LQG: Lessons from Models

Abhay Ashtekar

Institute for Gravitational Physics and Geometry, Penn State

Will discuss models developed by

AA, A. Corichi, D-W Chiou, W. Kaminski, J. Lewandowski, A. Laddha, L. Szulc,

T. Pawlowski, P. Singh, V. Taveras, K. Vandersloot, and M. Varadarajan.

(AA, short, pedagogical review of cosmological models, gr-qc/0702030)

1. Introduction

Over the past couple of years there has been considerable progress on the mathematical front of full LQG. Excellent control of kinematics and new approaches to dynamics, i.e., quantum constraints. (Thiemann's Talk)

But reliable physical predictions are yet to appear.
- What does full LQG have to say about the most interesting conceptual questions? (Fate of singularities; quantum nature of the big bang; information loss; ...)
- What is the *physical* meaning of ambiguities in the quantum Hamiltonian constraint?

Such questions can be answered in symmetry reduced models. Complete analysis available in the k=1 and k=0 models with a massless scalar field, with and without Λ; partial results for the k=-1 and Bianchi models and CGHS black holes (midi-superspaces).

Goal of the talk is 2-folds: To provide i) A few highlights of results obtained by an essentially complete analysis of symmetry reduced models (Cosmology and Black Holes); and ii) Examples of lessons they have for full LQG.

One's first reaction: Symmetry reduction gives only toy models! Full theory much richer and much more complicated.
But examples can be powerful!

- Full QED versus Dirac's hydrogen atom.
- Singularity Theorems versus first discoveries in simple models.
- 'Generic' BKL behavior versus homogeneous Bianchi models.

Do NOT imply that behavior found in examples is generic. Rather, they should not be dismissed a priori as being too special.

Organization:

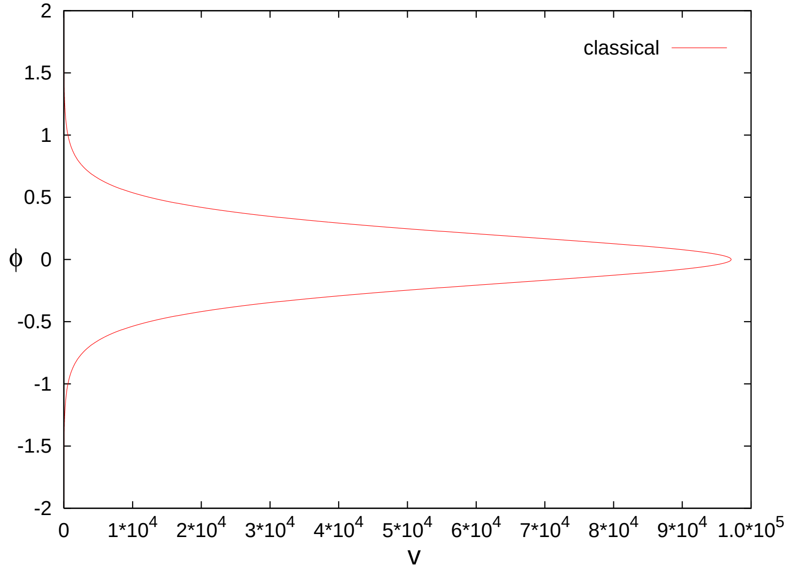
1. Introduction
2. Cosmological Models
3. The CGHS Model and Information Loss
4. Summay.

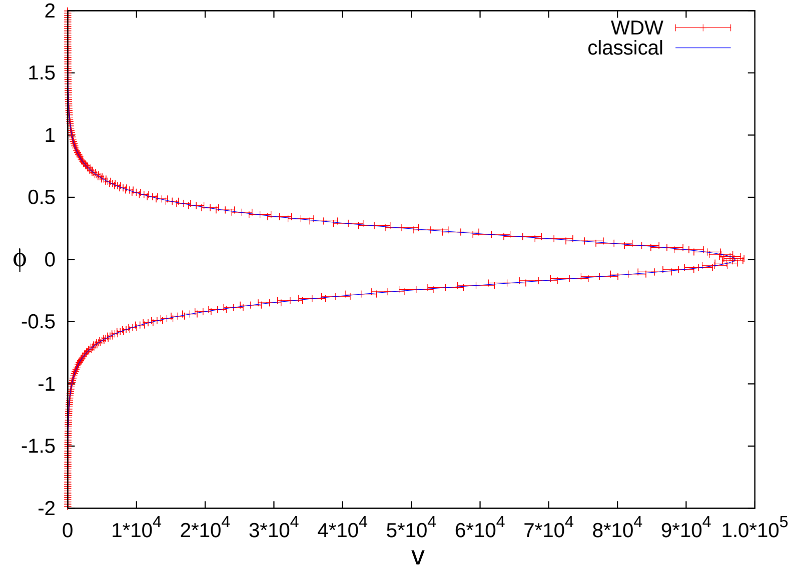
2. Cosmological Models

LQG program completed. Physical Hilbert space, Observables, Semi-Classical states,... Combination of analytical and numerical methods.

