Large scale correlations in Spin Foam Models for Quantum Gravity

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The problem of quantum gravity

Perturbative quantum gravity, viewed as an effective field theory, provides a consistent description of quantum gravity in the low-energy regime

(Donoghue PRL1994, Burgess LivRevRel2004)

- In this phenomenological approach, only the low-energy field content and the symmetries of the action have to be prescribed.
- Then, power-counting results allow to calculate perturbatively - including loops - using a non-renormalizable effective Lagrangian.

  fixed the typical energies of the process of interest and the required accuracy, the appropriate phenomenological Lagrangian contains only a finite number of terms.

- The theory is predictive, once a finite number of coupling constants has been measured.

A typical process of interest can be graviton scattering on Minkowski spacetime

Perturbative quantum gravity (as an effective field theory)

this approach naturally identifies the scale where it breaks down - the physical cutoff - and requires *new physics* to unfreeze at such scale.

Hence it can be viewed as an effective field theoretical description of the underlying physics at this scale.

The scale of interest here is $M_P \approx 10^{19}\mathrm{GeV}$

the underlying high-energy physics can be:

- (i) still an ordinary local quantum field theory defined on a manifold with a non-dynamical metric $(\mathcal{M}, \bar{g})$, or
- (ii) a quantum field theory defined on a *bare* manifold $\mathcal{M}$
- (iii) or even something else.

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- **(ii)** a quantum field theory defined on a *bare* manifold $\mathcal{M} \longleftarrow$ **LQG**
- (iii) or even something else.

mathematically well defined version of the Wheeler-DeWitt approach

The loop approach to quantum gravity

- important to probe the theory in the deep quantum regime, such as
  - at the big bang singularity
  - at the black hole singularity
- on the other hand

> it's crucial to understand if the theory admits a regime where large scale correlations are present and an ordinary quantum field theoretical description is available.

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Recently, some progress in this direction has been made

C. Rovelli, "Graviton propagator from background-independent quantum gravity", *Phys. Rev. Lett.* **97** (2006)

e. b., L. Modesto, C. Rovelli, and S. Speziale, "Graviton propagator in loop quantum gravity", *Class. Quant. Grav.* **23** (2006)

e. b., L. Modesto, C. Rovelli, "Towards perturbative quantum gravity from spinfoams: 3-point correlation functions", to appear

key point: dynamics of a state peaked on a classical geometry

Outline of (the rest of) the talk

1. Motivation from Effective Field Theory
2. Strategy and the general philosophy $\longrightarrow$ Rovelli's talk
3. Correlations at the vertex amplitude level
4. Large scale correlations
   - The dominant contribution
   - The perturbative action and measure
   - 2- and 3-area correlation functions
5. Correlations in perturbative quantum Regge-calculus
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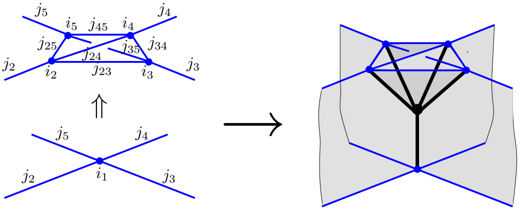
Boundary amplitude formalism at work

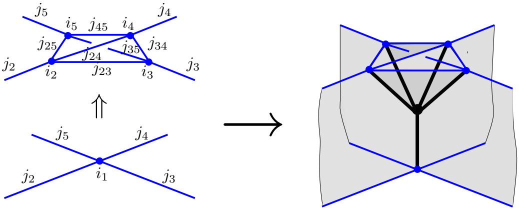
- In LQG, the Hamiltonian constraint acts on a spin-network state non-trivially only at nodes.
- In spinfoam models, the action of the Hamiltonian constraint at a node is given in terms of a spinfoam vertex amplitude.



Hamiltonian constraint corresponding to the Barrett-Crane spinfoam model acting on a state with a 4-valent node

Such transition amplitude can be written in the following way:

$$\langle \quad | \hat{H} | \quad \rangle = \left(\prod_f A_f(j_l) \right) A_v(j_l, i_n)$$



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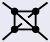
$\langle$  $|\hat{H}|$  $\rangle = \left(\prod_f A_f(j_l)\right) A_v(j_l, i_n)$

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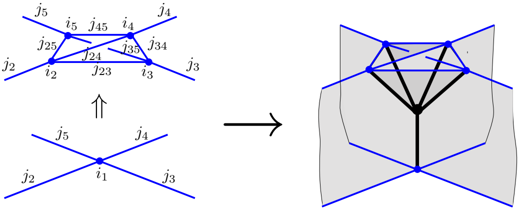

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To capture the role of the vertex amplitude, this formula is best written in the boundary amplitude formalism. The strategy is the following:

(i) cut out from the two-complex a 4-ball B_4 containing a spinfoam vertex v

(ii) introduce a vertex amplitude W_v to codify the dynamics in the region B_4; it is a map from the boundary Hilbert space $\mathcal{H}_{S^3}$ to $\mathbb{C}$;

(iii) introduce a state $\Psi_{S^3,q}[s] \in \mathcal{H}_{S^3}$ to describe the state on $S^3 = \partial B_4$. The role of the boundary state is to codify the dynamics *outside* B_4.

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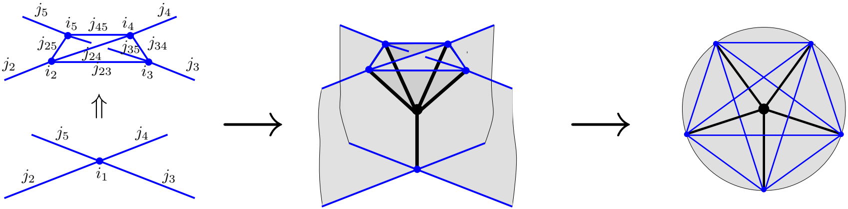

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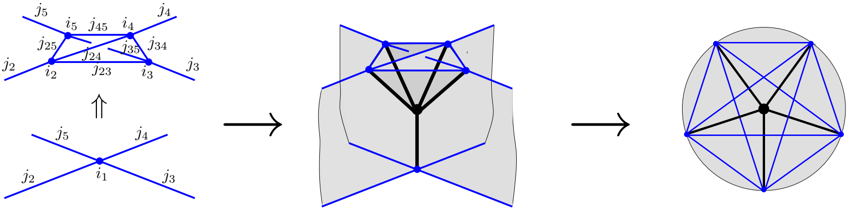

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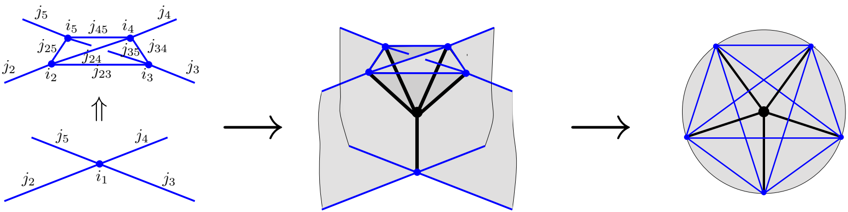

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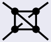
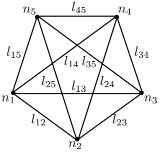
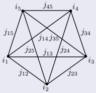
Transition amplitude $\langle$  $|\hat{H}|$  $\rangle = \left(\prod_f A_f(j_l)\right) A_v(j_l, i_n)$ (1)

As the boundary of the 4-ball $\mathcal{B}_4$ intersects the two-complex giving a graph Γ_5,

$\Gamma_5 =$ 

the boundary Hilbert space $\mathcal{H}_{S^3}$ is in fact an $\mathcal{H}_{\Gamma_5}$ which has the spin networks $|j_{12}, \ldots, j_{45}, i_1, \ldots, i_5\rangle$ as a basis. Hence, instead of equation (1), now we have the spinfoam vertex amplitude

$W_v(j_{mn}, i_n) = \langle W_v|$  $\rangle = \left(\prod_{m<n} A_f(j_{mn})\right) A_v(j_{mn}, i_n)$ (2)

Prescribing the action of the Hamiltonian constraint in this way, i.e. specifying the vertex amplitude, makes it easier to guarantee its crossing symmetry.

