Universe with cosmological constant in Loop Quantum Cosmology

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(work by Abhay Ashtekar, Eloisa Bentivegna, TP)

Purpose of the talk

- Isotropic flat universe with massless scalar field in LQC:
  Recent results for $\Lambda = 0$ *(A Ashtekar, TP, P Singh gr-qc/0607039)*:
  change of dynamics due to quantum geometric effects.
  - Existence of large semiclassical (contracting) universe preceding expanding one.
  - Bounce at energy density $\rho = \rho_c \approx 0.82\rho_{\mathrm{Pl}}$.
- Presented work: Extension to the case of nonvanishing cosmological constant (preliminary investigation in *gr-qc/0607039*).
  - Questions:
    - Does the qualitative picture (bounce, preexisting branch) remain ?
    - If yes, does ρ_c still play fundamental role ?
    - What new properties the models with Λ possess ?
  - Due to distinct mathematical properties of an evolution operator $+$ve and $-$ve Λ have to be investigated separately.

LQC quantization scheme

Considered model: flat isotropic (FRW) universe
Matter content: massless scalar field

- Basic variables:
  geometry: A_a^i, E_i^a in isotropic situation reexpressed in terms of coefficients c, p.
  matter: field ϕ and conjugate momentum p_ϕ.
- Quantization method following LQG:
  - Geometric DOF: triads p and holonomies h raised to operators. Matter DOF: standard (Schrodinger) quantization.
  - Kinematical Hilbert space:
    $\mathcal{H}_{\rm kin} = \mathcal{H}_g \otimes \mathcal{H}_\phi =: L^2(\bar{\mathbb{R}}_{\rm Bohr}, {\rm d}\mu_{\rm Bohr}) \otimes {\rm L}^2(\mathbb{R}, {\rm d}\phi)$
  - Basis of $\mathcal{H}_g$: eigenstates of $\hat{p}$ for convenience labeled by v s.t.
    $\hat{p}\,|v\rangle = 2 \cdot 3^{\frac{1}{6}} \pi\gamma\, {\rm sgn}(v) |v|^{\frac{2}{3}}\, |v\rangle$
- Quantization of Hamiltonian constraint $C_{\rm grav} + C_{\rm matt} = 0$: Its geometric components reexpressed in terms of holonomies (Thiemann method), next raised to operators.

Evolution operator

- The quantized constraint may be written in the form similar to Klein-Gordon equation:

$$\partial_\phi^2 \Psi(v,\phi) = -\Theta\Psi(v,\phi) = -\Theta_o\Psi(v,\phi) - \Theta_\Lambda\Psi(v,\phi)$$

  where Θ is divided onto two parts:
  - 'kinetic' part Θ_o corresponding to case $\Lambda = 0$
    $\Theta_o\Psi(v,\phi) = C^+(v)\Psi(v+4,\phi) + C^o(v)\Psi(v,\phi) + C^-(v)\Psi(v-4,\phi)$,
  - 'potential' term – modification due to cosmological constant
    $\Theta_\Lambda\Psi(v,\phi) = \Lambda C(v)\Psi(v,\phi)$.
- Both Θ_o and Θ_Λ symmetric on the domain $\mathcal{D}$ of finite combin. of $|v\rangle$.
- Reinterpretation of the system as free one evolving with respect to ϕ.
- Few important details:
  - No C-symmetry violation interactions $\Rightarrow$ states symmetric with respect to reflection in v.
  - Domain of v naturally splits onto family of sets preserved by action of Θ and reflection in v: $\mathcal{L}_\epsilon := \{v \in \mathbb{R} : v = \pm\epsilon + 4n, n \in \mathbb{Z}\}$. In consequence $\mathcal{H}_g = \oplus\mathcal{H}_\epsilon$, where $\mathcal{H}_\epsilon$ contains functions supported on $\mathcal{L}_\epsilon$ only

Observables

- Left-hand side negatively definite, thus we take only positive part of Θ. Two sectors: positive and negative frequency. We take the positive part: $i\partial_\phi \Psi(v,\phi) = \sqrt{|\Theta|}\Psi(v,\phi)$
- Dirac observables:
  - scalar field momentum: $\hat{p}_\phi \Psi(v,\phi) = -i\hbar\partial_\phi \Psi(v,\phi)$
  - volume at given ϕ: $|\hat{v}|_\phi \Psi(v,\phi') = \exp[i\sqrt{|\Theta|}(\phi' - \phi)]|v|\Psi(v,\phi)$