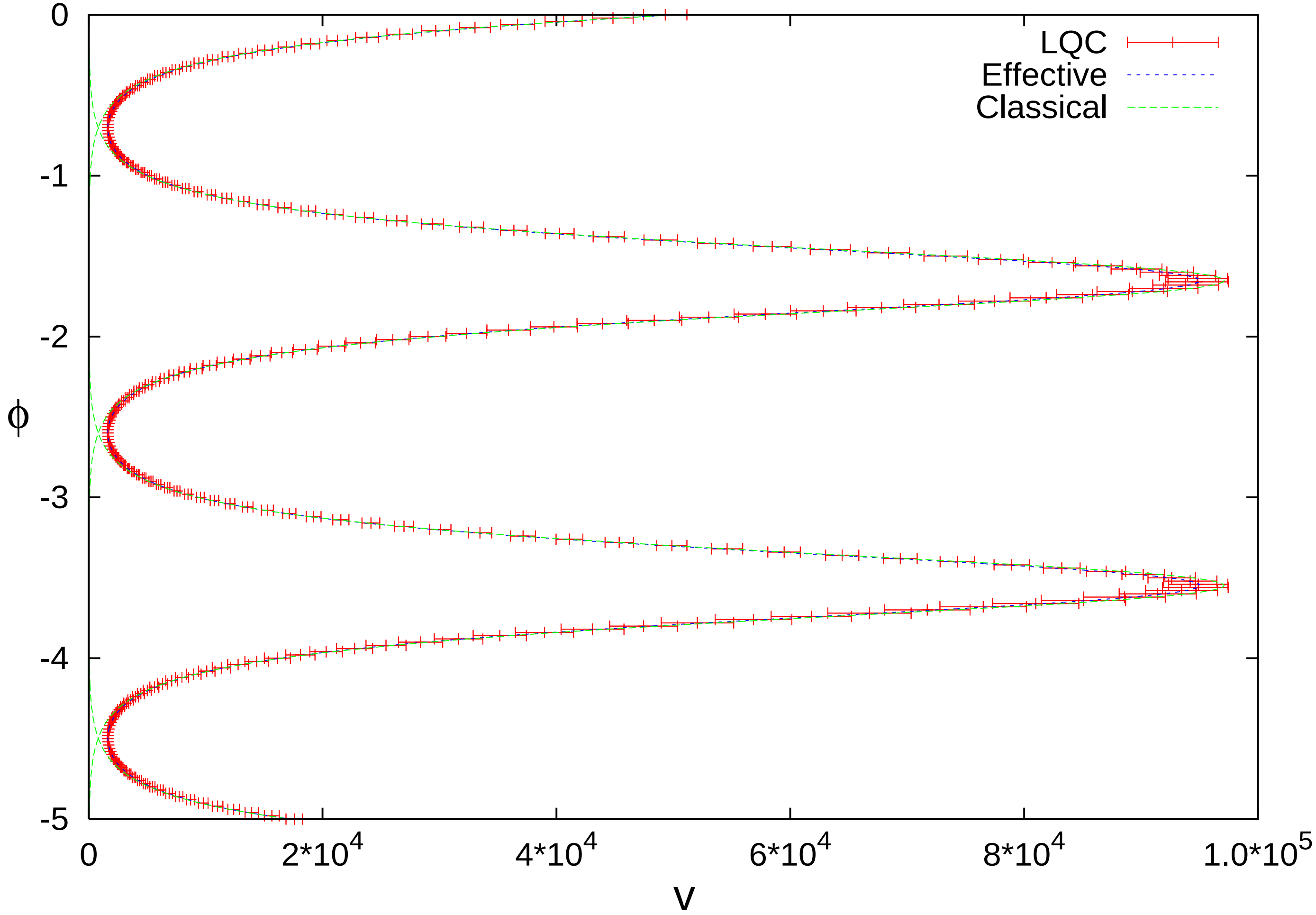
k=1 FRW with massless ϕ: Instructive because every classical solution is singular; scale factor not a good global clock (classical re-collapse). Provides a foundation for more complicated models.

