# The causaloid formalism – a tentative framework for QG.


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# Problem of Quantum Gravity

To find a theory which approximates GR and QT in appropriate limits....

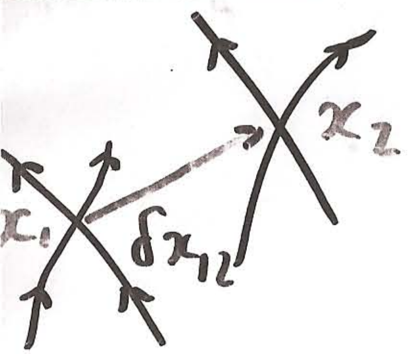
.. which is verified in future appropriate experiments.

# GR

conservative : deterministic

radical : non-fixed causal structure

## causal structure




need to solve for $g_{\mu\nu}$ to determine whether $\delta x_{12}$ timelike

causal structure not fixed

# QT

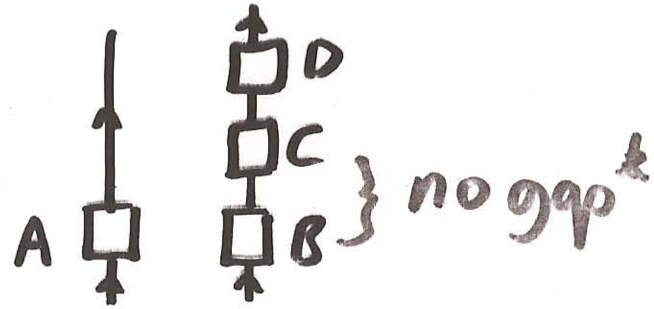
conservative : fixed causal structure

radical : inherently probabilistic

## causal structure

1) Background time $|\psi(t)\rangle = U(t)|\psi(0)\rangle$

2) Products between operators :



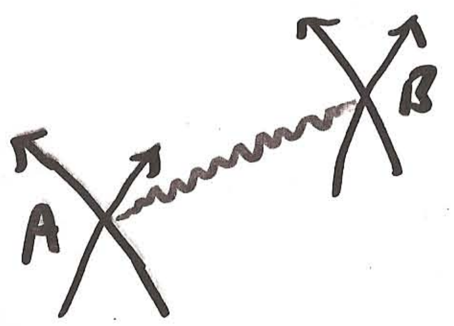

$A \otimes B$ spacelike $K_{AB} = K_A K_B$

$CB$ timelike* $K_{CB} = K_C \cdot K_B$

[D?B] timelike $K_{DB} = K_D K_B$

Will unify $\Rightarrow$ causaloid

In QG may even have <u>indefinite</u> causal structure




Maybe no matter-of-fact as to whether AB timelike or not unless measured

Want framework for theories which ~~are~~

1) are probabilistic
2) can accommodate indefinite causal structure.

(cannot assume background time)

## <u>The causaloid formalism</u>

# Operationalism in GR & QT

Einstein (1916)

"All our spacetime verifications invariably amount to a determination of space-time coincidences.... Moreover the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points ... "

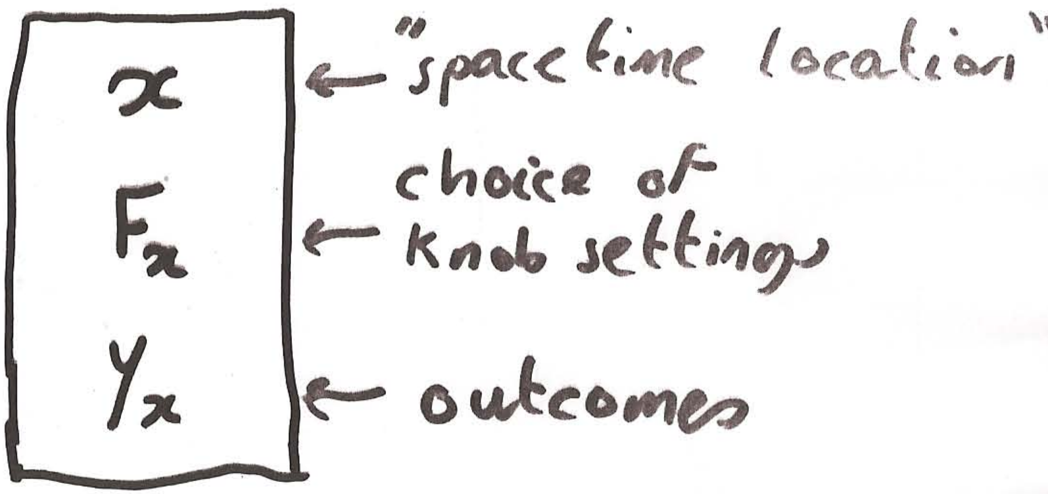
Heisenberg (1925) opening sentence:

