Algebraic Quantum Gravity

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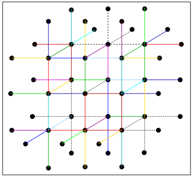
Albert – Einstein – Institute

Loops '07

Morelia 26.06.2007

Ref.: K.G., T. Thiemann, CQG **24** (2007) 2465-97, 2499-2564, 2565-88



Plan of the Talk

- Motivation
    - Status of the dynamics in Loop Quantum Gravity (LQG)
    - Status of the semiclassical properties of LQG
- The Master Constraint Programme
- Algebraic Quantum Gravity (AQG)
    - Conceptual setup of AQG
    - The kinematics & dynamics of AQG
    - Semiclassical limit of AQG
- Conclusions & Outlook

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Status of the Dynamics in LQG

- Starting point of LQG: Canonical formulation of GR
- Holonomies $\mathrm{A(e)}$ & Fluxes $\mathrm{E(S)}$
- Additionally one gets constraints:
  $\mathrm{G(A,E)} = 0, \quad \mathrm{D(A,E)} = 0, \quad \mathrm{H(A,E)} = 0$
- Kinematical Hilbert space of LQG: $\mathcal{H}_{\mathrm{kin}}$; Operators: $\widehat{\mathrm{G}}\,,\ \widehat{\mathrm{D}}\,,\ \widehat{\mathrm{H}}$

Quantum − Einstein − Equations

$$\widehat{\mathrm{G}}\psi_{\mathrm{phys}} = 0, \quad \widehat{\mathrm{D}}\psi_{\mathrm{phys}} = 0, \quad \widehat{\mathrm{H}}\psi_{\mathrm{phys}} = 0$$

- Solutions ψ_{phys}? Rediscover classical GR solutions in LQG?
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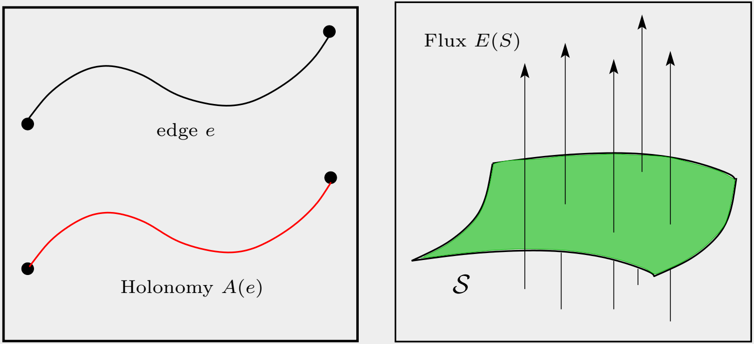
Elementary Phase Space Variables of LQG

Holonomies $A(e)$ und Fluxes $E(S)$



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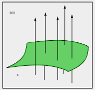

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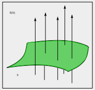

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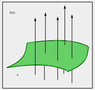

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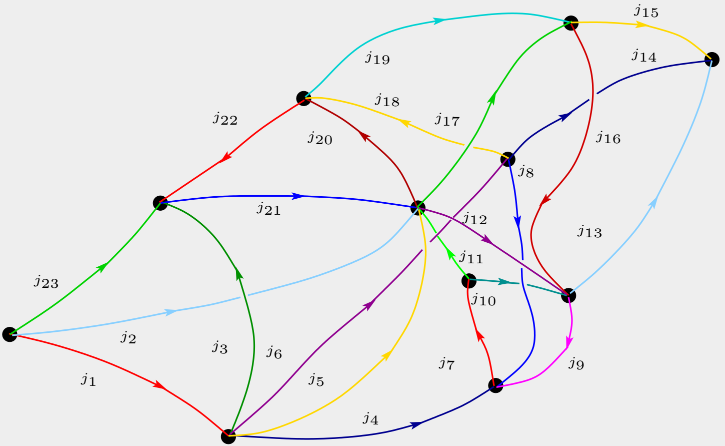
Basis of $\mathcal{H}_{kin}$

Spin network functions



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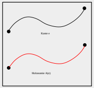

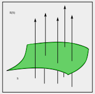

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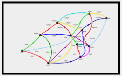

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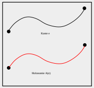

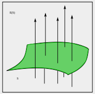

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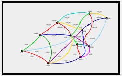

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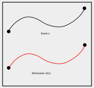

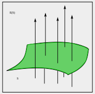

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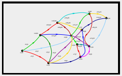

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Anomalies in the QT

$$\{\mathrm{H(N)}, \mathrm{H(N')}\} \propto \mathrm{D}(\vec{\mathrm{N}})$$
$$[\widehat{\mathrm{H}}(\mathrm{N}), \widehat{\mathrm{H}}(\mathrm{N'})] \propto ?$$