Classical and WDW solutions



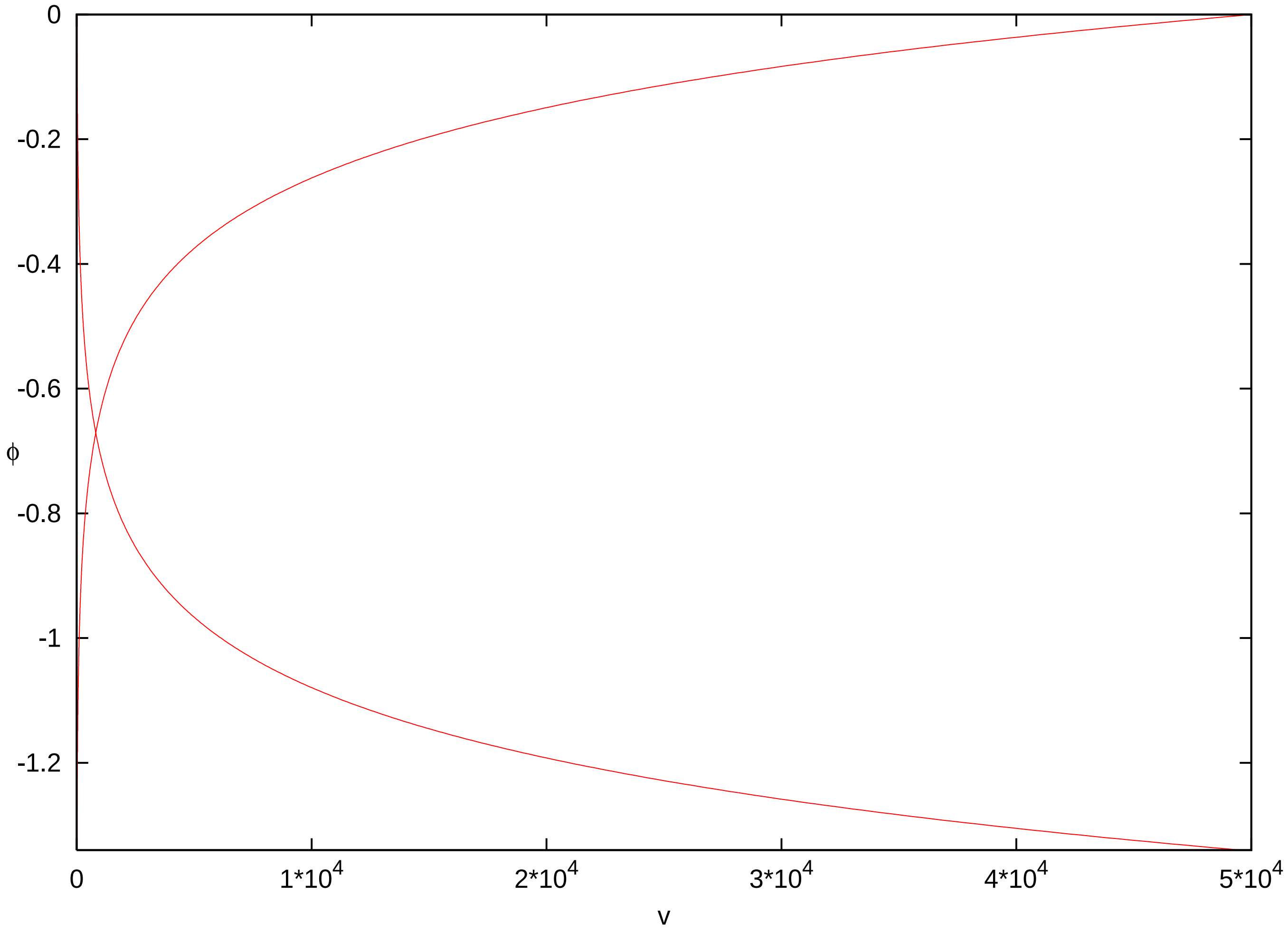


k=1, $\Lambda = 0$, LQC



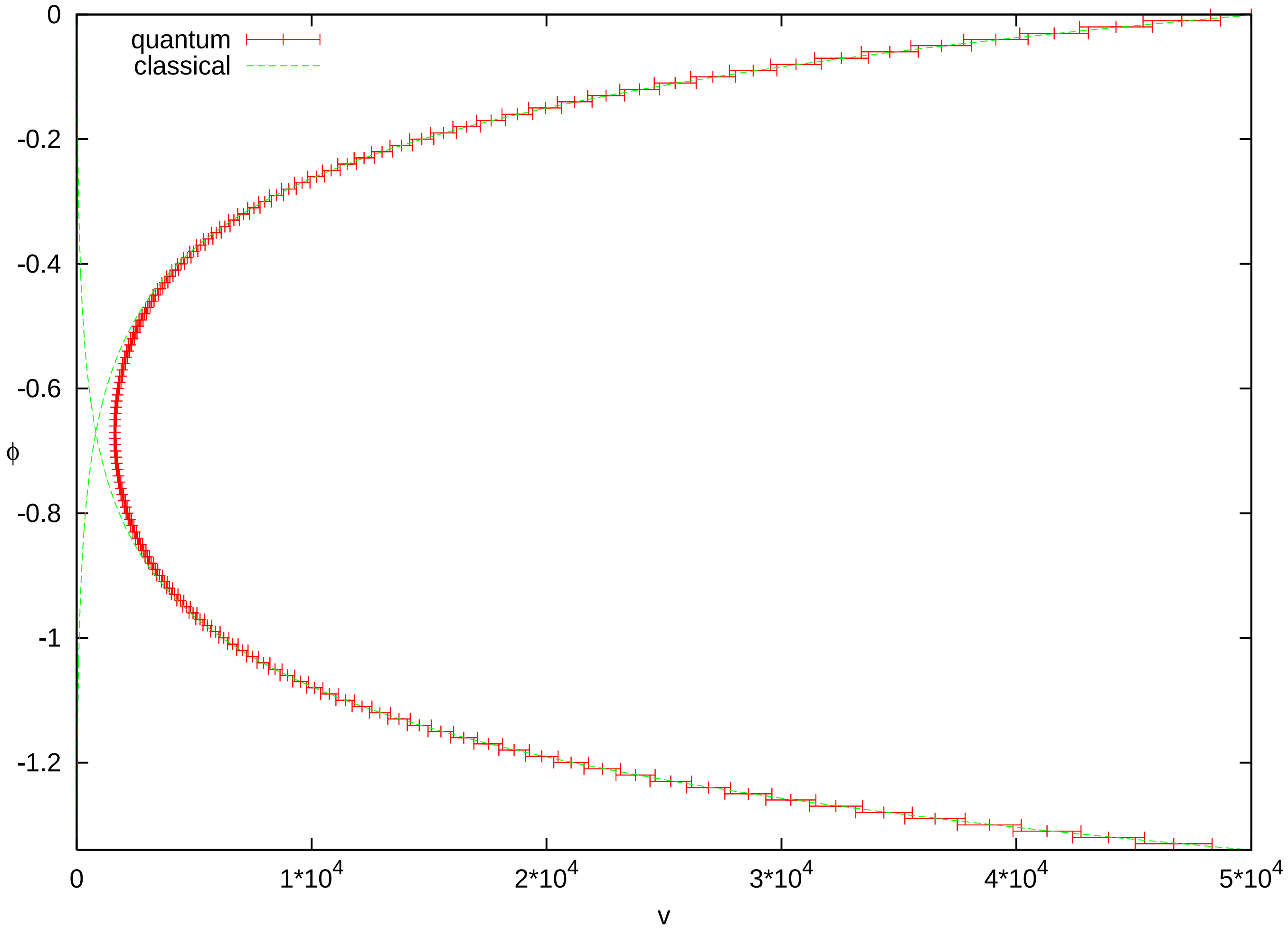
Expectations values and dispersions of $\hat{V}|_{\phi}$ & classical trajectories.