Correlations at the vertex amplitude level

The quantities we are interested in here are:
correlations of geometric operators at the vertex amplitude level

- such as area-area correlations on a state $\Psi_{\Gamma_5,q}(j_{mn}, i_n)$

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$$\langle\, \hat{A}_{m'n'}\, \hat{A}_{m''n''}\,\rangle_q = \frac{\sum_{j_{mn}} \sum_{i_n} W_v(j_{mn}, i_n)\, \hat{A}_{m'n'}\, \hat{A}_{m''n''}\, \Psi_{\Gamma_5,q}(j_{mn}, i_n)}{\sum_{j_{mn}} \sum_{i_n} W_v(j_{mn}, i_n)\, \Psi_{\Gamma_5,q}(j_{mn}, i_n)}$$

- or volume-volume correlation $\langle\, \hat{V}_{n'}\, \hat{V}_{n''}\,\rangle_q$.

Technically, these are correlations of *coloring*, as for instance

$$\hat{A}_{m'n'} \Psi_{\Gamma_5,q}(j_{mn}, i_n) = 8\pi G_N \sqrt{j_{m'n'}(j_{m'n'}+1)}\Psi_{\Gamma_5,q}(j_{mn}, i_n)\ .$$

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The idea is that

in order to have large scale correlations in a realistic situation

where $\left\{ \begin{array}{l} \text{- an appropriate boundary semiclassical state is chosen} \\ \text{- a sum over two-complexes is considered} \end{array} \right.$

correlations have to be present already at the level of vertex amplitudes

Boundary state

A spinnetwork state $|\Gamma_5, j_{mn}, i_n\rangle \in \mathcal{H}_{\Gamma_5}$ describes a quantum geometry on a manifold S^3 consisting of

- 5 chunks of space (one for each node of the graph Γ_5)

and, as each chunk meets the other four chunks, they identify in the whole

- 10 patches as prescribed by the connectivity the graph Γ_5.

This picture comes from the fact that the state $|\Gamma_5, j_{mn}, i_n\rangle$ is simultaneusly
- an eigenstate of the volume operator of a region containing a node of Γ_5
- an eigenstate of the area operator of a surface cut by a link of Γ_5

Here, we are interested in a state $|\Gamma_5, q\rangle$ on $\mathcal{H}_{\Gamma_5}$ which is peaked both on the intrisic and on the extrinsic geometry of S^3.

As such, we are looking for a *mildly* semiclassical state:

- same connectivity of the state described above
- **but** peaked both on area and on its conjugate momentum (and similarly for volume)

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We take the following ansatz: $|\Gamma_5, q\rangle = \sum_{i_n} \sum_{j_{mn}} C f(i_n) \Psi_{j_0,\phi_0}(j_{mn}) |\Gamma_5, j_{mn}, i_n\rangle$

- with $\Psi_{j_0,\phi_0}(j_{mn})$ given by

$$\Psi_{j_0,\phi_0}(j_{mn}) = \exp\left(-\frac{1}{2}\sum_{m<n}\sum_{p<q}\alpha_{(mn)(pq)}\frac{(j_{mn}-j^{(0)}_{mn})(j_{pq}-j^{(0)}_{pq})}{\sqrt{j^{(0)}_{mn}\, j^{(0)}_{pq}}}\right) e^{-i\sum_{m<n}\phi^{(0)}_{mn} j_{mn}}$$

- and for instance

$f(i_n) = \;\; 1$ for admitted intertwiners (given the j_{mn}) and $\;0$ otherwise

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Despite being very naive, we expect that this proposal should be quite general at least for $j_{mn}^{(0)} \gg 1$

Boundary state: large j_0

$$\Psi_{j_0,\phi_0}(j_{mn}) = \exp\Big(-\frac{1}{2}\sum_{m<n}\sum_{p<q}\alpha_{(mn)(pq)}\frac{(j_{mn}-j_{mn}^{(0)})(j_{pq}-j_{pq}^{(0)})}{\sqrt{j_{mn}^{(0)}\,j_{pq}^{(0)}}}\Big)e^{-i\sum\limits_{m<n}\phi_{mn}^{(0)}j_{mn}}$$

- we restrict attention to a symmetric situation with $j_{mn}^{(0)} = j_0$ and $\phi_{mn}^{(0)} = \phi_0$
- moreover, we assume $j_0 \gg 1$

due to the gaussian form of the state which is peaked on the value j_0 with dispersion $\sqrt{j_0}$, we have that j_{mn} is essentially restricted to be in the range

$$(1-\frac{1}{\sqrt{j_0}})j_0 \leq j_{mn} \leq (1+\frac{1}{\sqrt{j_0}})j_0$$

This peakedness property is a kinematical property

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We will discuss this in the following and show that,
for the Barrett-Crane model, it fixes the angle ϕ_0.

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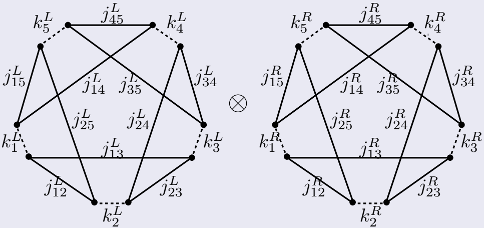
$$A_v(l_{mn}, i_n) = \sum_{j^L_{mn}, k^L_n} \sum_{j^R_{mn}, k^R_n} f^{l_{mn}, k_n}_{j^L_{mn}, k^L_n, j^R_{mn}, k^R_n}$$



$$f^{l_{mn}, i_n}_{j^L_{mn}, i^L_n, j^R_{mn}, i^R_n} = \Big(\prod_{m<n} c^{l_{mn}}_{j^L_{mn}, j^R_{mn}}\Big)\Big(\prod_n f^{i_n}_{i^L_n, i^R_n}(j_{m'n}, j^L_{m'n}, j^R_{m'n})\Big)$$

This model is defined taking a branching function defined in the following way:

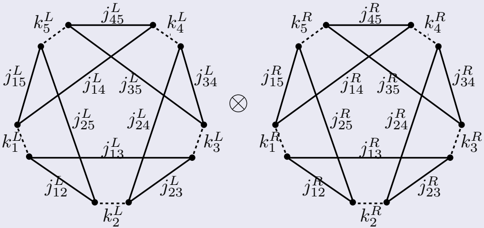
- on links

$$c^{l_{mn}}_{j^L_{mn}, j^R_{mn}} = \delta_{j^L_{mn}, j^R_{mn}} \; \delta_{l_{mn}, (j^L_{mn} + j^R_{mn})}$$

- on nodes

$$f^{k_n}_{k^L_n, k^R_n} = \begin{cases} \delta_{k^L_n, k^R_n} & \text{for } k_n, k^L_n, k^R_n \text{ admissible} \\ 0 & \text{otherwise} \end{cases}$$

$$A_v(l_{mn}, i_n) = \sum_{j^L_{mn},k^L_n} \sum_{j^R_{mn},k^R_n} f^{l_{mn},k_n}_{j^L_{mn},k^L_n,j^R_{mn},k^R_n}$$



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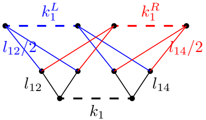
- on nodes

$$f^{k_1}_{k^L_1,k^R_1} =$$

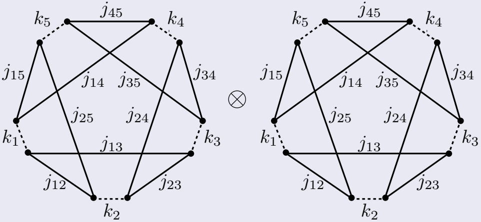


Eagle-Pereira-Rovelli model

The Barrett-Crane model

$$A_v(l_{mn}, i_n) = A_{\mathrm{BC}}(l_{12}/2, \ldots, l_{45}/2)$$

$$A_{\mathrm{BC}}(j_{12}, \ldots, j_{45}) = \sum_{k_1 \ldots k_5} \Big(\prod_{i=1}^{5} \bigcirc_{k_i} \Big)$$