"The present paper seeks to establish a basis for theoretical mechanics founded exclusively upon relationships between quantities which are in principle are observable".

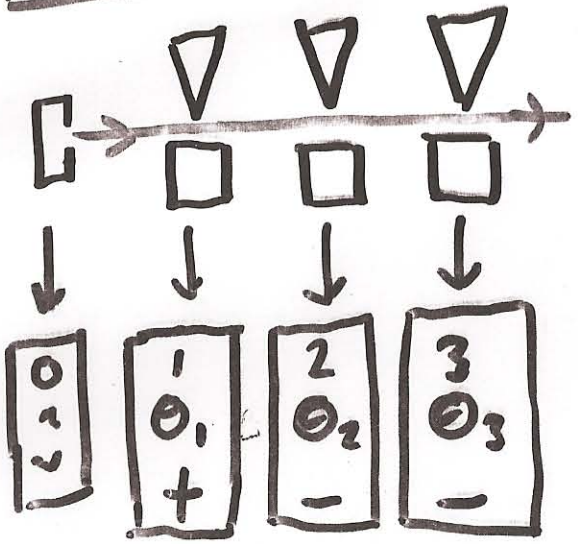
# Implementing Operationalism

## Operationalism as methodology

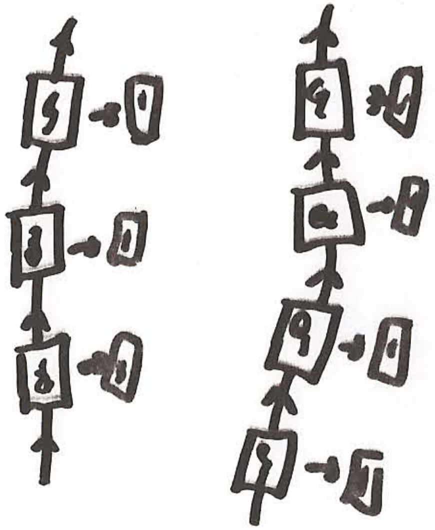