k=0, $\Lambda = 0$



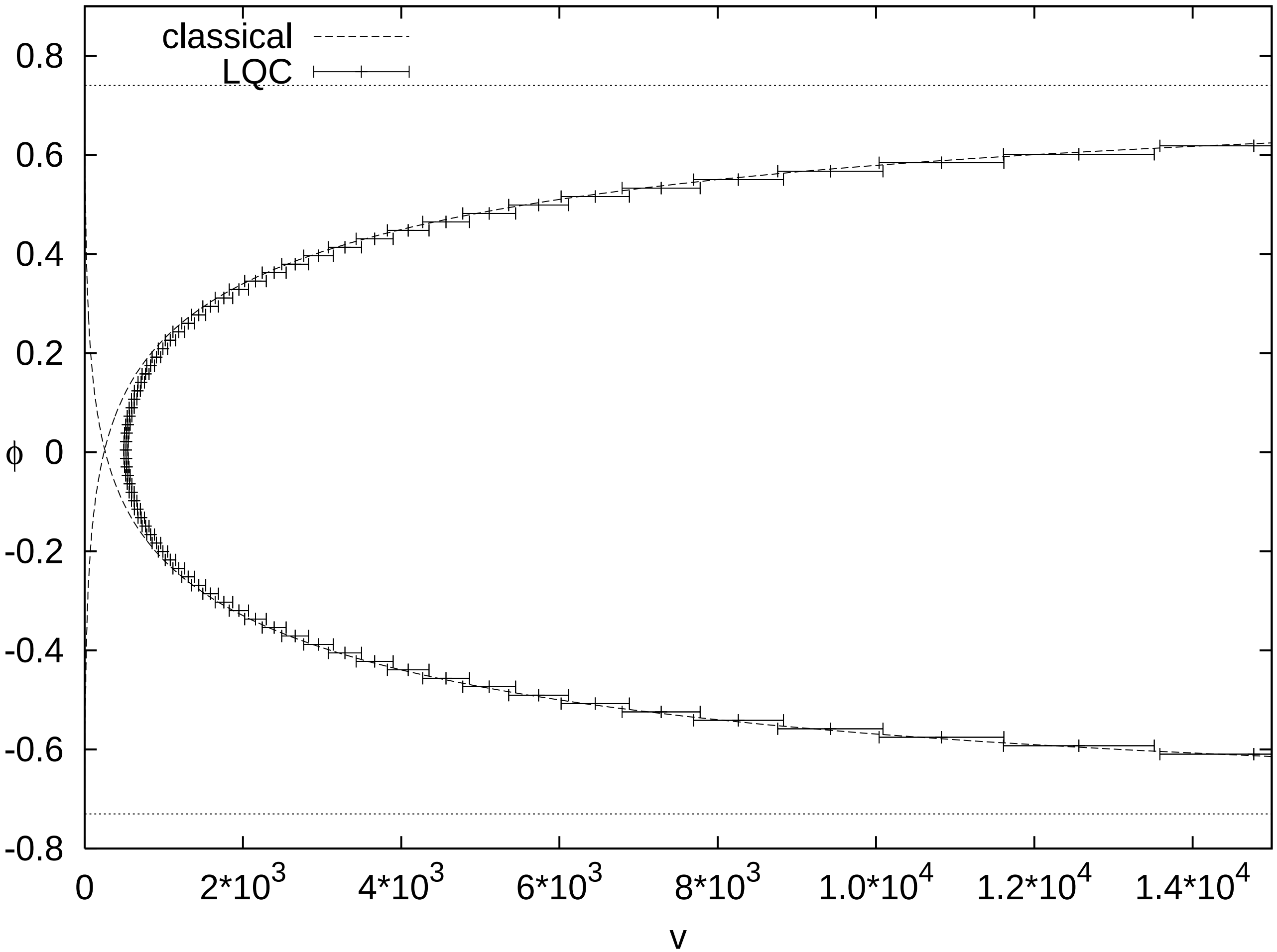
Classical trajectories.

k=0, $\Lambda = 0$: LQC



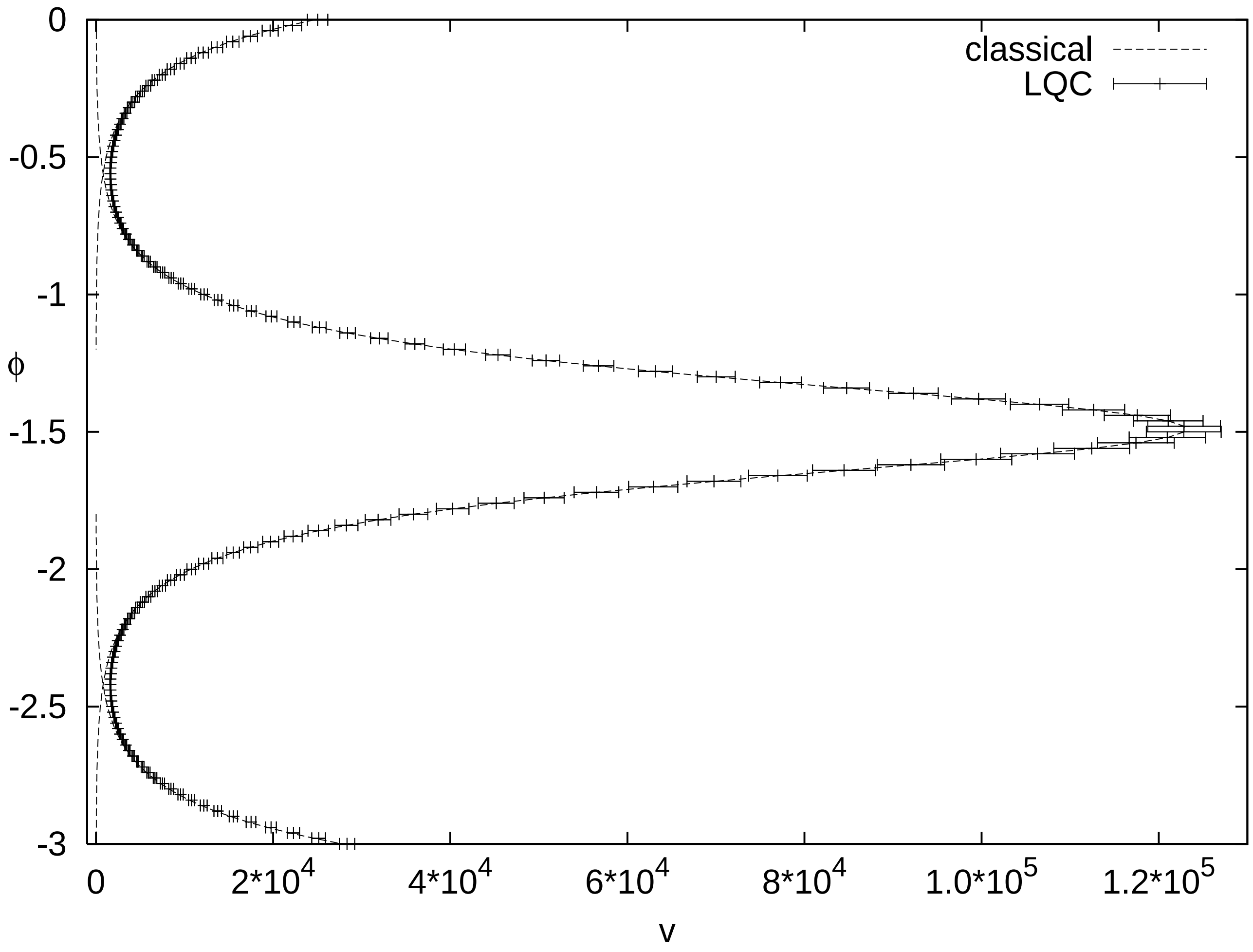
Expectations values and dispersions of $\hat{V}|_\phi$.