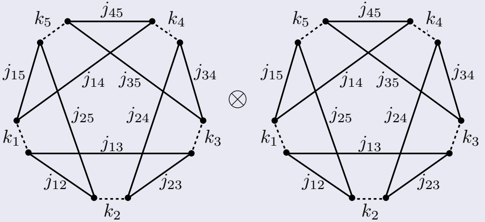


$$W_v(l_{mn}, i_n) = \Big(\prod_{m<n} A_f(l_{mn}) \Big) A_v(l_{mn}, i_n)$$

$$= \Big(\prod_{m<n} (2l_{mn} + 1)^{N_f} \Big) A_{\mathrm{BC}}(l_{mn}/2)$$

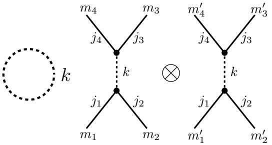
The face amplitude is generally taken with the exponent $N_f = 2$

$$A_{\mathrm{BC}}(j_{12},..,j_{45}) = \sum_{k_1\ldots k_5}\Big(\prod_{i=1}^{5} \bigcirc_{k_i}\Big)$$



Using formula

$\sum_k$



$$= \int_{SU(2)} d\mu(h)\ \mathscr{D}^{(j_1)}_{m_1 m'_1}(h)\mathscr{D}^{(j_2)}_{m_2 m'_2}(h)\mathscr{D}^{(j_3)}_{m_3 m'_3}(h)\mathscr{D}^{(j_4)}_{m_4 m'_4}(h)$$

we can express Barrett-Crane vertex amplitude as an integral over $SU(2)^5$

$$A_{\mathrm{BC}}(j_{12},..,j_{45}) = \int_{SU(2)^5} \prod_{1\le k\le 5} d\mu(h_k) \prod_{1\le m<n\le 5} \chi^{(j_{mn})}(h_m h_n^{-1})$$

Barrett-Crane vertex amplitude as an integral over $SU(2)^5$

$$A_{\mathrm{BC}}(j_{12},..,j_{45}) = \int_{SU(2)^5} \prod_{1\leq k\leq 5} d\mu(h_k) \prod_{1\leq m<n\leq 5} \chi^{(j_{mn})}(h_m h_n^{-1})$$

- group element $h_k \in SU(2) \to$ spherical coordinates $(\psi_k, \theta_k, \phi_k)$ on S^3
- group element $h_k \to$ a unit-vector v_k in $\mathbb{R}^4$:

Using Weyl representation formula with $(v_m, v_n) = \cos\Phi_{mn}$

$$A_{\mathrm{BC}}(j_{12},\ldots,j_{45}) = \int_{(S^3)^5} \prod_{1\leq k\leq 5} \frac{d\Omega_k}{2\pi^2} \prod_{1\leq m<n\leq 5} \frac{\sin(2j_{mn}+1)\Phi_{mn}}{\sin\Phi_{mn}}$$

$SO(4)$ invariance $\to$ fix coordinates on $(S^3)^5$ so that the integrand depends only on

$$u = (\psi_2, \psi_3, \theta_3, \psi_4, \theta_4, \phi_4, \psi_5, \theta_5, \phi_5)$$

and integrate trivially over $\psi_1, \theta_1, \phi_1,\ \theta_2, \phi_2,\ \phi_3$

$$A_{\mathrm{BC}}(j_{12},\ldots,j_{45}) = \int_D \prod_{i=1\ldots 9} du_i\, f(u) \prod_{m<n} \sin\big((2j_{mn}+1)\Phi_{mn}(u)\big)$$

Barrett-Crane vertex amplitude as a 9-d angular integral

$$A_{\mathrm{BC}}(j_{12},\ldots,j_{45}) = \int_D \prod_{i=1\ldots 9} du_i\, f(u) \prod_{m<n} \sin\left((2j_{mn}+1)\Phi_{mn}(u)\right)$$

where $f(u)$ is given by

$$f(u) = \frac{16\pi^4}{(2\pi^2)^5} \frac{(\sin u_1)^2\,(\sin u_2)^2 \sin u_3\ (\sin u_4)^2 \sin u_5\,(\sin u_7)^2 \sin u_8}{\prod_{1\le m<n\le 5} \sin\Phi_{mn}(u)}.$$

The ten angles Φ_{mn} between the vectors v_n, v_m in $\mathbb{R}^4$ can be written explicitly in terms of the nine angles $u = (\psi_2,\psi_3,\theta_3,\psi_4,\theta_4,\phi_4,\psi_5,\theta_5,\phi_5)$:

$$\Phi_{12} = \psi_2\,,\quad \Phi_{13} = \psi_3\,,\quad \Phi_{14} = \psi_4\,,\quad \Phi_{15} = \psi_5$$

$$\Phi_{23} = \cos^{-1}\left(\cos\psi_2\cos\psi_3 + \cos\theta_3\sin\psi_2\sin\psi_3\right)$$

$$\Phi_{24} = \cos^{-1}\left(\cos\psi_2\cos\psi_4 + \cos\theta_4\sin\psi_2\sin\psi_4\right)$$

$$\Phi_{25} = \cos^{-1}\left(\cos\psi_2\cos\psi_5 + \cos\theta_5\sin\psi_2\sin\psi_5\right)$$

$$\Phi_{34} = \cos^{-1}\left(\cos\psi_3\cos\psi_4 + (\cos\theta_3\cos\theta_4 + \cos\phi_4\sin\theta_3\sin\theta_4)\sin\psi_3\sin\psi_4\right)$$

$$\Phi_{35} = \cos^{-1}\left(\cos\psi_3\cos\psi_5 + (\cos\theta_3\cos\theta_5 + \cos\phi_5\sin\theta_3\sin\theta_5)\sin\psi_3\sin\psi_5\right)$$

$$\Phi_{45} = \cos^{-1}\left(\cos\psi_4\cos\psi_5 + (\cos\theta_4\cos\theta_5 + \cos(\phi_5-\phi_4)\sin\theta_4\sin\theta_5)\sin\psi_4\sin\psi_5\right)$$

Barrett-Crane vertex amplitude as a 9-d angular integral

$$A_{\rm BC}(j_{12},\ldots,j_{45}) = \int_D \prod_{i=1\ldots9} du_i\, f(u) \prod_{m<n} \sin\big((2j_{mn}+1)\Phi_{mn}(u)\big)$$

where $f(u)$ is given by

$$f(u) = \frac{16\pi^4}{(2\pi^2)^5}\,\frac{(\sin u_1)^2\,(\sin u_2)^2 \sin u_3\;(\sin u_4)^2 \sin u_5\;(\sin u_7)^2 \sin u_8}{\prod_{1\le m<n\le 5} \sin \Phi_{mn}(u)}\,.$$

While the integrand is well defined, the function $f(u)$ is singular for configurations u_i such that some $\Phi_{mn}(u)=0$.
These points correspond to degenerate configurations of the five vectors v_n, with two or more of them coinciding.

Barrett-Crane vertex amplitude as a 9-d angular integral

$$A_{\mathrm{BC}\varepsilon}(j_{12},\ldots,j_{45}) = \int_{D_\varepsilon} \prod_{i=1\ldots9} du_i\, f(u) \prod_{m<n} \sin\big((2j_{mn}+1)\Phi_{mn}(u)\big)$$

where $f(u)$ is given by

$$f(u) = \frac{16\pi^4}{(2\pi^2)^5}\,\frac{(\sin u_1)^2\,(\sin u_2)^2 \sin u_3\ (\sin u_4)^2 \sin u_5\ (\sin u_7)^2 \sin u_8}{\prod_{1\le m<n\le5} \sin \Phi_{mn}(u)}.$$

While the integrand is well defined, the function $f(u)$ is singular for configurations u_i such that some $\Phi_{mn}(u)=0$.
These points correspond to degenerate configurations of the five vectors v_n, with two or more of them coinciding.

