Statistical Properties of Quantum Graphity

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What is Quantum Graphity?

Idea: Erase edges from a complete graph ($N \sim 10^{180}$)

A single framework can accomodate different dimensions, topologies, geometries

It is a-priori background independent

Use Hilbert spaces and a Hamiltonian

$$\mathcal{H}_{total} = \otimes^{N(N-1)/2} \mathcal{H}_{edge}$$

$$\mathcal{H}_{edge} = \text{span}\,\{|0,0\rangle,\ |1,-1\rangle,\ |1,0\rangle,\ |1,+1\rangle\}.$$

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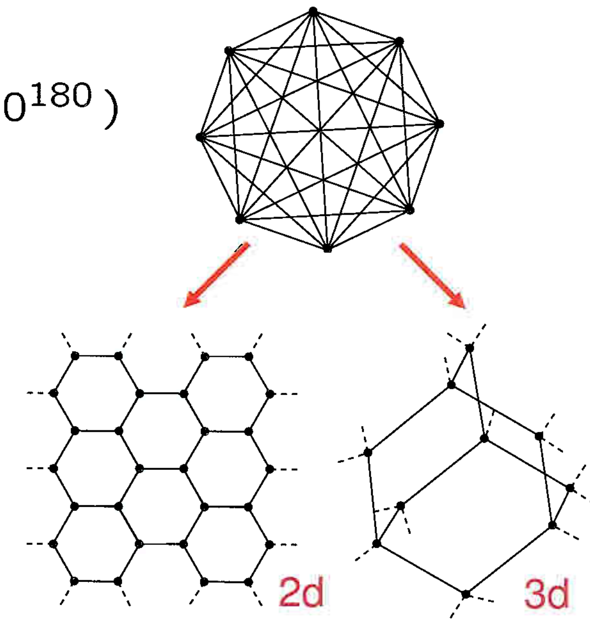


Motivation: Test the idea of 'geometrogenesis' - emergence of geometry

Explore conditions under which observed low-energy limit might appear

[see talks: causal dynamical triangulations, spin foams, group ft, others]

Attempt to understand early-universe cosmology

[see talks: loop quantum cosmology, others]

Hamiltonian

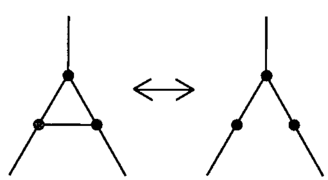
We guess a Hamiltonian s.t. the minimal energy configuration could be regular

$$\hat{H} = \begin{cases} \hat{H}_V = V \sum_a \left(v_0 - \sum_b J_{ab} \right)^2 \\ \hat{H}_{CD} = C \sum_a \left(\sum_b M_{ab} \right)^2 + D \sum_{ab} M_{ab}^2 \\ \hat{H}_B = - \sum_{\text{min loops}} B(L) \prod_{i=1}^{L} M_i^{\pm} \\ \hat{H}_{LQG} \end{cases}$$

$\hat{H}_V$ → sets preferred valence v_0

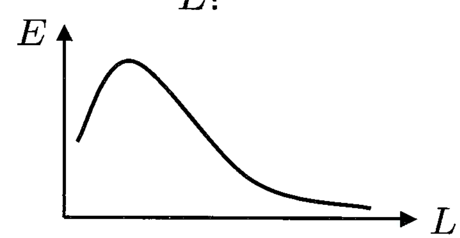
$\hat{H}_{CD}$, $\hat{H}_B$: on a fixed lattice, these terms give rise to an emergent U(1) gauge theory + scalars [e.g. Levin, Wen, hep-th/0507118]

$\hat{H}_{LQG}$ → changes graph, e.g.



$\hat{H}_B$ →

$$B(L) = \frac{1}{L!} B_0 B^L$$



sets preferred loop length L_*

Ensembles and Statistical Mechanics

Use canonical ensemble ($T = 1/\beta$) to study a simple system that is similar

$$Z = \sum_{config(N)} e^{-\beta H} \quad \rightarrow \quad (Z_1)^N = \left(\sum_{config(1)} e^{-\beta H_1} \right)^N$$

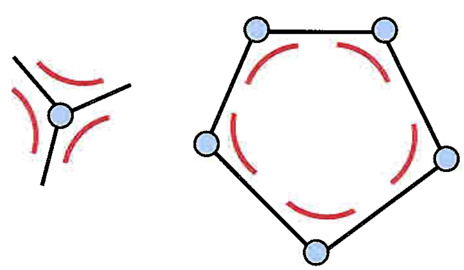
configurations: valence of a node, v from 0 to N

number and length of loops, $\{n_L(L)\}$

$$Z_1 = \sum_{v=0}^{N} \sum_{\{n_L(L)\}}{}' g\{v, n_L(L)\}\, e^{-\beta V(v_0 - v)^2} e^{\beta \sum_L n_L(L) B(L)}$$

counting function: e.g. $g\{v, n_L(L)\} = {}^N C_v$

Two constraints: $Arcs = \frac{v(v-1)}{2} = \sum_L n_L(L) L$



To avoid these constraints:

Introduce GCE, $e^{-\alpha A} = e^{-\alpha \sum_L n_L(L) L}$

Multiply Z_1 by $e^{-\xi\alpha |(\sum_L n_L(L) L) - v(v-1)/2|}$

The Grand Canonical Partition function

$$\mathcal{Z}_1 = \sum_{A=0}^{\infty} \left[\sum_{v} \sum_{\{n_L\}}' g\{v\}\, e^{-\beta V (v_0 - v)^2 + \alpha \xi v(v-1)/2} \prod_L e^{+\beta n_L(L) B(L) - \alpha(1+\xi) n_L(L) L} \right]$$

This can also be written in the product form $\mathcal{Z}_1 = \mathcal{Z}_{1V} \mathcal{Z}_{1B}$

$$\mathcal{Z}_{1V} = \sum_v {}^N C_v \, e^{-\beta V (v - v_0)^2 + \alpha \xi v (v-1)/2}$$ → evaluate numerically

$$\mathcal{Z}_{1B} = \prod_L \left[1 - e^{\beta B(L) - \alpha (1+\xi) L} \right]^{-1}$$ → similar to Bose gas

Analytically, can find e.g. that valence starts changing near $\beta V \sim \log N$

In early Universe, $\beta \sim 10^{13} E_P^{-1}$

so if $V \sim O(1) E_P$ and $N \sim 10^{180}$

graph must have been very near ground state

Summary

Q. Graphity is a discrete model in which to study geometrogenesis.

A graph is coupled to a heat bath. There is a Hamiltonian.
At low temperature, we would like the graph to form a low-dimensional geometry.

In this talk, a simple version of the model was studied using stat. mech. techniques

Outlook

Work in progress on describing the ground state using graph theory/combinatorics
on the transition bw ordered (high β) and disordered (low β) phases