k=0, $\Lambda > 0$, LQC



Expectations values and dispersions of $\hat{V}|_{\phi}$ & classical trajectories.
(Numerics are currently being improved.)

k=0, $\Lambda < 0$, LQC



Negative Cosmological constant: Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories. (Numerics are currently being improved.)

Lessons from analytic+numerical analysis

Does LQG have a good semi-classical limit?

• Quantum geometry cures ultra-violet problems near the singularity. But do those effects become negligible sufficiently quickly as to provide agreement with GR in the *very long* low curvature history of the universe?

The Green-Unruh concern in the k=1 case (based on the older form of the Hamiltonian constraint) removed by the 'improved LQC dynamics'. Classical recollapse valid even for universes which grow only to $a_{\rm max} \sim 25\ell_{\rm Pl}$!

Important test of viability of LQC!

Lessons from analytic+numerical analysis

Does LQG have a good semi-classical limit?

• Older μ_o versus newer $\bar{\mu}$ dynamics:
Both use the idea that the operator F_{ab}^i be replaced by holonomy around a loop of minimum area. Systematizes quantization unlike postulating $c \rightsquigarrow (\sin c\lambda/\lambda)$. μ_o evolution uses area w.r.t. the fiducial metric while $\bar{\mu}$ w.r.t. the *physical* metric.

In the μ_o-evolution: Bounce can occur at low densities, even density of water! Gross violation of semi-classical limit.

Sometimes argued that this is an artifact of homogeneity assumption. But then why doesn't this assumption also invalidate singularity resolution?

Lesson: Correct Semi-Classical Limit heavily constrains how ambiguities in the quantum Hamiltonian constraint are resolved!

Lessons from analytic+numerical analysis

• Quantum gravity scale set by curvature not by volume. In k=1 case, if the universe grows to $a_{\max} = 1$Mpc, bounce at $V \approx 10^{115} V_{\rm Pl}$! Such numbers are instructive.

• New repulsive force from quantum geometry responsible for singularity resolution. Modifications of the matter Hamiltonian ('inverse volume effects') play no role if the universe grows to a macroscopic size. Key factor: $F_{ab}^{i} \ \rightsquigarrow \ \mathrm{holonomy}$, *non-local*. Hint for the full theory?

• As in classical GR physical interpretation difficult to extract in a manifestly 3-diffeomorphism invariant framework. Gauge fixing of this invariance made rapid progress possible. Suggests a complementary strategy for the full theory: focus on non-linear neighborhoods of physically interesting phase space points, gauge-fix diff constraint there and study *physical* implications of the Hamiltonian constraint.

Analytical understanding: k=0, $\Lambda = 0$

- Simplified LQC: (AA, Corichi, & Singh)

The $\bar{\mu}$ LQC Hamiltonian constraint is a difference equation in v. Difficult to solve analytically.

Contains two functions $A(v)$ and $B(v)$ as coefficients. $A(v) = v$ for $v > 1$ and $B(v) \approx v^{-1}$ if inverse volume corrections are ignored. Well motivated approximations of the quantum constraint.

The resulting simplified model can be solved analytically.

(Contrast with the Bojowald model).

Analytical understanding: k=0, $\Lambda = 0$

• Simplified LQC: (AA, Corichi, & Singh)
The $\bar{\mu}$ LQC Hamiltonian constraint is a difference equation in v. Difficult to solve analytically.
Contains two functions $A(v)$ and $B(v)$ as coefficients. $A(v) = v$ for $v > 1$ and $B(v) \approx v^{-1}$ if inverse volume corrections are ignored. Well motivated approximations of the quantum constraint.

The resulting simplified model can be solved analytically.
(Contrast with the Bojowald model).

• b: Canonically conjugate to v. In the b representation, SLQC constraint is a differential equation: $\partial_\phi^2 \Psi(b,\phi) = 12\pi G \left[(\sin \lambda b/\lambda)\ \partial_b\right]^2 \Psi(b,\phi)$
where $\lambda^2 =$ Area gap.

• Similar to the WDW Theory: $\partial_\phi^2 \Psi(b,\phi) = 12\pi G \left[b\partial_b\right]^2 \Psi(b,\phi)$

• In fact the *physical* Hilbert spaces of two theories are naturally isomorphic. Difference: Expressions of the Dirac observables $\hat{V}|_\phi$.

Lessons: SLQC vs WDW theory

On a dense sub-space of the physical Hilbert space,

- SLQC exhibits a bounce in the sense that
$\langle \hat{V}_\phi \rangle \to \infty$ as $\phi \to \pm\infty$. Quantum bounce is generic!

- WDW Theory mimics classical GR in the sense that:
$\langle \hat{V}_\phi \rangle \to 0$ as $\phi \to -\infty$ and $\langle \hat{V}_\phi \rangle \to \infty$ as $\phi \to \infty$ (or vice versa) .

Lessons: SLQC vs WDW theory

On a dense sub-space of the physical Hilbert space,

• SLQC exhibits a bounce in the sense that
$\langle \hat{V}_\phi \rangle \to \infty$ as $\phi \to \pm\infty$. Quantum bounce is generic!