$\Rightarrow$ introduce a quantity $A_{\mathrm{BC}\varepsilon}(j_{mn})$ defined as integral on a D_ε
$D_\varepsilon = \{$domain D with ball of radius ε excised around degenerate configs$\}$
The original vertex amplitude can be obtained taking the limit $\varepsilon \to 0$.

$$A_{\mathrm{BC}}(j_{12},\ldots,j_{45}) = \lim_{\varepsilon\to0} A_{\mathrm{BC}\varepsilon}(j_{12},\ldots,j_{45})$$

Barrett-Crane vertex amplitude as a 9-d angular integral

$$A_{\mathrm{BC}\varepsilon}(j_{12},\ldots,j_{45}) = \int_{D_\varepsilon} \prod_{i=1\ldots 9} du_i \, f(u) \prod_{m<n} \sin\big((2j_{mn}+1)\Phi_{mn}(u)\big)$$

The product $\prod \sin\big((2j_{mn}+1)\Phi_{mn}(u)\big)$ can be written as a sum of 2^{10} terms

$$\prod_{1\leq m<n\leq 5} \sin\big((2j_{mn}+1)\Phi_{mn}(u)\big) = \frac{1}{(2\,i)^{10}} \prod_{m<n} \Big(e^{+i(2j_{mn}+1)\Phi_{mn}(u)} - e^{-i(2j_{mn}+1)\Phi_{mn}(u)}\Big) =$$

$$= -\frac{1}{2^{10}} \sum_{b=0}^{2^{10}-1} (-1)^{\sum b_{mn}} \exp\Big(i \sum_{m<n} (-1)^{b_{mn}} (2j_{mn}+1)\Phi_{mn}(u)\Big)$$

where we have introduced the integer $b = 0,..,1023$ and the binary digit notation with $b_{mn} \in \{0,1\}$ so that

$$\begin{array}{lcl}
b=0 & \to & (b_{12},b_{13},b_{14},b_{15},b_{23},b_{24},b_{25},b_{34},b_{35},b_{45}) = (0,0,0,0,0,0,0,0,0,0) \\
b=1 & \to & (b_{12},b_{13},b_{14},b_{15},b_{23},b_{24},b_{25},b_{34},b_{35},b_{45}) = (1,0,0,0,0,0,0,0,0,0) \\
b=2 & \to & (b_{12},b_{13},b_{14},b_{15},b_{23},b_{24},b_{25},b_{34},b_{35},b_{45}) = (0,1,0,0,0,0,0,0,0,0) \\
b=3 & \to & (b_{12},b_{13},b_{14},b_{15},b_{23},b_{24},b_{25},b_{34},b_{35},b_{45}) = (1,1,0,0,0,0,0,0,0,0) \\
 & \ldots & \\
b=1023 & \to & (b_{12},b_{13},b_{14},b_{15},b_{23},b_{24},b_{25},b_{34},b_{35},b_{45}) = (1,1,1,1,1,1,1,1,1,1)
\end{array}$$

Barrett-Crane vertex amplitude as (the limit of) a sum of 2^{10} terms

$$A_{\mathrm{BC}}(j_{12},\dots,j_{45}) = \lim_{\varepsilon\to 0} -\frac{1}{2^{10}} \sum_{b=0}^{2^{10}-1} (-1)^{\sum b_{mn}} A^{(b)}_{\mathrm{BC}\varepsilon}(j_{12},\dots,j_{45})$$

with $A^{(b)}_{\mathrm{BC}\varepsilon}$ given by a 9-d angular integral

$$A^{(b)}_{\mathrm{BC}\varepsilon}(j_{12},\dots,j_{45}) = \int_{D_\varepsilon} \prod_{i=1\dots 9} du_i\, f(u)\ \exp\Big(i \sum_{m<n} (-1)^{b_{mn}} (2j_{mn}+1)\Phi_{mn}(u)\Big)$$

These formulae for the Barrett-Crane vertex amplitude will have a major role in the following analysis.

$$\Big[\quad \text{recall that} \quad W_v(l_{mn}, i_n) = \Big(\prod_{m<n} (2l_{mn}+1)^{N_f} \Big) A_{\mathrm{BC}}(l_{mn}/2) \quad \Big]$$

Outline of the talk

Large scale correlations

Restrict attention to the *integer spin* subspace of $\mathcal{H}_{\Gamma_5}$, i.e. consider only $SO(3)$ representations l_{mn}. Dynamics $\rightarrow$ Barrett-Crane model.

$$\Psi_{l_0,\phi_0}(l_{mn}) = \exp\Big(-\frac{1}{2}\sum_{m<n}\sum_{p<q}\alpha_{(mn)(pq)}\frac{(l_{mn}-l_{mn}^{(0)})(l_{pq}-l_{pq}^{(0)})}{\sqrt{l_{mn}^{(0)}\, l_{pq}^{(0)}}}\Big)\, e^{-i\sum\limits_{m<n}\phi_{mn}^{(0)} l_{mn}}$$

choose $l_{mn}^{(0)} = l_0 \gg 1$ and $\phi_{mn}^{(0)} = \phi_0$

$$l_{mn} = l_0 + \delta l_{mn} = (1+k_{mn}) l_0 \quad \text{with} \quad k_{mn} = \frac{\delta l_{mn}}{l_0} = O(\frac{1}{\sqrt{l_0}})$$

for $l_0 \gg 1$ and $k_{mn} \sim O(1/\sqrt{l_0})$ we have $A_{mn} \approx 8\pi G_N l_0 (1 + k_{mn} + O(k^2))$

area-area correlations

$$\langle\, \hat{A}_{m'n'}\, \hat{A}_{m''n''}\, \rangle_q =$$

$$= (8\pi G_N)^2 \frac{\sum_{l_{mn}} W_v(l_{mn})\, \sqrt{l_{m'n'}(l_{m'n'}+1)}\sqrt{l_{m''n''}(l_{m''n''}+1)}\; \Psi_{l_0,\phi_0}(l_{mn})}{\sum_{l_{mn}} W_v(l_{mn})\; \Psi_{l_0,\phi_0}(l_{mn})}$$

Large scale correlations

$$\Psi_{l_0,\phi_0}((1+k_{mn})l_0) = C(l_0)\; e^{-\frac{1}{2}\sum\sum \alpha_{(mn)(pq)} l_0\, k_{mn}\, k_{pq}}\; e^{-i\sum \phi_0 l_0 k_{mn}}\; e^{-i\sum \phi_0 l_0}$$

$$l_{mn} = (1+k_{mn})l_0 \quad \text{with} \quad l_0 \gg 1 \quad \text{and} \quad k_{mn} = \frac{\delta l_{mn}}{l_0} = O(\frac{1}{\sqrt{l_0}})$$

area eigenvalue $\rightarrow \quad A_{mn} \approx 8\pi G_N l_0 (1 + k_{mn} + O(k^2))$

$$\langle\, \hat{A}_{m'n'}\, \hat{A}_{m''n''}\,\rangle_q \approx 8\pi G_N)^2 l_0^2 \;\times$$

$$\times \;\; \frac{\int \prod dk_{mn} W_v((1+k_{mn})l_0)\,(1+k_{m'n'})\,(1+k_{m''n''})\,\Psi_{l_0,\phi_0}((1+k_{mn})l_0)}{\int \prod dk_{mn} W_v((1+k_{mn})l_0)\,\Psi_{l_0,\phi_0}((1+k_{mn})l_0)}$$

Quantity appearing in correlation formula

$$\int \prod_{m<n} dk_{mn} P(k_{mn})\; A^{(b)}_{\mathrm{BC}\varepsilon}((1+k_{mn})l_0/2)\; \Psi_{l_0,\phi_0}((1+k_{mn})l_0)$$

with $P(k_{mn})$ a polynomial in k_{mn}

Contribution of A_{BC} to correlations

Quantity appearing in correlation formula

$$\int \prod_{m<n} dk_{mn} P(k_{mn})\, A^{(b)}_{\mathrm{BC}\varepsilon}((1+k_{mn})l_0/2)\, \Psi_{l_0,\phi_0}((1+k_{mn})l_0) =$$

$$= \int_{D_\varepsilon} \prod_{i=1}^{9} du_i f(u) \int \prod_{m<n} dk_{mn} P(k_{mn})\, C(l_0) e^{-\frac{1}{2}\sum\sum \alpha_{(mn)(pq)} l_0 k_{mn} k_{pq}} \times$$

$$\times\, e^{i\sum\left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi_0\right) l_0 k_{mn}}\; e^{i\sum\left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi_0\right) l_0}$$

$$= \int_{D_\varepsilon} \prod_{i=1}^{9} du_i f(u) e^{i\sum\left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi_0\right) l_0} P\Big(-i\frac{\partial}{\partial\phi^{(0)}_{mn}}\Big) \times$$

$$\times \tilde{C}(l_0) e^{-\frac{1}{2}\sum\sum \alpha^{-1}_{(mn)(pq)} l_0 \left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi^{(0)}_{mn}\right)\left((-1)^{b_{pq}}\Phi_{pq}(u)-\phi^{(0)}_{pq}\right)}\Big|_{\phi_0}$$