Record proximate data on cards




### Ex 1

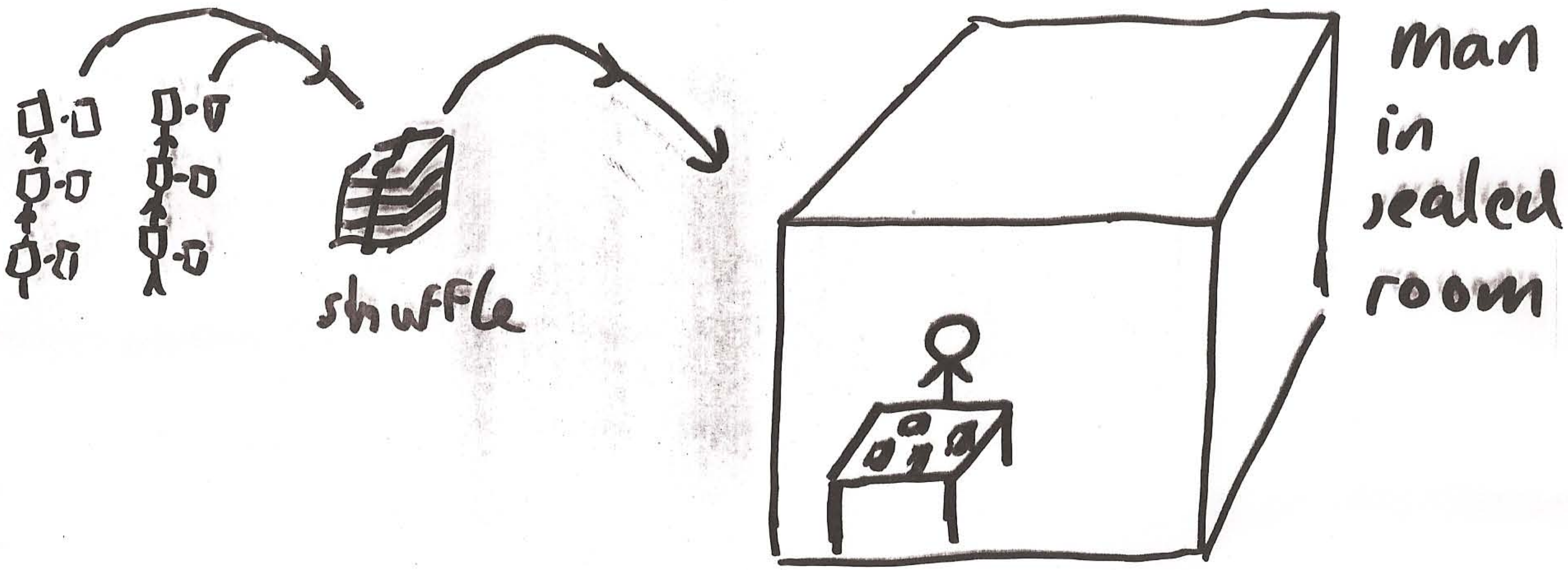



### Ex 2



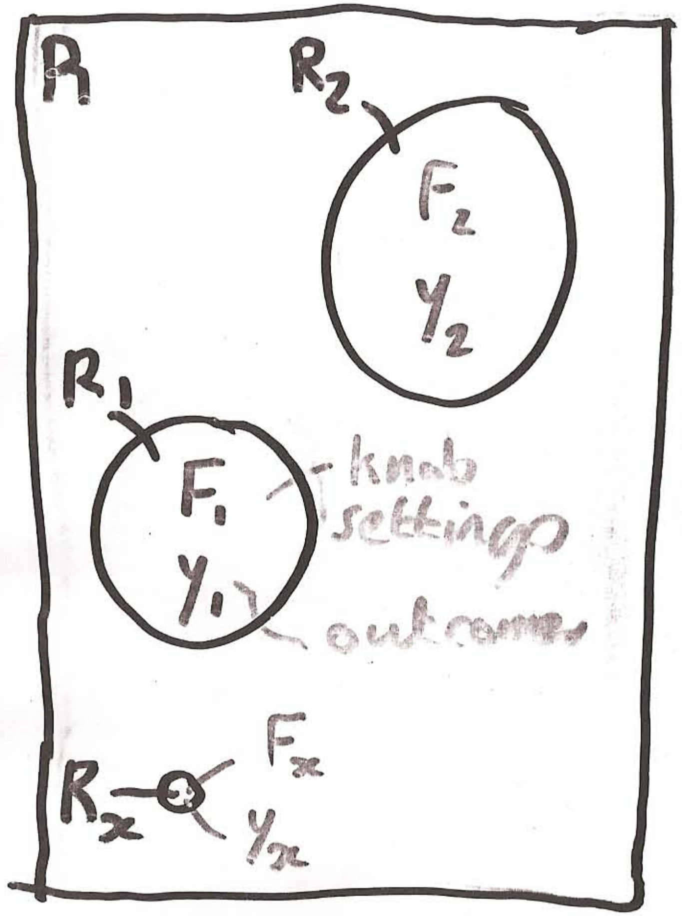
probes drifting in space

# Analysis of cards




repeat many times for each $F(x) \left(\equiv F_x\right)$

How can man in sealed room analyse cards?