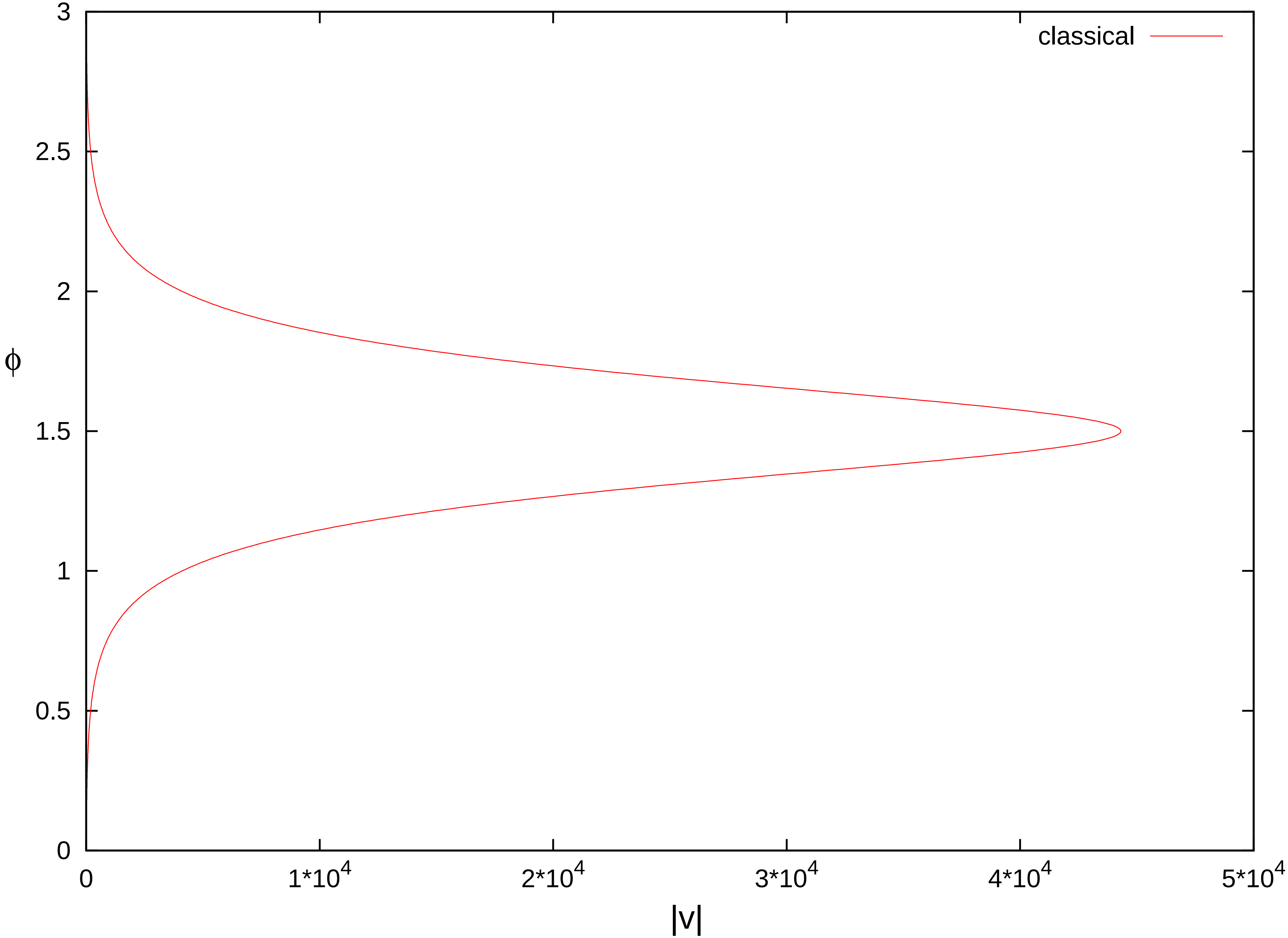
$\Lambda < 0$

Work by: E Bentivegna, TP

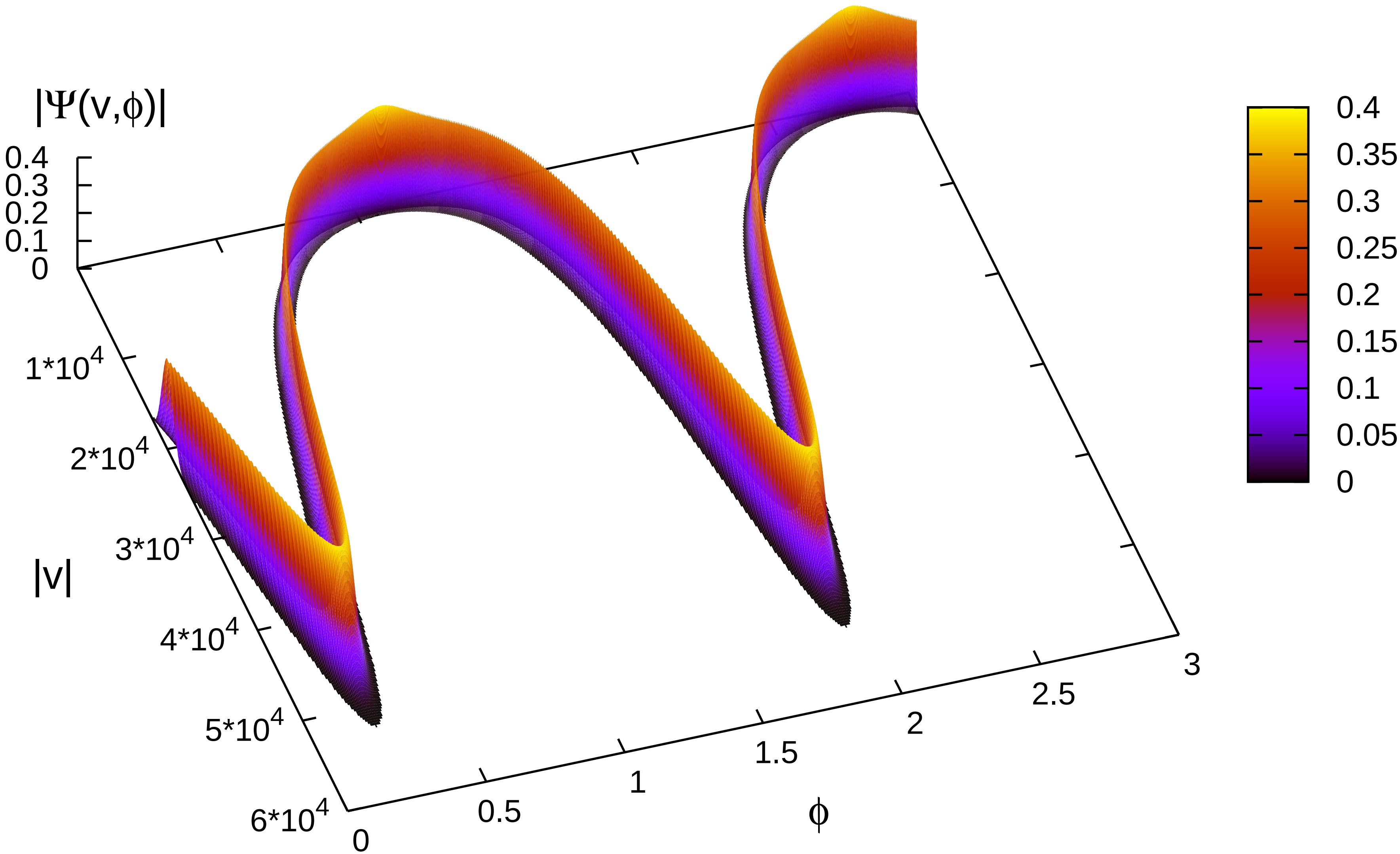
- Classically recollapsing system. Recollapse when energy density of ϕ satisfies: $\rho_\phi + \Lambda/8\pi G = 0$
- Θ_Λ: approximately $\propto v^2$ potential. Θ is positively definite, essentially self-adjoint (J Lewandowski's talk), its spectrum is discrete.
- Selection of eigenstates:
  - Eigenfunctions are solutions to difference equation $\Theta\psi_\omega(v) = \omega^2\psi_\omega(v)$.
  - All $\psi_\omega(v)$ grow/decay exponentially for $|v| > v_r(\omega)$. Normalizable ones exist only for discrete family ω_n of eigenvalues. Each subspace of normalizable ψ_ω is 1-dimensional.
  - We select basis e_n of normalized ψ_ω.
- Physical states: $\Psi(v,\phi) = \sum_n \tilde{\Psi}_n e_n(v) \exp[i\omega_n(\phi - \phi_o)]$
- Choice: Gaussian states sharply peaked about $\omega^\star = \hbar^{-1}p_\phi^\star$ and some large $v^\star$ for some initial ϕ

