Quantum Extensions of Classically Singular Spacetimes – The CGHS Model

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CGHS Model

Action: $$S(g,\phi,f) = \frac{1}{2G}\int d^2x\sqrt{|g|}\left[e^{-2\phi}\left(R + 4\nabla^a\phi\nabla_a\phi + 4\kappa^2\right) + G\nabla^a f\nabla_a f\right]$$

- Free Field Equation for f $\Box f = 0$
- Dilaton is completely determined by stress energy due to f.

- Field Redefinitions: $\Phi = e^{-2\phi} \quad \Theta = e^{2\rho-2\phi} \quad g_{ab} = e^{2\rho}\eta_{ab}$

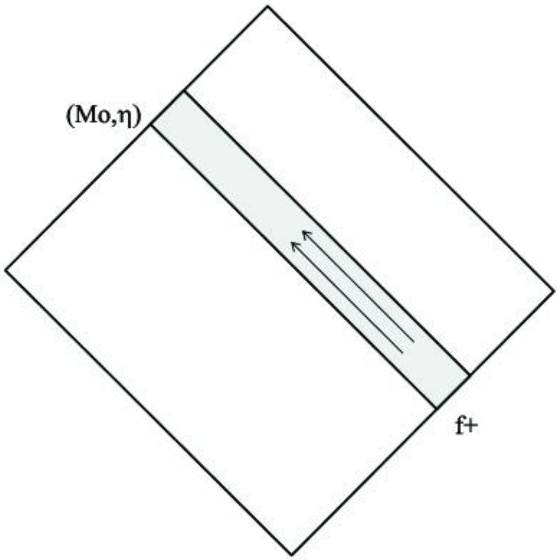
- Equations of motion

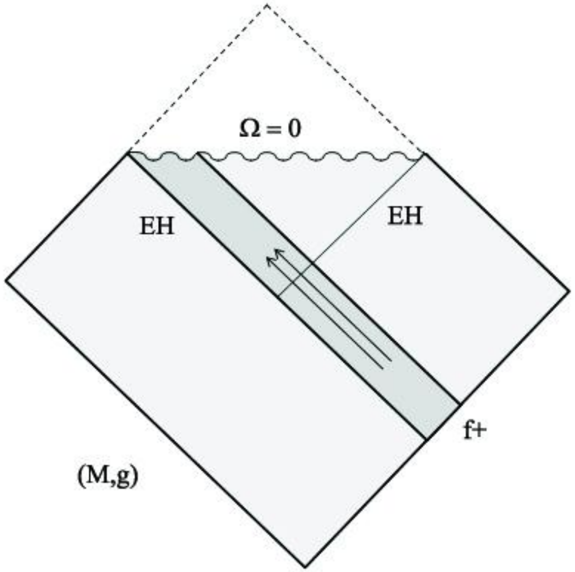
$$\begin{aligned}
\partial_+\partial_-\Phi + \kappa^2\Theta &= 2GT_{+-} \\
\Phi\partial_+\partial_-\ln\Theta &= 2GT_{+-} \\
\hline
-\partial_+^2\Phi + \partial_+\Phi\partial_+\ln\Theta &= 2GT_{++} \\
-\partial_-^2\Phi - \partial_-\Phi\partial_-\ln\Theta &= 2GT_{--}
\end{aligned}$$

Callan, Giddings, Harvey, and Strominger (1992)

BH Collapse Solutions in CGHS

- Black Hole Solutions
- Physical spacetime has a singularity.
- True DOFs in f_+ and f_-.

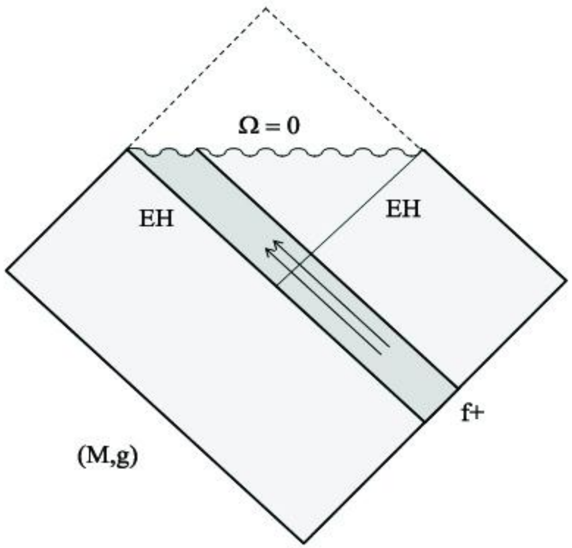




Hawking Effect

- Trace anomaly $$\langle T \rangle = \frac{N\hbar}{24} R$$
- Conservation Law $$\nabla^a \langle T_{ab} \rangle = 0$$
- Hawking Radiation $$T = \frac{\kappa \hbar}{2\pi}$$

$$\langle T^f_{--} \rangle_{\bar{g}} = \frac{N\hbar\kappa^2}{48} \left[1 - \frac{1}{(1 + \frac{GM}{\kappa} e^{\kappa(y^- - y_0^+)})^2} \right]$$