as $l_0 \to \infty$ we have

$$\prod_{m<n} \delta\Big((-1)^{b_{mn}}\Phi_{mn}(u) - \phi_0\Big)$$

Contribution of A_{BC} to correlations $\quad l_0 \to \infty$

Quantity appearing in correlation formula

$$\int \prod_{m<n} dk_{mn} P(k_{mn})\, A^{(b)}_{\mathrm{BC}\varepsilon}((1+k_{mn})l_0/2)\, \Psi_{l_0,\phi_0}((1+k_{mn})l_0) =$$

$$= \int_{D_\varepsilon} \prod_{i=1}^{9} du_i f(u) e^{i\sum \left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi_0\right) l_0} P(-i\frac{\partial}{\partial\phi^{(0)}_{mn}}) \times$$

$$\times \tilde{C}(l_0) e^{-\frac{1}{2}\sum\sum \alpha^{-1}_{(mn)(pq)} l_0 \left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi^{(0)}_{mn}\right)\left((-1)^{b_{pq}}\Phi_{pq}(u)-\phi^{(0)}_{pq}\right)}\Big|_{\phi_0}$$

as $l_0 \to \infty$ we have

$$\prod_{m<n} \delta\Big((-1)^{b_{mn}}\Phi_{mn}(u) - \phi_0\Big)$$

as $0 \le \Phi_{mn}(u) \le \pi$ and $0 \le \phi_0 \le 2\pi$, only the term with $b = 0$ can contribute
$\to$ look for solutions of the equation

$$\Phi_{mn}(u) = \phi_0 \quad \forall m < n$$

$\to$ only two solutions $\bar{u}^0$ and $P\bar{u}_0$

Quantity appearing in correlation formula

as $l_0 \to \infty$ we have

$$\prod_{m<n} \delta\Big((-1)^{b_{mn}} \Phi_{mn}(u) - \phi_0\Big)$$

as $0 \leq \Phi_{mn}(u) \leq \pi$ and $0 \leq \phi_0 \leq 2\pi$, only the term with $b = 0$ can contribute
$\to$ look for solutions of the equation

$$\Phi_{mn}(u) = \phi_0 \quad \forall m < n$$

$\to$ only two solutions $\bar{u}^0$ and $P\bar{u}_0$ (exchange $\bar{v}_4 \leftrightarrow \bar{v}_5$, i.e. $\bar{\phi}_4 \leftrightarrow \bar{\phi}_5$)

Solution $\bar{u}^0 = \big(\bar{\psi}_2, \bar{\psi}_3, \bar{\theta}_3, \bar{\psi}_4, \bar{\theta}_4, \bar{\phi}_4, \bar{\psi}_5, \bar{\theta}_5, \bar{\phi}_5\big)$ with

$$\bar{\psi}_2 = \bar{\psi}_3 = \bar{\psi}_4 = \bar{\psi}_5 = \cos^{-1}(-\frac{1}{4}) \ , \ \bar{\theta}_3 = \bar{\theta}_4 = \bar{\theta}_5 = \cos^{-1}(-\frac{1}{3}) \ , \ \bar{\phi}_4 = 2\pi - \cos^{-1}(-\frac{1}{2}) \ , \ \bar{\phi}_5 = \cos^{-1}(-\frac{1}{2})$$

corresponds to vectors $\bar{v}_n$ having a 4-dim span and angle between them
$\bar{\Phi}_{mn} = \cos^{-1}(\bar{v}_m, \bar{v}_n) = \cos^{-1}(-\frac{1}{4}) \quad \Rightarrow \quad \phi_0 = \cos^{-1}(-\frac{1}{4})$.

Quantity appearing in correlation formula

$$\int \prod_{m<n} dk_{mn} P(k_{mn})\, A^{(b)}_{\mathrm{BC}\varepsilon}((1+k_{mn})l_0/2)\, \Psi_{l_0,\phi_0}((1+k_{mn})l_0) =$$

$$= \int_{D_\varepsilon} \prod_{i=1}^{9} du_i f(u) e^{i\sum\left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi_0\right)l_0} P\big(-i\frac{\partial}{\partial\phi^{(0)}_{mn}}\big)\times$$

$$\times \tilde{C}(l_0) e^{-\frac{1}{2}\sum\sum \alpha^{-1}_{(mn)(pq)} l_0 \left((-1)^{b_{mn}}\Phi_{mn}(u)-\phi^{(0)}_{mn}\right)\left((-1)^{b_{pq}}\Phi_{pq}(u)-\phi^{(0)}_{pq}\right)}\Big|_{\phi_0}$$

For large (but finite) l_0, to the integral $\int du$ contribute only the u such that

$$|(-1)^{b_{mn}}\Phi_{mn}(u) - \phi_0| \lesssim \frac{1}{\sqrt{l_0}}$$

$\Rightarrow$ only $b=0$
$\Rightarrow$ only u belonging to a ball $\mathcal{B}_{\bar{u}^0}$ centered in $\bar{u}^0$ and of radius $1/\sqrt{l_0}$ or to $\mathcal{B}_{P\bar{u}^0}$

$$D_\varepsilon = \mathcal{B}_{\bar{u}^0} \cup \mathcal{B}_{P\bar{u}^0} \cup \mathcal{R}_\varepsilon$$

Contribution of $A_{\rm BC}$ to correlations

Decomposing the angular domain D_ε in the following way

$$D_\varepsilon = \mathcal{B}_{\bar{u}^0} \cup \mathcal{B}_{P\bar{u}^0} \cup \mathcal{R}_\varepsilon ,$$

with $\mathcal{B}_{\bar{u}^0}$ a ball of radius $1/\sqrt{l_0}$ centered in $\bar{u}^0$, we have

$$A_{\rm BC}(j_{mn}) = -\frac{1}{2^{10}}\Big(2A^{(0)}_{{\rm BC}\mathcal{B}_{\bar{u}^0}} + \lim_{\varepsilon\to 0} A^{(0)}_{{\rm BC}\mathcal{R}_\varepsilon} + \lim_{\varepsilon\to 0}\sum_{b=1}^{2^{10}-1}(-1)^{\sum b_{mn}} A^{(b)}_{{\rm BC}D_\varepsilon}\Big)$$

$A^{(b\neq 0)}_{{\rm BC}D_\varepsilon}$ and $A^{(0)}_{{\rm BC}\mathcal{R}_\varepsilon}$ contribute only in a exponentially suppressed way

$$\int\prod dk_{mn} P(k_{mn})\, A^{(b\neq 0)}_{{\rm BC}D_\varepsilon}((1+k_{mn})l_0/2)\,\Psi_{l_0,\phi_0}((1+k_{mn})l_0) = o(1/l_0^N) \;\; \forall N>0$$

$$\int\prod dk_{mn} P(k_{mn})\, A^{(0)}_{{\rm BC}\mathcal{R}_\varepsilon}((1+k_{mn})l_0/2)\,\Psi_{l_0,\phi_0}((1+k_{mn})l_0) = o(1/l_0^N) \;\; \forall N>0$$

On the other hand, $A^{(0)}_{{\rm BC}\mathcal{B}_{\bar{u}^0}}(j_{mn})$ is suppressed only as a power of $1/l_0$

$$\int\prod dk_{mn} P(k_{mn})\, A^{(0)}_{{\rm BC}\mathcal{B}_{\bar{u}^0}}((1+k_{mn})l_0/2)\,\Psi_{l_0,\phi_0}((1+k_{mn})l_0) = O(1/l_0^{\bar{n}})$$

Perturbative action and measure

Use method of the stationary phase:

$$\int_D \prod_{i=1,..,d} du_i \, f(u) \, e^{i\lambda S(u)} = \sum_n \left(\frac{2\pi}{\lambda}\right)^{\frac{d}{2}} \frac{f(\bar{u}_n)\, e^{i\lambda S(\bar{u}_n)}}{\sqrt{\left|\det\left(\frac{\partial^2 S}{\partial u_i \partial u_j}\Big|_{\bar{u}_n}\right)\right|}} e^{\pm i\frac{\pi}{4}} + O\left(\left(\frac{1}{\lambda}\right)^{\frac{d}{2}+1}\right)$$