$$\tilde{\Psi}_n = \exp(-(\omega_n - \omega^\star)^2/2\sigma^2)$$

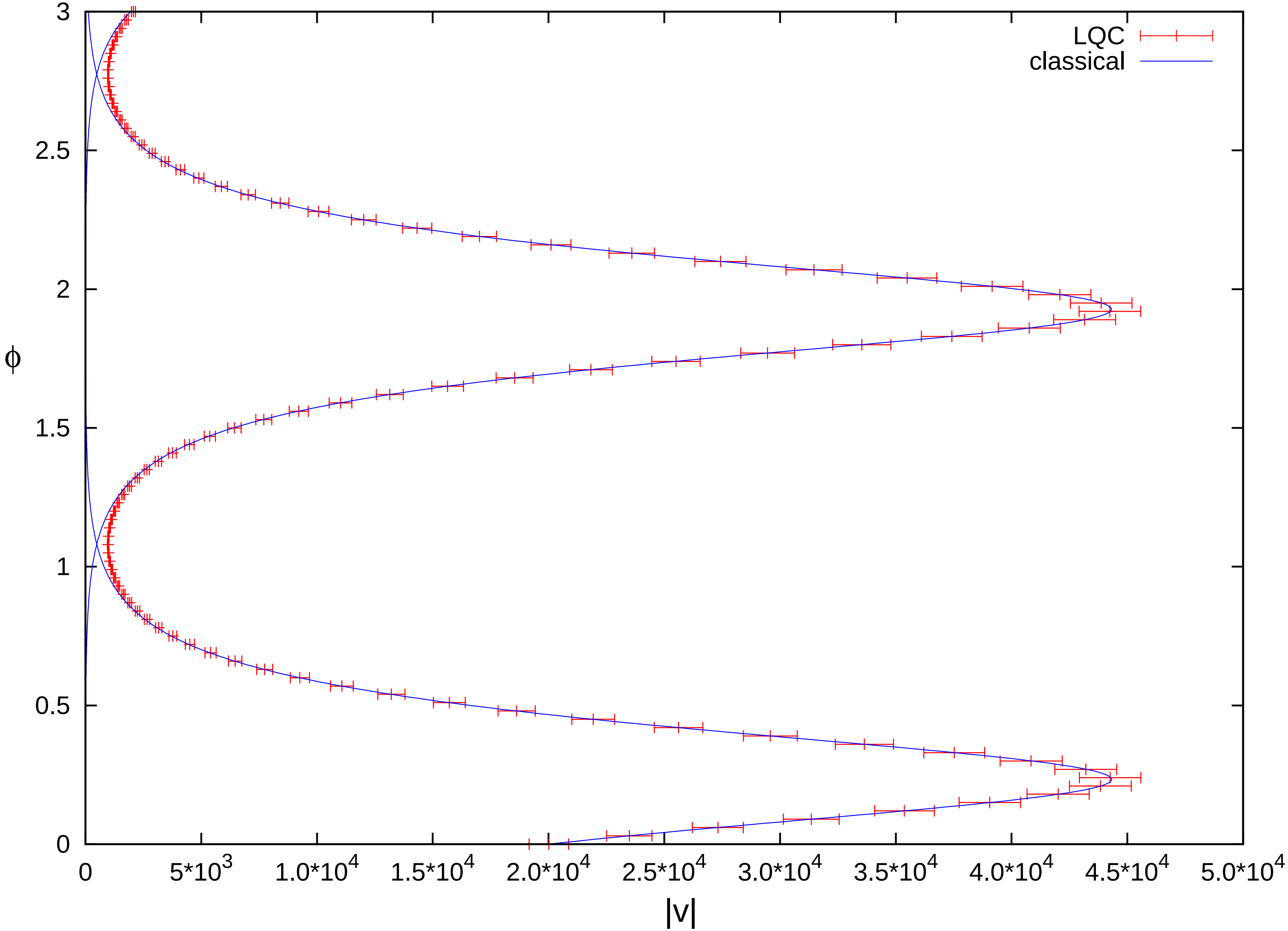
$\Lambda < 0$: *classical trajectory*



$\Lambda < 0$: *wave function*



$\Lambda < 0$: *quantum trajectory*



$\Lambda < 0$ – *results*

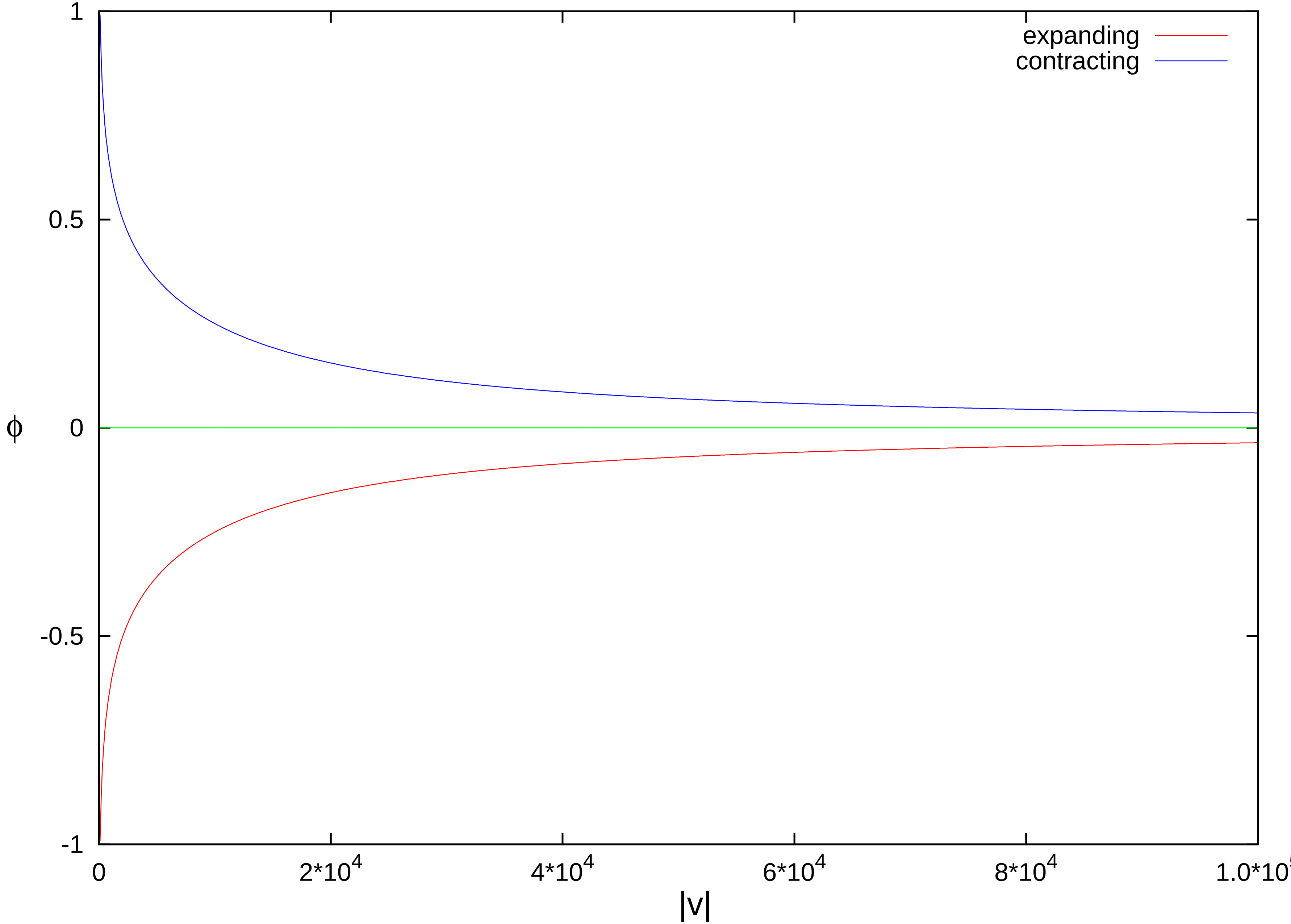
- State remains sharply peaked throughout the evolution.
- Expectation values follow classical trajectory till (total) energy density becomes comparable to ρ_c. In particular classical recollapse at point predicted by classical theory.
- Bounce exactly at $\rho_\phi + \Lambda/8\pi G = \rho_c$ joins two large semiclassical sectors.
- Resulting evolution is periodic (with period depending on Λ).

$\Lambda > 0$

Work by: A Ashtekar, E Bentivegna, TP

- Classically two distinct classes: ever-expanding and ever-contracting. In both classes v reaches infinity for finite $\phi = \phi_o$. Solutions parametrized by proper time end there. Can they be extended ?
  - Yes: One can introduce new variable such that domain of v is compact in it and equation of motion (wrt. ϕ) is analytic in it. In consequence EOM can be analytically extended.
  - Behavior of energy density shows that no 'new' regions of domain produced. Extension simply identifies $v = +\infty$ with $v = -\infty$.
  - Extended solutions: at infinity universe transits from expanding to contracting phase.
- On the quantum level: Θ_Λ is approximately $\propto -v^2$ potential (unbounded from below). Hamiltonians of such system are usually not self-adjoint. Is that the case here ? If yes, can we identify the extensions ?

$\Lambda > 0$: *classical trajectory*



Self-adjoint extensions: general

General mathematical theorem (see eg. Simon, Reed).

- Consider operator Θ symmetric on the domain $\mathcal{D}$.
- Deficiency subspaces $\mathcal{K}_\pm$: spaces of solutions to equation