$R_x$ is an elementary region

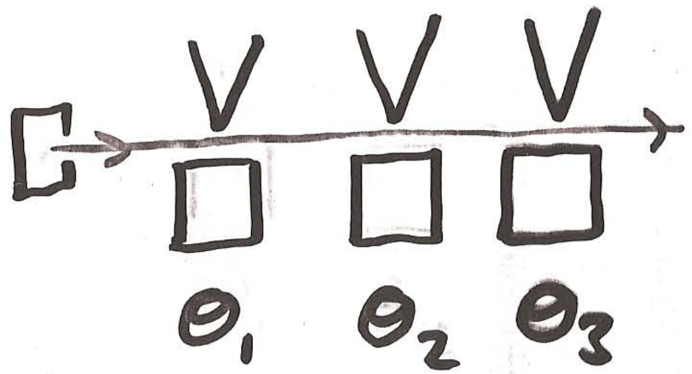
$$R_1 = \bigcup_{x \in \Theta_1} R_x$$

Want to be able to calculate probabilities like

$$\text{prob}(Y_2 | Y_1, F_1, F_2)$$

without requiring definite causal structure.

# When are probabilities well defined?




$\text{prob}(t_3 | t_2, \theta_2, \theta_3)$ well defined - can calculate in QT

$\text{prob}(t_3 | t_1, \theta_1, \theta_3)$ not well defined - cannot calculate in QT

Know whether prob is w.d. by refering to causal structure.

# Objective for theory.

i) to be able to say whether prob$(Y_2 | Y_1, F_1, F_2)$ is well defined

ii) if it is w.d., to be able to calculate it.

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In causaloid formalism $\exists$ vectors

$$\underline{r}_{\alpha_1 \alpha_2} \qquad \underline{r}_{\alpha_1 -}$$

these are dual to states.

i) Prob w.d. iff $\underline{r}_{\alpha_1 \alpha_2}$ parallel to $\underline{r}_{\alpha_1 -}$

ii) Prob given by

$$\underline{r}_{\alpha_1 \alpha_2} = p\, \underline{r}_{\alpha_1 -}$$

# Physical Compression

## Example from QT

Spin $\frac{1}{2}$

$$\hat{\rho} = \begin{pmatrix} p_{z+} & a^* \\ a & p_{z-} \end{pmatrix} \Longleftrightarrow \underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

$$a = p_{x+} + i p_{y+} - \frac{(1+i)}{2}(p_{z+} + p_{z-})$$

$$\text{prob}_\alpha = tr(A_\alpha \hat{\rho})$$

$$= \underline{r}_\alpha \cdot \underline{p}$$

$$\begin{pmatrix} \vdots \\ p_\alpha \\ \vdots \end{pmatrix} \quad \underline{p} = \begin{pmatrix} \vdots \\ p_\ell \\ \vdots \end{pmatrix}$$

$$\Lambda_\alpha^{\ \ell} \equiv \underline{r}$$

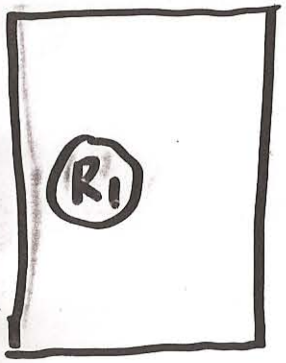
The decompressi

matrix

# Three levels of physical compression

## Level one linear

"local"



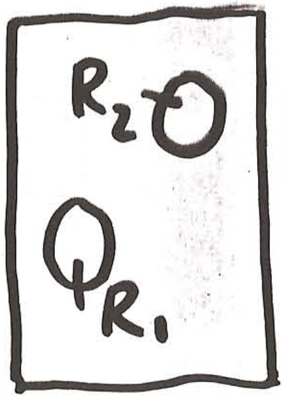

$$\begin{pmatrix}\vdots\\ p_{\alpha_1}\\ \vdots\end{pmatrix} \rightarrow \begin{pmatrix}\vdots\\ p_{\ell_1}\\ \vdots\end{pmatrix}$$

$\Lambda^{\ell_1}_{\alpha_1}$ decompression matrix

$\equiv \underline{\Gamma}_{\alpha_1}\Big|_{\ell_1}$

## Level two linear

"composite regions"




$$\begin{pmatrix}\vdots\\ p_{\ell_1 \ell_2}\\ \vdots\end{pmatrix} \rightarrow \begin{pmatrix}\vdots\\ p_{k_1 k_2}\\ \vdots\end{pmatrix}$$