Asymptotic analysis of $A^{(0)}_{\mathrm{BC}\mathcal{B}_{\bar{u}_0}}$ for $j_0 \gg 1$

$$A^{(0)}_{\mathrm{BC}\mathcal{B}_{\bar{u}_0}}((1+k_{mn})j_0) = \int_{\mathcal{B}_{\bar{u}_0}} \prod_{i=1,..,9} du_i \, f(u)\, e^{\,i\,2j_0 \sum (1+k_{mn}+\frac{1}{2j_0})\Phi_{mn}(u)}$$

$\Rightarrow$ fixed the fluctuations k_{mn}, find the stationary points $\bar{u}_i$ of the phase $S(u)$

$$S(u) = \sum_{m<n} (1+k_{mn}+\frac{1}{2j_0})\Phi_{mn}(u)$$

i.e. $\bar{u}_i \in \mathcal{B}_{\bar{u}^0}$ such that $0 = \sum_{m<n}\left(1+k_{mn}+\frac{1}{2j_0}\right) \frac{\partial \Phi_{mn}}{\partial u_i}\Big|_{\bar{u}}$

Strategy: solve for $k_{mn}=0$, then perturbatively in $k_{mn} \ll 1$

Perturbative action and measure

Fixed the fluctuations k_{mn}, find the stationary points $\bar{u}_i$ of the phase $S(u)$:

- $k_{mn} = 0$, look for angles $\bar{u}_i^{(0)}$ in $\mathcal{B}_{\bar{u}_0}$ such that $0 = \sum_{m<n} \frac{\partial \Phi_{mn}}{\partial u_i}\Big|_{\bar{u}^{(0)}}$

$\bar{u}_0$ determined as a solution of $\Phi_{mn}(u) = \phi_0$ is an isolateted stationary point of $S^{(0)}(u) = \sum_{m<n} \Phi_{mn}(u)$ in $\mathcal{B}_{\bar{u}^0}$, i.e. in a ball of radius $1/\sqrt{j_0}$ around $\bar{u}^0$.

- perturbation theory in $k_{mn} \ll 1$ around the stationary point $\bar{u}^0$

Solution as series, determined order by order $\left[\text{recall that } k_{mn} = O(1/\sqrt{j_0})\right]$

$$\bar{u} = \bar{u}^0 + \bar{u}^{(1)} + \frac{1}{2}\bar{u}^{(2)} + \frac{1}{3!}\bar{u}^{(3)} + \dots \quad \text{with} \quad \bar{u}^{(n)} = O(k^n)$$

Phase evaluated at stationary point

$$S(\bar{u}^0 + \bar{u}^{(1)} + \frac{1}{2}\bar{u}^{(2)} + \frac{1}{3!}\bar{u}^{(3)} + \dots) = \sum_{m<n} \Phi_{mn}(\bar{u}^0) + \sum_{m<n} k_{mn}\Phi_{mn}(\bar{u}^0) + \sum_{m<n} \frac{1}{2j_0}\Phi_{mn}(\bar{u}^0) +$$

$$+ \frac{1}{2}\sum_i \sum_j \Big(\sum_{m<n} \Phi_{mn,ij}(\bar{u}^0)\Big)\bar{u}_i^{(1)}\bar{u}_j^{(1)} + \frac{1}{3!}\left(\sum_i\sum_j\sum_l \Big(\sum_{m<n} \Phi_{mn,ijl}(\bar{u}^0)\Big)\bar{u}_i^{(1)}\bar{u}_j^{(1)}\bar{u}_l^{(1)} + \right.$$

$$\left. + 3\sum_i\sum_j \Big(\sum_{m<n} k_{mn}\Phi_{mn,ij}(\bar{u}^0)\Big)\bar{u}_i^{(1)}\bar{u}_j^{(1)}\right) + O(k^4)$$

Perturbative action and measure

The dependence on $\bar{u}^{(2)}$ in $S(\bar{u}^0 + \bar{u}^{(1)} + \frac{1}{2}\bar{u}^{(2)} + \frac{1}{3!}\bar{u}^{(3)} + \ldots)$ appears only starting from the forth order in k_{mn}. This is a straightforward consequence of the function $S(u)$ being evaluated at a stationary point.

$\Rightarrow$ to obtain the value of $S(\bar{u})$ up to $O(k^3)$ we need to compute the stationary configuration $\bar{u}$ only to first order in k_{mn}:

$$\bar{u} = \begin{pmatrix} \cos^{-1}(-\frac{1}{4}) \\ \cos^{-1}(-\frac{1}{4}) \\ \cos^{-1}(-\frac{1}{3}) \\ \cos^{-1}(-\frac{1}{4}) \\ \cos^{-1}(-\frac{1}{3}) \\ 2\pi - \cos^{-1}(-\frac{1}{2}) \\ \cos^{-1}(-\frac{1}{4}) \\ \cos^{-1}(-\frac{1}{3}) \\ \cos^{-1}(-\frac{1}{2}) \end{pmatrix} + \begin{pmatrix} \frac{\sqrt{3/5}}{8}(-18k_{12} + 7k_{13} + 7k_{14} + 7k_{15} + 7k_{23} + 7k_{24} + 7k_{25} - 8k_{34} - 8k_{35} - 8k_{45}) \\ \frac{\sqrt{3/5}}{8}(7k_{12} - 18k_{13} + 7k_{14} + 7k_{15} + 7k_{23} - 8k_{24} - 8k_{25} + 7k_{34} + 7k_{35} - 8k_{45}) \\ \frac{1}{2\sqrt{2}}(k_{12} + k_{13} - k_{14} - k_{15} - 4k_{23} + 2k_{24} + 2k_{25} + 2k_{34} + 2k_{35} - 4k_{45}) \\ \frac{\sqrt{3/5}}{8}(7k_{12} + 7k_{13} - 18k_{14} + 7k_{15} - 8k_{23} + 7k_{24} - 8k_{25} + 7k_{34} - 8k_{35} + 7k_{45}) \\ \frac{1}{2\sqrt{2}}(k_{12} - k_{13} + k_{14} - k_{15} + 2k_{23} - 4k_{24} + 2k_{25} + 2k_{34} - 4k_{35} + 2k_{45}) \\ -\frac{\sqrt{3}}{4}(k_{13} + k_{14} - 2k_{15} + k_{23} + k_{24} - 2k_{25} - 2k_{34} + k_{35} + k_{45}) \\ \frac{\sqrt{3/5}}{8}(7k_{12} + 7k_{13} + 7k_{14} - 18k_{15} - 8k_{23} - 8k_{24} + 7k_{25} - 8k_{34} + 7k_{35} + 7k_{45}) \\ \frac{1}{2\sqrt{2}}(k_{12} - k_{13} - k_{14} + k_{15} + 2k_{23} + 2k_{24} - 4k_{25} - 4k_{34} + 2k_{35} + 2k_{45}) \\ \frac{\sqrt{3}}{4}(k_{13} - 2k_{14} + k_{15} + k_{23} - 2k_{24} + k_{25} + k_{34} - 2k_{35} + k_{45}) \end{pmatrix} + O(k^2)$$

Substituting in $S(\bar{u}^0 + \bar{u}^{(1)} + \ldots)$, we find

$$S(\bar{u}(k_{mn})) = S_0 + \sum_{m<n} B_{mn} k_{mn} + \frac{1}{2} \sum_{m<n} \sum_{p<q} K_{(mn)(pq)} k_{mn} k_{pq} +$$
$$+ \frac{1}{3!} \sum_{m<n} \sum_{p<q} \sum_{r<s} I_{(mn)(pq)(rs)} k_{mn} k_{pq} k_{rs} + O(k^4)$$

Perturbative action and measure

The dependence on $\bar{u}^{(2)}$ in $S(\bar{u}^0 + \bar{u}^{(1)} + \frac{1}{2}\bar{u}^{(2)} + \frac{1}{3!}\bar{u}^{(3)} + \ldots)$ appears only starting from the forth order in k_{mn}. This is a straightforward consequence of the function $S(u)$ being evaluated at a stationary point.