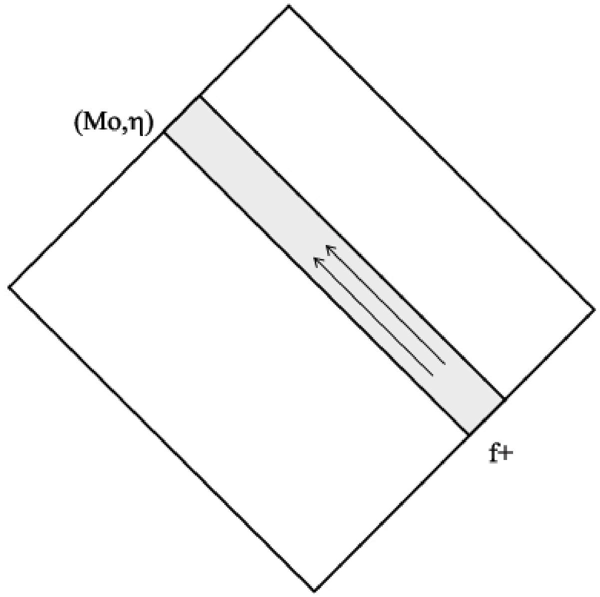
• WDW Theory mimics classical GR in the sense that:
$\langle \hat{V}_\phi \rangle \to 0$ as $\phi \to -\infty$ and $\langle \hat{V}_\phi \rangle \to \infty$ as $\phi \to \infty$ (or vice versa) .

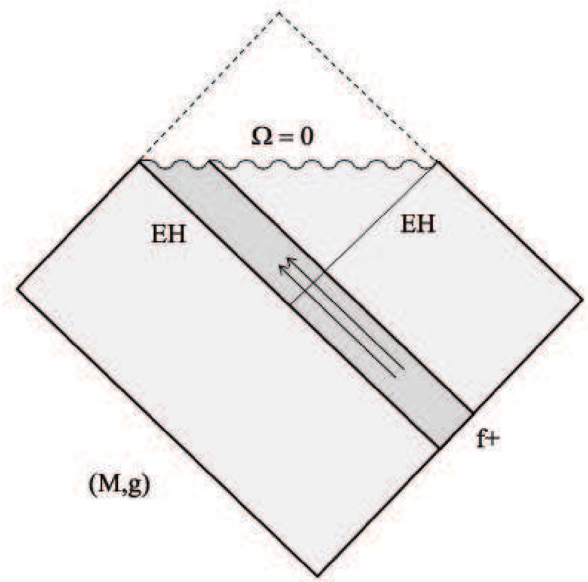
• Start with the same physical state at $\phi = \phi_o$ and evolve using SLQC or WDW theory. Then:

- Certain predictions of SLQC approach those of the WDW theory as the area gap λ goes to zero:
  Given $\Delta\phi$ and $\epsilon > 0$, there exists a $\delta > 0$ such that $\forall \lambda < \delta$, 'physical predictions of the two theories are within ϵ of each other.'
- However, approximation is *not* uniform. The WDW theory is *not* the limit of SLQC:
  Given $N > 0$ however large, there exists a ϕ such that
  $\langle \hat{V}_\phi \rangle_{\rm SLQC} - \langle \hat{V}_\phi \rangle_{\rm WDW} > N$.
  LQC is *fundamentally* discrete.

3. CGHS black holes & information loss

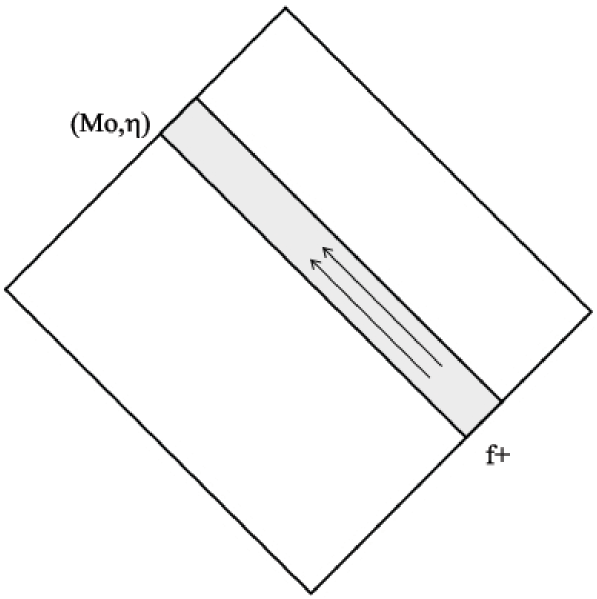
• 1+1 dim model; closely related to spherically symmetric reduction of GR with a massless scalar field f.
TDOF: massless scalar field f on a fiducial Minkowski space (M_o, η).
Physical space-time $(M, g = \Omega\eta)$. Ω determined by f. Singularity at $\Omega = 0$. BH formation due to Gravitational collapse of the f_+ mode.