$$\Theta\varphi_\pm(v) = \pm i\varphi_\pm(v)$$

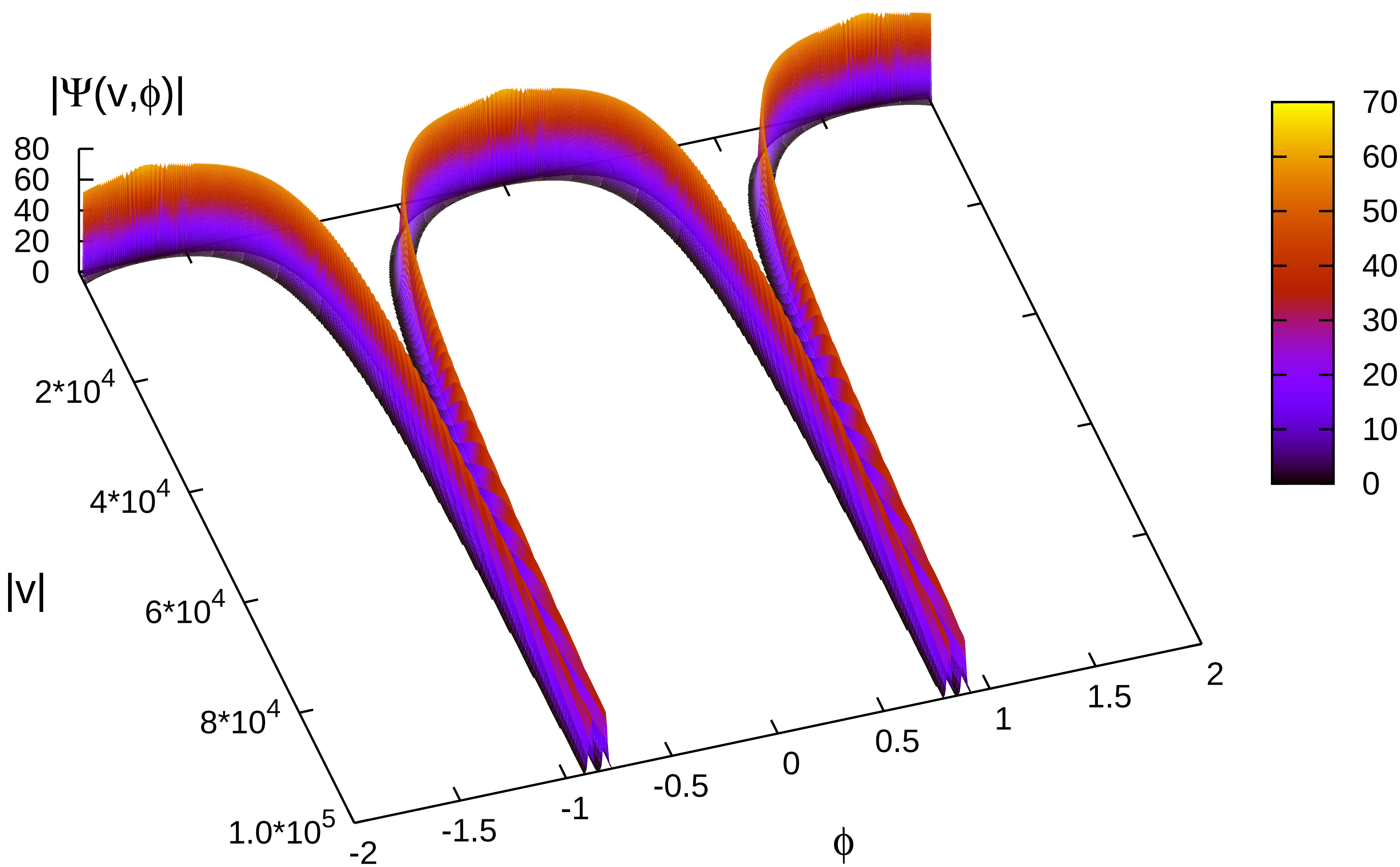
  which belong to $\mathcal{H}_g$. Their dimensions: $k_\pm$ – deficiency indexes.
- Depending on $k_\pm$ following cases possible:
  - $k_+ = k_- = 0$: Θ is essentially self-adjoint (unique extension).
  - $k_+ \neq k_-$: no self-adjoint extensions.
  - $k_+ = k_- \neq 0$: many self-adjoint extensions
- 3rd case: All the extensions can be constructed as follows:
  - Take the set of all partial isometries (operators preserving inner product with possible nontrivial kernel) $U_\alpha : \mathcal{K}_+ \to \mathcal{K}_-$.
  - Each U_α defines a self-adjoint extension of Θ to the domain $\mathcal{D}_\alpha = \{\psi + a(\varphi_+ + U_\alpha(\varphi_+)); \psi \in \mathcal{D}, a \in \mathbb{C}\}$ where φ_+ is in initial space (not in kernel) of U_α.
  - $\mathcal{D}_\alpha$ are the only extensions of $\mathcal{D}$.