$\Rightarrow$ coefficients:

$$S_0 = 10\cos^{-1}(-1/4)\ ,\quad B_{mn} = \cos^{-1}(-1/4)$$

$$K_{(mn)(pq)} = \frac{\sqrt{3/5}}{4}\begin{pmatrix}
-9 & 7/2 & 7/2 & 7/2 & 7/2 & 7/2 & 7/2 & -4 & -4 & -4 \\
7/2 & -9 & 7/2 & 7/2 & 7/2 & -4 & -4 & 7/2 & 7/2 & -4 \\
7/2 & 7/2 & -9 & 7/2 & -4 & 7/2 & -4 & 7/2 & -4 & 7/2 \\
7/2 & 7/2 & 7/2 & -9 & -4 & -4 & 7/2 & -4 & 7/2 & 7/2 \\
7/2 & 7/2 & -4 & -4 & -9 & 7/2 & 7/2 & 7/2 & 7/2 & -4 \\
7/2 & -4 & 7/2 & -4 & 7/2 & -9 & 7/2 & 7/2 & -4 & 7/2 \\
7/2 & -4 & -4 & 7/2 & 7/2 & 7/2 & -9 & -4 & 7/2 & 7/2 \\
-4 & 7/2 & 7/2 & -4 & 7/2 & 7/2 & -4 & -9 & 7/2 & 7/2 \\
-4 & 7/2 & -4 & 7/2 & 7/2 & -4 & 7/2 & 7/2 & -9 & 7/2 \\
-4 & -4 & 7/2 & 7/2 & -4 & 7/2 & 7/2 & 7/2 & 7/2 & -9
\end{pmatrix}$$

$$I_{(12)(12)(12)} = -\frac{189}{80}\sqrt{\frac{3}{5}}\ ,\quad I_{(12)(12)(13)} = +\frac{347}{160}\sqrt{\frac{3}{5}}\ ,\quad I_{(12)(12)(34)} = -\frac{14}{5}\sqrt{\frac{3}{5}}\ ,\quad I_{(12)(23)(13)} = -\frac{141}{20}\sqrt{\frac{3}{5}}\ ,$$

$$I_{(12)(23)(34)} = +\frac{39}{20}\sqrt{\frac{3}{5}}\ ,\quad I_{(12)(13)(14)} = -\frac{453}{160}\sqrt{\frac{3}{5}}\ ,\quad I_{(12)(23)(45)} = -\frac{3}{10}\sqrt{\frac{3}{5}}$$

Substituting in $S(\bar{u}^0 + \bar{u}^{(1)} + \ldots)$, we find

$$S(\bar{u}(k_{mn})) = S_0 + \sum_{m<n} B_{mn}k_{mn} + \frac{1}{2}\sum_{m<n}\sum_{p<q} K_{(mn)(pq)}k_{mn}k_{pq} +$$

$$+\frac{1}{3!}\sum_{m<n}\sum_{p<q}\sum_{r<s} I_{(mn)(pq)(rs)}k_{mn}k_{pq}k_{rs} + O(k^4)$$

Perturbative action and measure

Recall stationary phase formula:

$$\int_D \prod_{i=1,..,d} du_i\, f(u)\, e^{i\lambda S(u)} = \sum_n \left(\frac{2\pi}{\lambda}\right)^{\frac{d}{2}} \frac{f(\bar{u}_n)\, e^{i\lambda S(\bar{u}_n)}}{\sqrt{\left|\det\left(\frac{\partial^2 S}{\partial u_i \partial u_j}\Big|_{\bar{u}_n}\right)\right|}} e^{\pm i\frac{\pi}{4}} + O\left(\left(\frac{1}{\lambda}\right)^{\frac{d}{2}+1}\right)$$

We have that, perturbatively in k_{mn},

$$\left|\det \frac{\partial^2 S}{\partial u_i \partial u_j}\right|_{\bar{u}} = \frac{512}{177147}\sqrt{\frac{5}{3}}\left(10 - \frac{3}{2}k_{12} + \frac{17}{2}(k_{13}+k_{14}+k_{15}) + 11(k_{23}+k_{24}+k_{25}+k_{34}+k_{35}+k_{45})\right) + O(k^2)$$

and

$$f(\bar{u}) = \frac{128}{405}\frac{\sqrt{2}}{\pi^6}\left(1 - \frac{1}{10}\left(\frac{21}{4}k_{12} + \frac{1}{4}(k_{13}+k_{14}+k_{15}) - (k_{23}+k_{24}+k_{25}+k_{34}+k_{35}+k_{45})\right)\right) + O(k^2)$$

Notably, the ratio appearing in stationary phase formula

$$\frac{f(\bar{u})}{\sqrt{\left|\det\left(\frac{\partial^2 S}{\partial u_i \partial u_j}\Big|_{\bar{u}}\right)\right|}} = \frac{12\sqrt{2}}{5\pi^6}\left(\frac{3}{5}\right)^{\frac{3}{4}}\left(1 - \frac{9}{20}\sum_{m<n} k_{mn}\right) + O(k^2)\,,$$

is symmetric under exchange of the ten k_{mn}.

From W_v to perturbative action and measure

$$W_v(l_0+\delta l_{mn}) = \Big(\prod_{m<n}(2l_0)^{N_f}(1+\frac{\delta l_{mn}}{l_0}+1/(2l_0))^{N_f}\Big)A_{\mathrm{BC}}((1+\frac{\delta l_{mn}}{l_0})l_0/2)$$

$$\approx \mathcal{N}\, l_0^{10(N_f-\frac{9}{20})}\Big(1+(N_f-\frac{9}{20})\sum_{m<n}\frac{\delta l_{mn}}{l_0}+O\big(\delta l^2/l_0^2\big)\Big)\,e^{iS_{l_0}(\delta l_{mn})}+R(l_0+\delta l_{mn})$$

where $R(l_0+\delta l_{mn})$ gives an exponentially suppressed contribution to the correlation functions and $S_{l_0}(\delta l_{mn})$ is given by

$$S_{l_0}(\delta l_{mn}) = 10\phi_0 l_0+\sum_{m<n}\phi_0\delta l_{mn}+\frac{1}{2}\sum_{m<n}\sum_{p<q}\frac{K_{(mn)(pq)}}{l_0}\delta l_{mn}\delta l_{pq}+$$
$$+\frac{1}{3!}\sum_{m<n}\sum_{p<q}\sum_{r<s}\frac{I_{(mn)(pq)(rs)}}{l_0^2}\delta l_{mn}\delta l_{pq}\delta l_{rs}+O(\delta l^4/l_0^3)$$

numerical coefficients $K_{(mn)(pq)}$ and $I_{(mn)(pq)(rs)}$ $\rightarrow$ computed explicitly

2- and 3-area correlation functions

$$A_{mn} = 8\pi G_N \sqrt{l_{mn}(l_{mn}+1)} = 8\pi G_N l_0 \Big(1 + \frac{\delta l_{mn}}{l_0} + \frac{1}{2l_0} + O(1/l_0^2)\Big)$$

Correlations of coloring at the vertex amplitude level

$$\langle \frac{\delta l_{mn}}{l_0} \rangle = 0 + O(1/l_0) \quad , \quad \langle \frac{\delta l_{mn}}{l_0} \frac{\delta l_{pq}}{l_0} \rangle = \frac{1}{l_0} (iK - \alpha)^{-1}_{(mn)(pq)} + O(1/l_0^2)$$

To compute $\langle \frac{\delta l_{mn}}{l_0} \frac{\delta l_{pq}}{l_0} \frac{\delta l_{rs}}{l_0} \rangle$ improvement of the boundary state needed

$$\Psi_{l_0,\phi_0}(l_0 + \delta l_{mn}) = \Big(1 + c_1 \sum_{m<n} \frac{\delta l_{mn}}{l_0} + O(\delta l^2 / l_0^2)\Big) \times$$
$$\times\, e^{-\frac{1}{2} \sum_{m<n} \sum_{p<q} \frac{\alpha_{(mn)(pq)}}{l_0} \delta l_{mn} \delta l_{pq}} \, e^{-i \sum_{m<n} \phi^{(0)}_{mn} (l_0 + \delta l_{mn})}$$

in order to be of the same order of the contribution from the measure in W_v

$$\langle \frac{\delta l_{mn}}{l_0} \frac{\delta l_{pq}}{l_0} \frac{\delta l_{rs}}{l_0} \rangle = \frac{1}{l_0^2} \sum_{'} i I_{m'n'p'q'r's'} (iK-\alpha)^{-1}_{m'n'mn} (iK-\alpha)^{-1}_{p'q'pq} (iK-\alpha)^{-1}_{r's'rs} +$$
$$+ \frac{1}{l_0^2} (c_1 + N_f - \frac{9}{20}) \Big(\sum_{m'<n'} (iK-\alpha)^{-1}_{m'n'mn} (iK-\alpha)^{-1}_{pq\,rs} + \text{perm.}) + O(1/l_0^2)$$