$\Lambda^{k_1 k_2}_{\ell_1 \ell_2}$ 2nd level decomp. matrices

$\Lambda^{k_1 k_2 k_3}_{\ell_1 \ell_2 \ell_3}$

$\vdots$

## Level three

$$\begin{bmatrix}\Lambda^{\ell_1}_{\alpha_1}\\ \vdots\\ \Lambda^{k_1 k_2}_{\ell_1 \ell_2}\\ \vdots\\ \Lambda^{k_1 k_2 k_3}_{\ell_1 \ell_2 \ell_3}\\ \vdots\end{bmatrix} \rightarrow \begin{bmatrix}\text{subset of } \Lambda\text{'s}\\ \&\\ \text{RULES}\\ \text{for}\\ \text{decomp.}\end{bmatrix}$$

$= \underline{\Lambda}$ The causaloid

## Third level physical compression

$$\left[\begin{array}{l} \Lambda_{\alpha_1}^{\ell_1} \quad \forall R_1 \\ \Lambda_{\ell_1 \ell_2}^{k_1 k_2} \quad \forall R_1, R_2 \text{ (disjoint)} \\ \Lambda_{\ell_1 \ell_2 \ell_3}^{k_1 k_2 k_3} \quad \forall R_1, R_2, R_3 \text{ (disjoint)} \\ \vdots \end{array}\right] \quad \underline{\Lambda} = (\underbrace{\Lambda\text{'s}}_{\text{subset}}, \underbrace{\text{RULES}}_{\text{RULES for decompression}})$$

We use identities to implement 3rd level (de)compression

# Identities for level three compression

$$\Lambda^{k_1 k_2 k_3}_{\ell_1 \ell_2 \ell_3} = \Lambda^{k_1}_{\ell_1} \Lambda^{k_2 k_3}_{\ell_2 \ell_3} \quad \text{if } \Omega_{123} = \Omega_1 \times \Omega_2$$

$$\Lambda^{k_1 k_2 k_3}_{\ell_1 \ell_2 \ell_3} = \sum_{k' \in \Omega_{2\dagger}} \Lambda^{k_1 k_2'}_{\ell_1 k_2'} \Lambda^{k_2' k_3}_{\ell_2 \ell_3} \quad \text{if } \begin{array}{l} \Omega_{123} = \Omega_{12} \times \Omega_{2\dagger} \\ \Omega_{23} = \Omega_{2\dagger} \times \Omega_{\dagger 3} \end{array}$$

$k_{x'}\, k_{x''} \cdots k_{x'''}$

$l_{x'}\, l_{x''} \cdots l_{x'''}$

Associated with each collection of elem. regs. is a 2nd level decompr. matrix

$l_x$

$\alpha_x$

Associated with each elem. reg. is a 1st level decompr. matrix.

...entary

...ions.

is exponential no. of objects are related by ...d level comp.

...ic structure ~~more~~ richer than a graph ...ch as a causal network)

# The causaloid Product

$$\underline{\Gamma}_{\alpha_1\alpha_2} = \underline{\Gamma}_{\alpha_1} \otimes^{\Lambda} \underline{\Gamma}_{\alpha_2}$$

defined by

$$\underline{\Gamma}_{\alpha_1\alpha_2}\Big|_{k_1k_2} = \sum_{l_1 l_2 \in \Omega_1 \times \Omega_2} \underline{\Gamma}_{\alpha_1}\Big|_{l_1} \underline{\Gamma}_{\alpha_2}\Big|_{l_2} \Lambda_{l_1 l_2}^{k_1 k_2}$$

When there is no extra compression at 2nd level

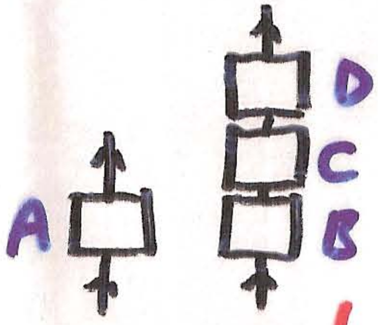
$\otimes^{\wedge}$ equiv to $\otimes$ — trivial 2nd level compression