- Most difficult part: $\widehat{H}\psi_{phys} = 0$
- $\mathcal{H}_{\mathrm{kin}}$, Uniqueness LOST – Theorem
- Unitary repres. of finite diffeomorphisms
  $\widehat{\mathrm{U}}(\mathrm{D})$ is not weakly continuous $\longrightarrow$ infinites. $\widehat{D}$ $\nexists$ in $\mathcal{H}_{kin}$
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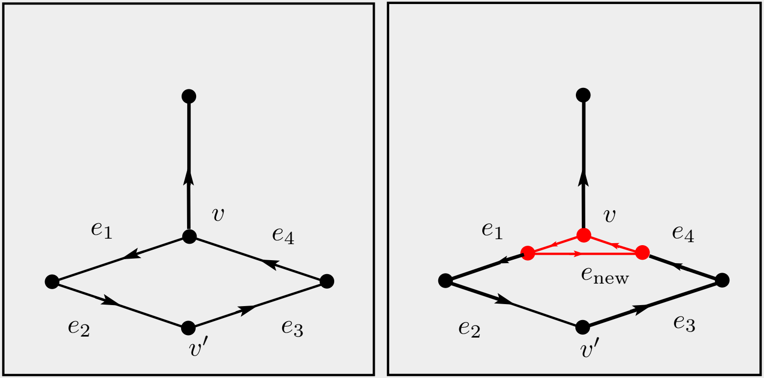
Graph – changing Operators

Graph – changing Hamiltonian Constraint Operator $\widehat{\mathrm{H}}$



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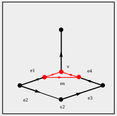

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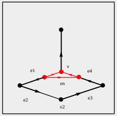

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Master Constraint Programme

The Master Constraint **M**

- **M** consists of weighted, spatially diff-inv. sum [Thiemann 2003]

$$\mathbf{M} = \int_\sigma d^3x \, \frac{\delta^{jk} G_j G_k + q^{ab} D_a D_b + H^2}{\sqrt{|\det(E)|}}(x)$$

- $\mathbf{M} = 0 \quad \Leftrightarrow \quad \mathrm{G} = 0 \quad \wedge \quad \mathrm{D} = 0 \quad \wedge \quad \mathrm{H} = 0$
- Weighting: Infinitesimal diffeomorphism – operators exist
- Trivial algebra: $\{\mathbf{M}, \mathbf{M}\} = 0$, QT: $[\widehat{\mathbf{M}}, \widehat{\mathbf{M}}] = 0$
- Anomalies? $\widehat{\mathbf{M}}\,\psi = 0 \to 0 \notin$ in $\sigma(\widehat{\mathbf{M}})$
- Two possibilities to quantise **M**
  - Graph – changing: Semiclassically problematic, $\mathcal{H}_{\mathrm{diff}}$ [WIP Bahr]
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- $\mathbf{M} = 0 \quad \Leftrightarrow \quad \mathrm{G} = 0 \quad \wedge \quad \mathrm{D} = 0 \quad \wedge \quad \mathrm{H} = 0$
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Master Constraint Programme

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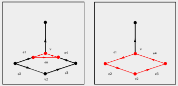

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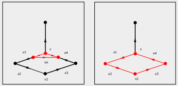

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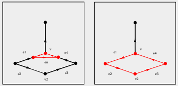

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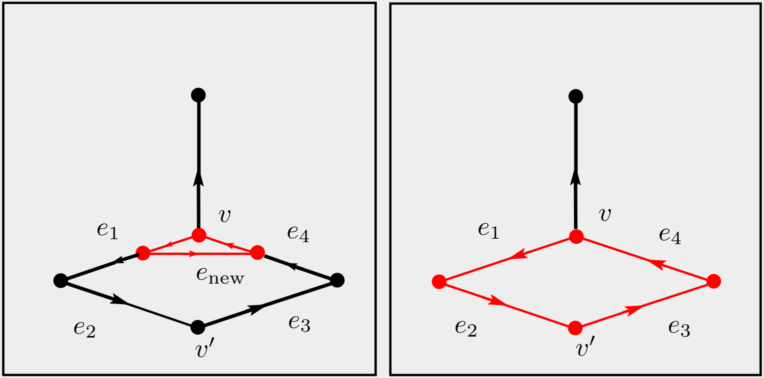
Comparison

Graph – changing und graph – preserving $\widehat{\mathbf{M}}$



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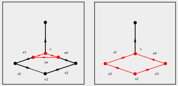

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Semiclassical Techniques

Coherent States

- Certain sector of $\mathcal{H}_{kin}$ with almost classical behaviour
- Such so called coherent states exist in $\mathcal{H}_{kin}$ [Winkler,Thiemann 2001]
- Good semiclassical approximation of $\widehat{\mathrm{A}}(\mathrm{e})$, $\widehat{\mathrm{E}}(\mathrm{S})$
  [More details Bahr's talk]
- Our Aim: Testing the semiclassical limit of dynamical operators in $\mathcal{H}_{kin}$
  - $\widehat{H}$ Graph – changing problematic:
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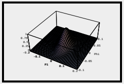