Outline of the talk

Correlations in perturbative quantum Regge-calculus

In the previous section, we made no use of results from Regge-calculus

Correlations of areas at the single-4-simplex level can be computed in perturbative area-Regge-calculus too

$$\langle \delta A_{uvz}\, \delta A_{rst} \rangle_0 = \frac{\int \prod d\delta A\ \mu(\delta A)\ e^{iS(A_0+\delta A)}\ \delta A_{uvz}\, \delta A_{rst}\ \Psi_0[\delta A]}{\int \prod d\delta A\ \mu(\delta A)\ e^{iS(A_0+\delta A)}\ \Psi_0[\delta A]}$$

The action for a single 4-simplex in length-Regge calculus is given by

$$S(L_{rs}) = \tfrac{1}{8\pi G_N} \textstyle\sum_{1<u<v<z<5} A_{uvz}(L_{rs}) \big(\pi - \theta_{uvz}(L_{rs})\big)$$

For the configuration $L_{rs} = L_0$, the change of variables $\delta L_{rs} \to \delta A_{uvz}$ is well-defined and the action, perturbatively, is given by

$$S(A_0 + \delta A_{uvz}) = \frac{1}{8\pi G_N}\Big(10(\pi - \cos^{-1}(1/4))A_0 + \sum (\pi - \cos^{-1}(1/4))\delta A_{uvz} +$$
$$+ \frac{1}{2}\sum \frac{\tilde{K}_{uvz\,rst}}{A_0}\delta A_{uvz}\delta A_{rst} + \frac{1}{3!}\sum \frac{\tilde{I}_{uvz\,rst\,hkl}}{A_0^2}\delta A_{uvz}\delta A_{rst}\delta A_{hkl} + O(\delta A^4/A_0^3)\Big)$$

The perturbative Area-Regge action $S(A_0 + \delta A)$ can be compared to the perturbative action coming from $A_{\mathrm{BC}\mathcal{B}_{\bar{u}_0}}(j_0 + \delta j_{mn})$

$$S(A_0 + \delta A_{uvz}) = \frac{1}{8\pi G_N}\Big(10(\pi - \cos^{-1}(1/4))A_0 + \sum(\pi - \cos^{-1}(1/4))\delta A_{uvz} + \\ + \frac{1}{2}\sum \frac{\tilde{K}_{uvz\,rst}}{A_0}\delta A_{uvz}\delta A_{rst} + \frac{1}{3!}\sum \frac{\tilde{I}_{uvz\,rst\,hkl}}{A_0^2}\delta A_{uvz}\delta A_{rst}\delta A_{hkl} + O(\delta A^4/A_0^3)\Big)$$

$$S_{2j_0}(2\delta j_{mn}) = 10\phi_0\, 2j_0 + \sum_{m<n}\phi_0\, 2\delta j_{mn} + \frac{1}{2}\sum_{m<n}\sum_{p<q}\frac{K_{(mn)(pq)}}{(2j_0)}(2\delta j_{mn})(2\delta j_{pq}) + \\ + \frac{1}{3!}\sum_{m<n}\sum_{p<q}\sum_{r<s}\frac{I_{(mn)(pq)(rs)}}{(2j_0)^2}(2\delta j_{mn})(2\delta j_{pq})(2\delta j_{rs}) + O(\delta j^4/j_0^3)$$

and they match up to third order once we identify

$$2j_0 \equiv \frac{A_0}{8\pi G_N} \quad \text{and} \quad 2\delta j_{mn} \equiv \frac{\delta A_{uvz}}{8\pi G_N}$$

Outline of the talk

1. Motivation from Effective Field Theory
2. Strategy and the general philosophy $\rightarrow$ Rovelli's talk
3. Correlations at the vertex amplitude level
4. Large scale correlations
   - The dominant contribution
   - The perturbative action and measure
   - 2- and 3-area correlation functions
5. Correlations in perturbative quantum Regge-calculus
6. **Conclusions**

Conclusions

- We have computed $2-$ and $3-$area correlation functions for the Barrett-Crane vertex
- The result matches with the perturbative calculation in quantum area-Regge calculus up to third order
- Do we have to expect contributions from higher-curvature Regge terms? $\longrightarrow$ go beyond 3^{rd} order
- Does the EPR vertex pass the area-correlation test?
- Compute volume-volume correlations in EPR model

General reasoning behind the calculations presented:

correlations on a semiclassical state in the full theory can be compared to correlations on the vacuum state of the perturbative theory around *a* classical solution

$\longrightarrow$ an elementary QM example

An elementary QM example

Problem: study correlations of position

$$\langle x(t_2)x(t_1)\rangle = \frac{\int D[x(t)]\; x(t_1)\,x(t_2)\; e^{iS[x(t)]/\hbar}}{\int D[x(t)]\; e^{iS[x(t)]/\hbar}}$$

for a particle in a Coulomb potential

$$S[x(t)] = \int_{-\infty}^{+\infty} dt\,\left(\frac{1}{2}M\dot{x}^i(t)\dot{x}^i(t) - \frac{-\alpha}{\sqrt{x^i(t)x^i(t)}}\right) \qquad \text{with} \quad \alpha > 0$$

warning: non-gaussian path integral involved

hint 1: compute it perturbatively around a classical solution, $x(t) = x_{\rm cl}(t) + \xi(t)$

$$S_{x_{\rm cl}}[\xi(t)] = \int_{-\infty}^{+\infty} dt\,\left(\frac{1}{2}M\dot{\xi}^i(t)\dot{\xi}^i(t) - K(t)_{ij}\xi^i(t)\xi^j(t) - I(t)_{ijk}\xi^i(t)\xi^j(t)\xi^k(t) + \dots\right)$$

hint 2: assume $x_{\rm cl}(t) =$ circular orbit of radius $R \gg \frac{\hbar}{M\alpha}$

Problem*: the result of the path integral can be defined using canonical methods
$$W_T[x_1, x_2] = \sum_{n,l,m} \psi_{nlm}(x_2)\, e^{iE_nT}\, \psi^*_{nlm}(x_1)\,.$$

Use it to compute correlations non-perturbatively. Identify the regime which corresponds to the perturbative calculation above.

Correlations on the non-perturbative vacuum state

$$\langle x(t_2)x(t_1)\rangle_0 = \text{``}\frac{\displaystyle\int D[x(t)]\; x(t_1)\, x(t_2)\; e^{iS[x(t)]/\hbar}}{\displaystyle\int D[x(t)]\; e^{iS[x(t)]/\hbar}}\text{''} =$$

$$= \frac{\int dx_1 \int dx_2\; \psi_0^*(x_2)\, x_2\, W_T(x_1,x_2)\, x_1\, \psi_0(x_1)}{\int dx_1 \int dx_2\; \psi_0^*(x_2)\, W_T(x_1,x_2)\, \psi_0(x_1)} = \sum_{nlm} e^{iE_n T} \left|\langle 0|\hat{x}|nlm\rangle\right|^2$$

Regime which corresponds to the perturbative calculation $\rightarrow$

Correlations on a semiclassical state

$$\psi_{q_1}(x_1) = C \exp\left(-\frac{1}{2}\alpha_{ij}(x_1^i - x_{\mathrm{cl}}^i(t_1))(x_1^i - x_{\mathrm{cl}}^i(t_1))\right)\, e^{ip_{\mathrm{cl}}(t_1)x_1}\;,\;\; |x_{\mathrm{cl}}(t_1)| \gg \frac{\hbar}{M\alpha}$$

$$\langle x(t_2)x(t_1)\rangle_q = \frac{\int dx_1 \int dx_2\; \psi_{q_2}^*(x_2)\, x_2\, W_T(x_1,x_2)\, x_1\, \psi_{q_1}(x_1)}{\int dx_1 \int dx_2\; \psi_{q_2}^*(x_2)\, W_T(x_1,x_2)\, \psi_{q_1}(x_1)} \approx \langle \xi(t_2)\xi(t_1)\rangle_0$$