In QT

$A \otimes B$ trivial

$[D ? B]$ trivial

$C\,B$ non-trivial




non-trivial 2nd level comp. $\Longleftrightarrow$ causal adjacency

# Calculating probabilities

$$p \equiv \text{prob}\left(Y_2^{\alpha_2} \mid Y_1^{\alpha_1}, F_1^{\alpha_1}, F_2^{\alpha_2}\right) = \frac{\underline{\Gamma}_{\alpha,\alpha_2} \cdot \not{p}(R_1 \cup R_2)}{\sum_{\beta_2} \underline{\Gamma}_{\alpha,\beta_2} \cdot \not{p}(R_1 \cup R_2)}$$

$\not{p}(R_1 \cup R_2) \Leftarrow$ what happens outside $R_1 \cup R_2$ so prob must be independent of this. Hence

i) Prob is well defined iff

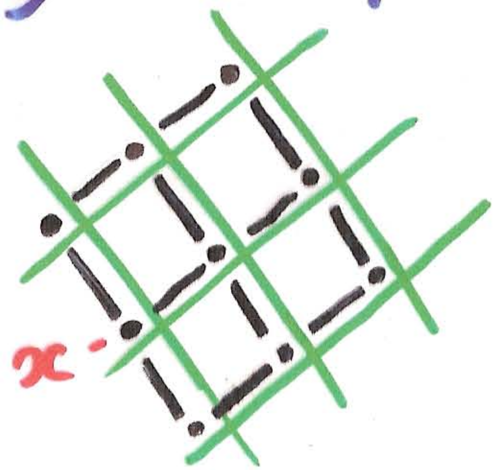
$$\underline{\Gamma}_{\alpha,\alpha_2} \text{ parallel to } \underline{\Gamma}_{\alpha_1 -} \equiv \sum_{\beta_2} \underline{\Gamma}_{\alpha,\beta_2}$$

ii) Then prob given by

$$\underline{\Gamma}_{\alpha,\alpha_2} = p\, \underline{\Gamma}_{\alpha_1 -}$$

# QI in causaloid formalism (and CPr

System of pairwise interacting qubits (bits)




Causaloid given by specifying only

$x \; \bullet \qquad \Lambda_{\alpha_x}^{l_x}$

$x' \quad x \qquad \Lambda_{l_x l_{x'}}^{k_x k_{x'}}$

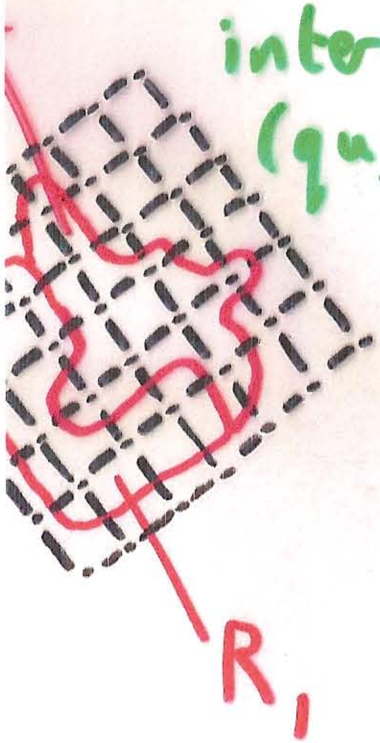
(glossing over some details here)

Massive 3rd level compression:

of order $2^M$ possible $\Lambda$'s $\quad M$ # nodes

Only order $M$ listed in causaloid

Pairwise interacting (qu)bits.




## Other formalisms for QT with arbitrary regions.

1) Yakir Aharonov et al. (esp. elaborations due to Sandu Popescu) Time-symmetric QT.

2) Robert Oeckl General boundary formalism

3) ⋮

# Towards Quantum Gravity

$$\begin{tikzcd} QT \arrow[r] & QG \\ CProbT \arrow[u, "\text{``quantization''}"] \arrow[r, "\text{``GR-ization''}"'] & ProbGR \arrow[u] \end{tikzcd}$$

$\uparrow$ "quantization" is needed locally (level one comp.)

$\rightarrow$ "GR-ization" is likely to relate to the way thin
are connected (level 3 and level 2,

# Prob GR

Prob GR should be

1) Operational - relate data
2) probabilistic
3) locally formulated

# Conclusions.

1) Causaloid formalism admits probabilistic theories with indefinite causal structure.

