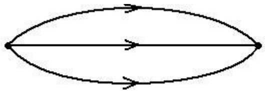

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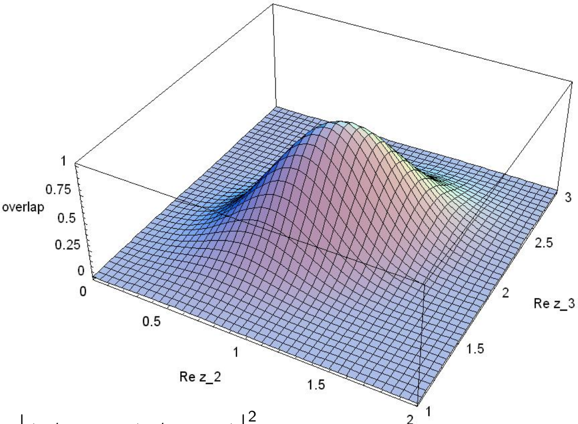
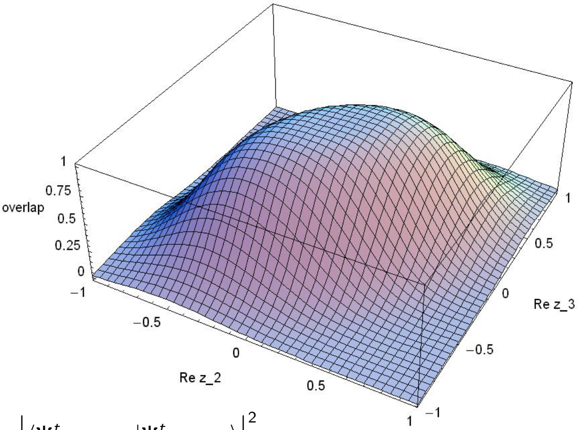
The sunset graph: Gaussian peak



$$\text{overlap} = \frac{\left| \langle \Psi^t_{(w_2, w_3, \chi)} | \Psi^t_{(z_2, z_3, \theta)} \rangle \right|^2}{\left\| \Psi^t_{(w_2, w_3, \chi)} \right\|^2 \left\| \Psi^t_{(z_2, z_3, \theta)} \right\|^2}, \quad w_2 = 1, \; w_3 = 2, \; \chi = \theta = 0, \; t = 0.2$$

The sunset graph: plateau structure



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General properties

- In all examples: Gauge-invariant states are peaked around gauge-invariant data.
- In all examples: States with data corresponding to degenerate gauge orbits (e.g. $[\vec{g}] = [1, 1, \ldots, 1]$) have significantly broader peak: plateau structure. No Gaussian anymore!
- In fact one can show this: At the maximum of peak, all derivatives vanish until order 4!
  - - in general for flower graphs
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Summary

- Gauge-invariant coherent states for $G = U(1)$ and $G = SU(2)$ obtained by projecting CCS to gauge-inv. subspace. They have been investigated by analytical and numerical methods
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