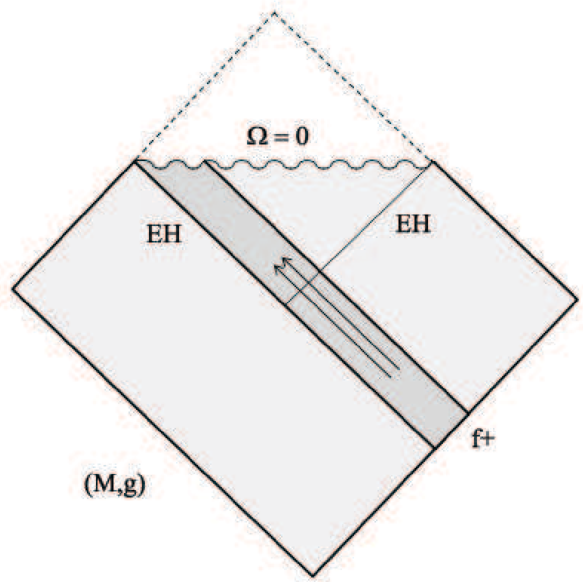




3. CGHS black holes & information loss

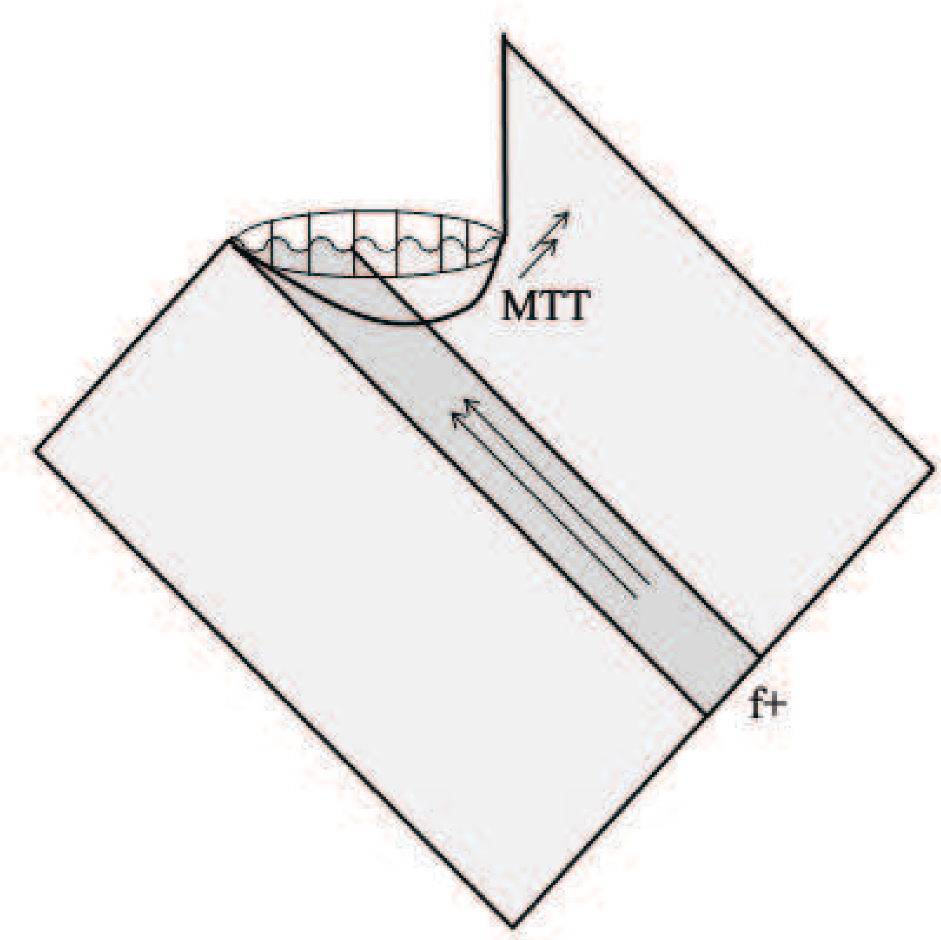
• 1+1 dim model; closely related to spherically symmetric reduction of GR with a massless scalar field f.
TDOF: massless scalar field f on a fiducial Minkowski space (M_o, η).
Physical space-time $(M, g = \Omega\eta)$. Ω determined by f. Singularity at $\Omega = 0$. BH formation due to Gravitational collapse of the f_+ mode.





• QFT on the BH space-time: Hawking radiation for f_-
Flux of energy across $\mathcal{I}_R^+$: Trace anomaly + Conservation of $\hat{T}_{ab}$.
Back reaction: Semi-classical gravity. A great deal of neat work in the 90's. But inconclusive.

Semi-Classical Gravity



Older Penrose diagram based on numerical simulations of Piran & Strominger and Lowe.

Quantum Gravity: Canonical Quantization

• A consistent framework exists. But complete solution not yet available. However, can make successive approximations.

• Solution by bootstrapping. In the quantum equations determining $\hat{\Omega}$ from $\hat{T}_{ab}$ of $\hat{f}$, use $\hat{T}_{ab}$ assuming the metric is η $\rightsquigarrow$ first approximation $\hat{\Omega}_1$.

• Expectation value $\rightsquigarrow$ precisely the classical BH solution.
Operator $\hat{g}_1 = \hat{\Omega}_1 \eta$ perfectly fine but its expectation value $\langle \hat{g}_1 \rangle$ vanishes along a space-like line $\rightsquigarrow$ classical singularity.

• Hawking radiation:the vacuum state $|O\rangle$ of the f_- mode on (M_o, η) interpreted on $\mathcal{I}_R^-$ of $\langle \hat{g}_1 \rangle$!

Quantum Gravity: Canonical Quantization

• A consistent framework exists. But complete solution not yet available. However, can make successive approximations.

• Solution by bootstrapping. In the quantum equations determining $\hat{\Omega}$ from $\hat{T}_{ab}$ of $\hat{f}$, use $\hat{T}_{ab}$ assuming the metric is η $\rightsquigarrow$ first approximation $\hat{\Omega}_1$.

• Expectation value $\rightsquigarrow$ precisely the classical BH solution. Operator $\hat{g}_1 = \hat{\Omega}_1 \eta$ perfectly fine but its expectation value $\langle \hat{g}_1 \rangle$ vanishes along a space-like line $\rightsquigarrow$ classical singularity.

• Hawking radiation:the vacuum state $|O\rangle$ of the f_- mode on (M_o, η) interpreted on $\mathcal{I}_R^-$ of $\langle \hat{g}_1 \rangle$!