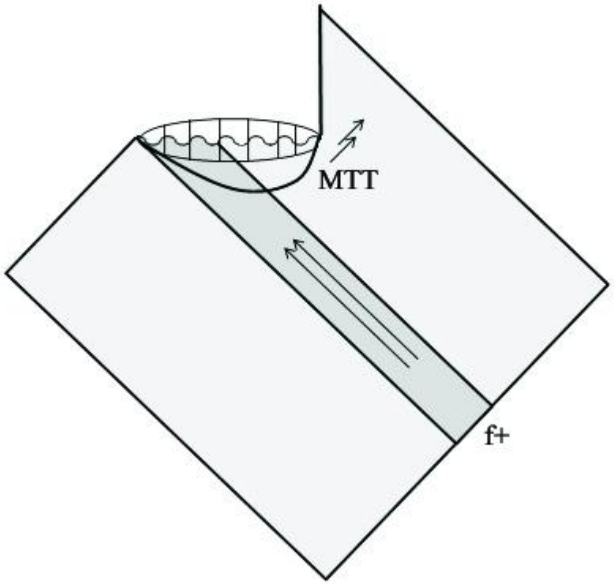


Giddings and Nelson (1992)

Numerical Work

•Incorporated the backreaction into an effective term in the action

•Equations discretized and solved numerically. Evolution breaks down at the singularity and near the endpoint of evaporation.



Lowe (1993), Piran & Strominger (1993)

Quantum Theory

Operator Equations:

$$\partial_+\partial_-\hat{\Phi} + \kappa^2\hat{\Theta} = 2G\hat{T}_{+-}$$
$$\hat{\Phi}\partial_+\partial_-\ln\hat{\Theta} = 2G\hat{T}_{+-}$$

+ Boundary conditions

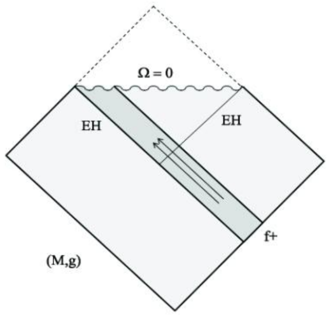
- For Φ an operator valued distribution and Θ a positive operator
- $\hat{\Omega} = \hat{\Phi}\hat{\Theta}^{-1}$ well defined everywhere even though $\langle\hat{\Omega}\rangle$ may vanish
- Ideally we would like to be able to specify $\hat{T}_{+-}(\hat{\Theta}, \hat{\Phi})$ (work in progress)
- Even without $\hat{T}_{+-}(\hat{\Theta}, \hat{\Phi})$ one can proceed by making successive approximations to the full quantum theory

Bootstrapping

- 0^{th} order – Put $\hat{T}_{+-} = 0$

  Compute $\langle \hat{g}^{ab} \rangle = \langle \hat{\Omega} \rangle \eta^{ab}$ in the state $|0_-\rangle \otimes |C_{f_+}\rangle$

  This yields the BH background.



- 1^{st} order – Interpret the vacuum on the $\mathscr{I}_R^+$ of the BH background metric, this is precisely the Hawking effect.

- 2^{nd} order - Semiclassical gravity (mean field approximation) :

  Ignore fluctuations in Φ and Θ, but not f.

$$\partial_+\partial_-\langle\hat{\Phi}\rangle + \kappa^2\langle\hat{\Theta}\rangle = 2G\langle\hat{T}_{+-}\rangle$$

$$\langle\hat{\Phi}\rangle\partial_+\partial_- \ln\langle\hat{\Theta}\rangle = 2G\langle\hat{T}_{+-}\rangle$$

  Use $\langle\hat{T}_{+-}\rangle$ determined from the trace anomaly.

  Agreement with analytic solution near $\mathscr{I}_R^+$ obtained by asymptotic analysis.

Asymptotic Analysis

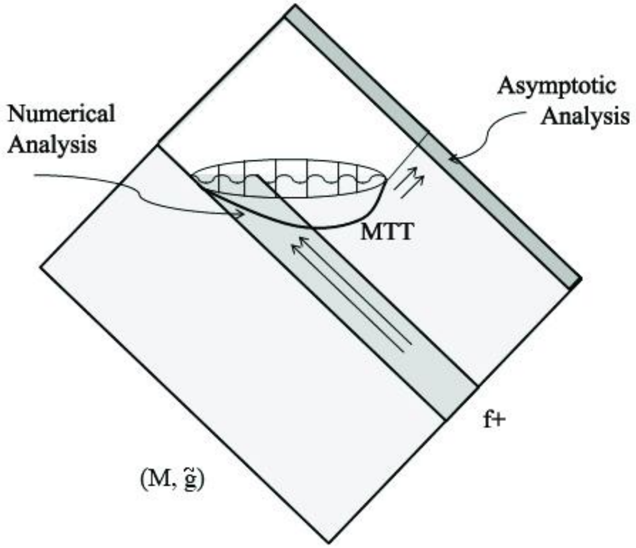
•Expand Φ and Θ in inverse powers of x^+.

•Idea: We should have a decent control of what is going on near $\mathcal{I}_R^+$ since curvatures and fluxes there are small

$$\partial_- M_B = -\langle \hat{T}_{--} \rangle$$

•The Hawking flux and the Bondi mass go to 0 and the physical metric approaches the flat one.

• $\mathcal{I}_R^+$ agrees with background $\mathcal{I}_R^+$.



Summary

- To 0^{th} order we recover the black hole background.
- To higher orders in the truncation, not only do we recover the Hawking effect but we obtain a self-consistent system of semiclassical equations.
- $\mathfrak{I}_R^+$ of the semiclassical metric coincides with that of the background metric thus there is no information loss.
- The pure state resembles a thermal state in the past of $\mathfrak{I}_R^+$
- The quantum spacetime is larger than the classical spacetime.

Future Work

- Need to extend QFT on CST to QFT on QG.