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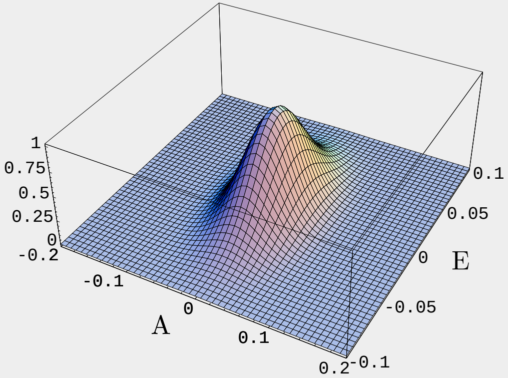
Coherent States

Gaußian states concentrated around a classical phase space point (A, E)



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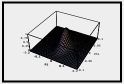

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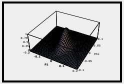

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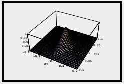

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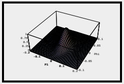

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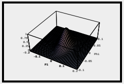

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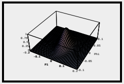

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Algebraic Quantum Gravity (AQG)

Conceptual Ideas of AQG

- LQG needs all embedded finite graphs in σ
- $\widehat{\mathrm{M}}$ can be quantised graph – preserving
- Embedding independent formulation of AQG:
  - One fundamental infinite algebraic graph
  - Difference with graphs in LQG:
    Information about topology & differential structure of spatial manifold σ are absent
- Idea: Def. of the algebra and dynamical operators on algebraic level
- Consequence: No graph – dependence anymore
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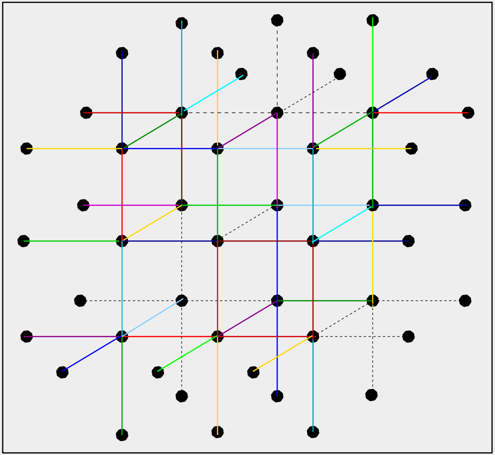
Algebraic Graph

Algebraic graph with cubic topology



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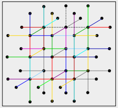

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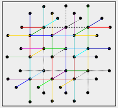

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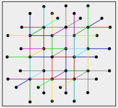

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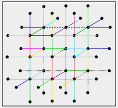

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Kinematical & Dynamical Setup of AQG

Mathematical Framework of AQG

- LQG: embedded algebra $\leftrightarrow$ AQG: abstract algebra
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  - Kinem. Hilbert space $\mathcal{H}_{ITP}$
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- Semiclassics: One needs to provide the following data:
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  - Analogous defintion of algebraic coherent states
- AQG: Topology, differential strucure of σ and background metric which is approximated is encoded in semiclassical states

Kinematical & Dynamical Setup of AQG

Mathematical Framework of AQG

- LQG: embedded algebra $\leftrightarrow$ AQG: abstract algebra
- Quantisation:
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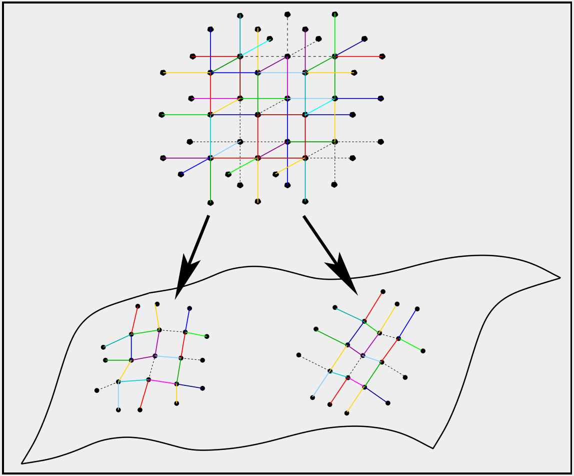
Fundamental Algebraic Graph

Information on the embedding are encoded in the coherent states



Semiclassical Analysis of $\widehat{\mathbf{M}}$

Semiclassical Limit of the Dynamics of AQG

- Semiclassical limit of $\widehat{\mathbf{M}}$ wrt algebraic coherent states of an algebraic cubic graph
- One gets expansion of $\langle \Psi^{\hbar}_{(A,E)} \mid \widehat{\mathbf{M}} \mid \Psi^{\hbar}_{(A,E)} \rangle$ wrt $\hbar$
- Result in leading order

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