Self-adjoint extensions: $\Lambda > 0$

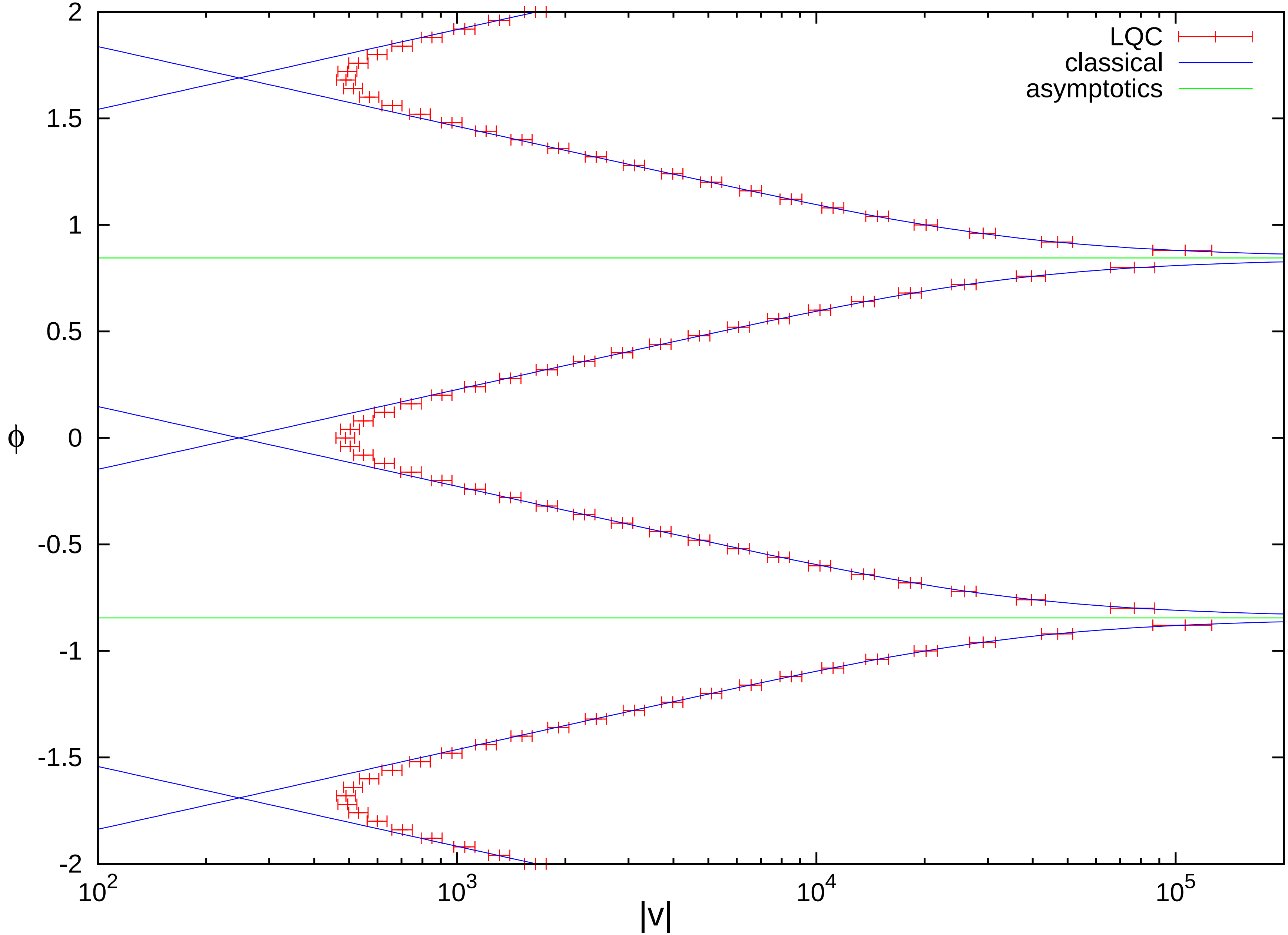
Method presented on previous slide can be applied to case $\Lambda > 0$ (Analysis done thus far for $\mathcal{H}_{\epsilon=0}$ only).

- The analysis:
  - Functions $\varphi_\pm$ found numerically, as (normalizable) solutions to difference equation $\Theta\varphi_\pm = \pm i\varphi_\pm$.
  - Due to symmetry in v for $\epsilon = 0$ $\varphi_\pm$ are determined by their initial values at $v = 0$ only. Thus we have $k_+ = k_- = 1$.
  - Choose normalized $\varphi_\pm$. Then all the partial isometries are of the form $U_\alpha \varphi_+ = e^{i\alpha}\varphi_-$.
  - Since $\mathcal{D}$ – finite combinations of $|v\rangle$ the behavior at $v \to \infty$ of wave functions is for each extension identical to behavior of $a(\varphi_+ + U_\alpha \varphi_+)$.
  - In consequence basis of $\mathcal{D}_\alpha$ can be selected out of eigenfunctions converging to $a(\varphi_+ + U_\alpha\varphi_+)$ at $v \to \infty$.
- Results:
  - All extensions of Θ have discrete spectra. We choose $+$ve parts.
  - Then the physical states have form analogous to the one for $\Lambda < 0$. We can repeat the analysis done for that case.