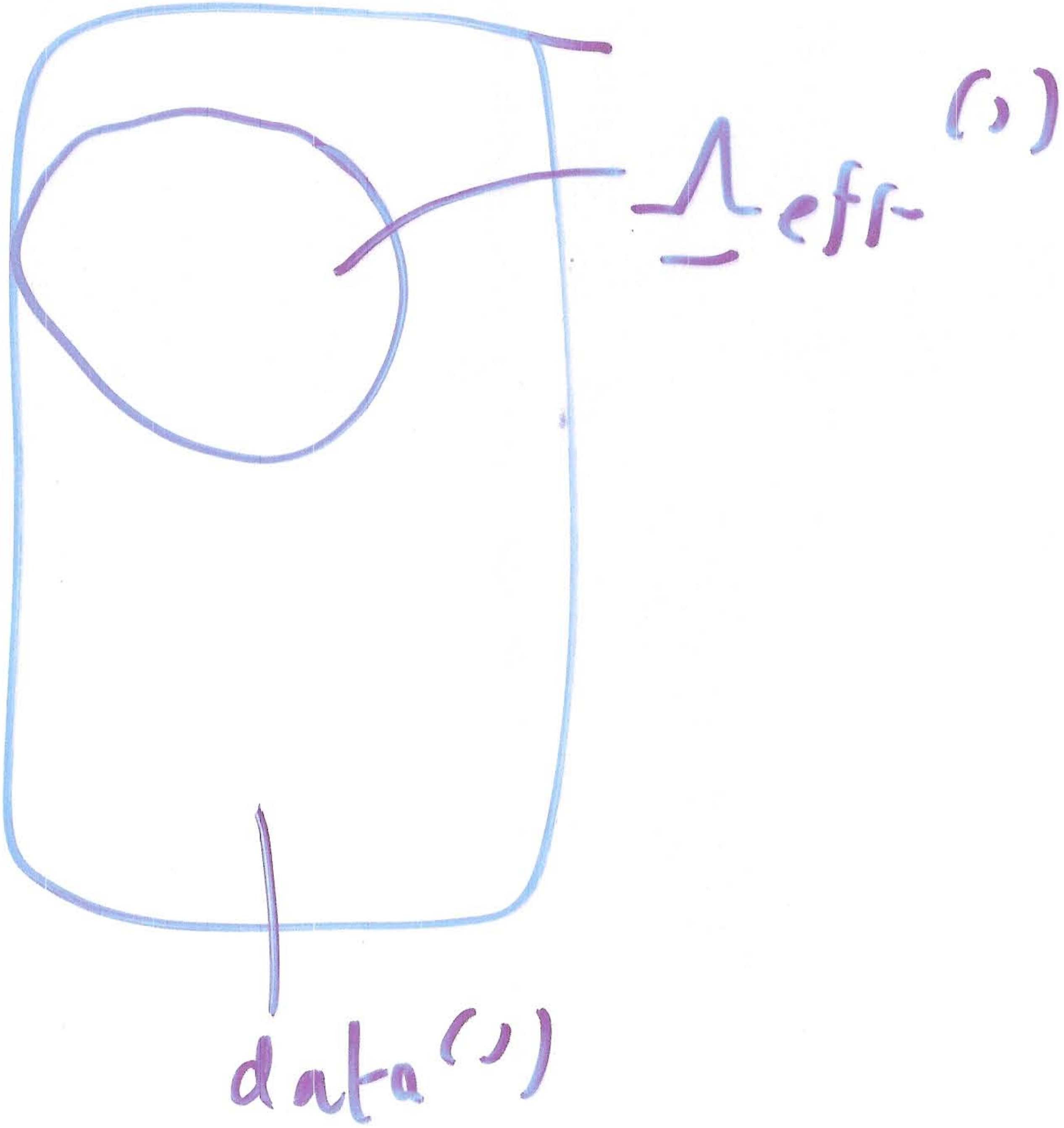
2) Mathematizes role played by causal structure

3) Can formulate QT in framework

4) can hope to formulate ProbGR

5) Route to QG sketched out

6) No background time


Δeff ()
data ()