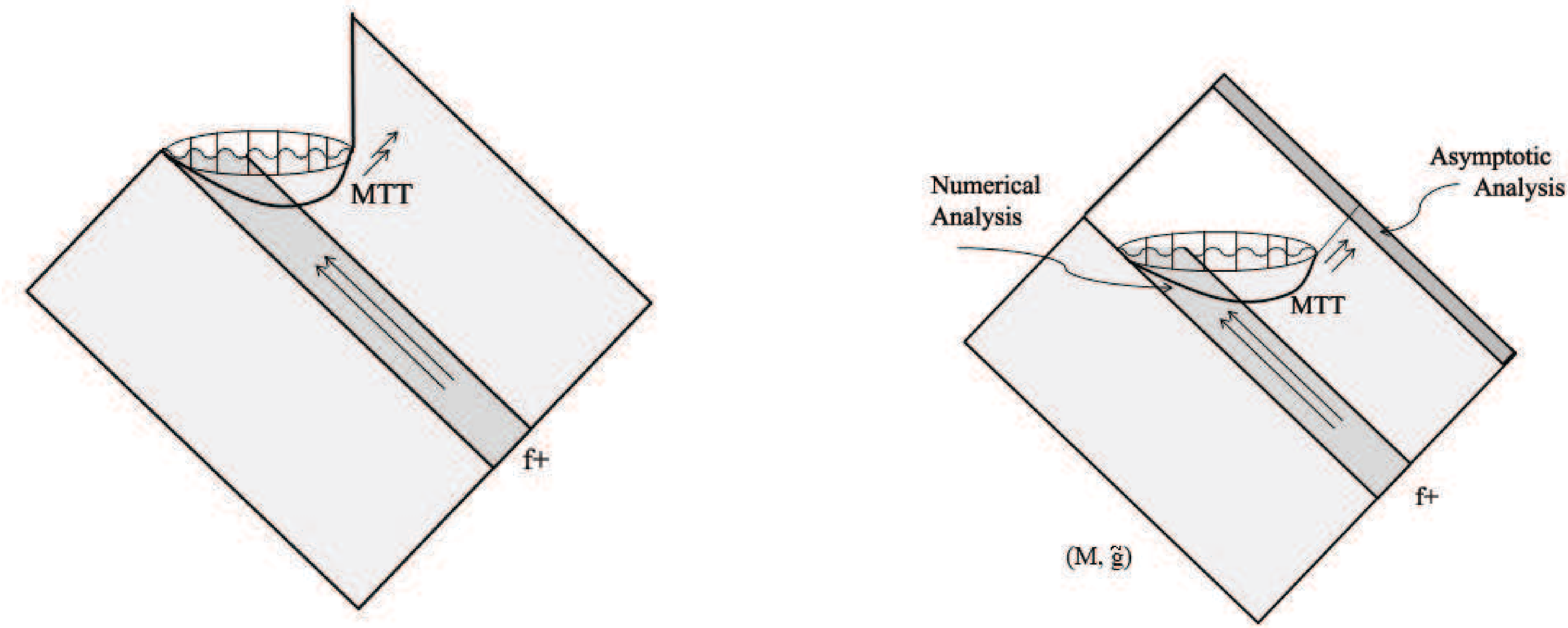
• *Mean field approximation:* In the untruncated, full quantum equations, ignore fluctuations of geometry but not of $\hat{f}$. Replace $\hat{\Omega}$ by its expectation value. Closed system, no bootstrapping $\rightsquigarrow$ Semi-classical gravity *as a systematic approximation of the full quantum theory.*

• *A second New element:* Analytical solution near $\mathcal{I}_R^+$ through asymptotic expansion.

Full semi-classical picture

• $\mathcal{I}_R^+$ of the semi-classical metric coincides with that of η $\Rightarrow$ Pure state on $\mathcal{I}_R^+$; No information loss! But the pure state resembles thermal state in the past.

The old and the new Penrose diagrams



• In contrast to the cosmological models, a region in which no smooth classical metric can approximate quantum geometry. But semi-classical regions in the distant past and distant future. *Genuine quantum bounce.*

Lessons from quantum CGHS

• In non-perturbative canonical quantization, ambiguities in the equations determining $\hat{\Omega}$ from $\hat{f}$. Simple minded choices (we first made!!) lead to local Lorentz violation in the semi-classical equations. LLI is likely to be a powerful tool in resolving ambiguities in full LQG.

• To complete the theory, need to extend QFT in 2-d curved space-times to QFT on 2-d quantum geometry. Specifically: Need a satisfactory extension of the trace anomaly calculation of QFT in curved space-times. Concrete challenge for quantum geometry.

4. Summary

• In mini-superspaces (Cosmology), complete control over the *physical sector* of the theory. In midi-superspaces (BHs) good control. In both cases physically interesting classical singularities resolved and quantum evolution deterministic. Contrast with other approaches.

Cosmology: Semi-classical bounce; BHs: Genuinely quantum bounce. Analysis suggests several interesting directions for full LQG.

4. Summary

• In mini-superspaces (Cosmology), complete control over the *physical sector* of the theory. In midi-superspaces (BHs) good control. In both cases physically interesting classical singularities resolved and quantum evolution deterministic. Contrast with other approaches.

Cosmology: Semi-classical bounce; BHs: Genuinely quantum bounce. Analysis suggests several interesting directions for full LQG.

• What happens generically? A singularity resolution theorem for all classical, space-like singularities? Nature of resolution? Semi-classical bounce? Quantum bounce? Just quantum foam? Key questions for the full theory.

• Confusion in the literature: Example

✓ Certain assumptions (e.g. homogeneity) $\Rightarrow$ Bounce.

✓ Perturbations off homogeneity remain small $\Rightarrow$ Bounce prevails.

!!! Perturbations grow near the singularity $\Rightarrow$ Bounce would disappear.

From $A \Rightarrow B$ *cannot conclude* $\mathrm{not}A \;\Rightarrow\; \mathrm{not}B$!

(Recall singularity theorems and BKL behavior. More recently, the ‘unreasonable’ success of the PPN framework.)

Inaugural Conference for

INSTITUTE FOR GRAVITATION AND THE COSMOS

PENN STATE, AUGUST 9-11

Hotel Reservation and Online Abstract Submission

http://www.gravity.psu.edu/igc/

DEADLINES FAST APPROACHING!!