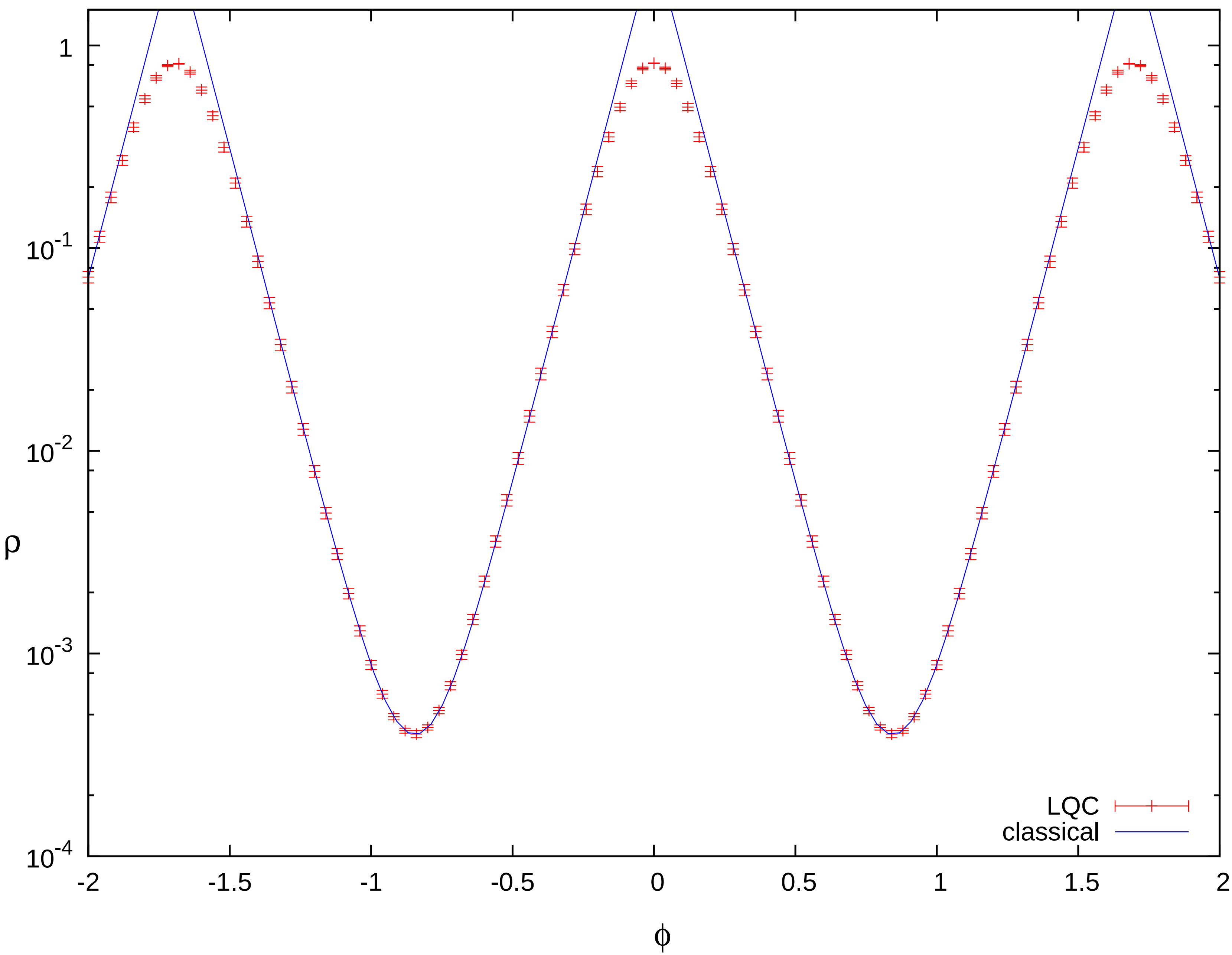
$\Lambda > 0$: *wave function*



$\Lambda > 0$: *quantum trajectory*



$\Lambda > 0$: *energy density*



$\Lambda > 0$ – *results*

We can construct Gaussian states sharply peaked about $\omega^\star$ and analyze exp. values of observables.
The results are the same for all extensions:

- States remain sharply peaked through the evolution.
- States follow classical trajectory until total energy density approaches critical, when gravity becomes repulsive and state bounces.
- Bounce joins deterministically contracting and expanding sectors.
- For all extensions the expanding universe after reaching infinite volume (or, equivalently $\rho = \Lambda/8\pi G$) reflects back into contracting one.
- Due to quantum bounce and reflection at infinity we again have cyclic evolution.

Summary

- For both signs of Λ the results of $\Lambda = 0$ hold. Constructed semiclassical wave packets have the following properties:
  - They follow appropriate classical trajectories till energy density becomes Planckian, in which case we observe bounce.
  - Bounce at $\rho_\phi + \Lambda/8\pi G$ always joins two large semiclassical sectors (epochs) of considered universe.
  - For $\Lambda < 0$ evolution operator has unique self-adjoint extension, whereas for $\Lambda > 0$ (in the sector of $\epsilon = 0$) there exists one-parameter family of extensions. Latter property is expected to be shared by eg. hyperbolic $k = -1$ model.
  - Due to bounce and, either classical recollapse or reflection at infinity evolution is periodic in ϕ.
- Remaining problem: Analysis done so far for $\epsilon = 0$ only. Work in progress for generic ϵ. Also many extensions, however set of them may be in principle bigger.