Loop Quantum Gravity (LQG)

From secured land to unknown territory

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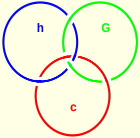
Loops’07, Morelia



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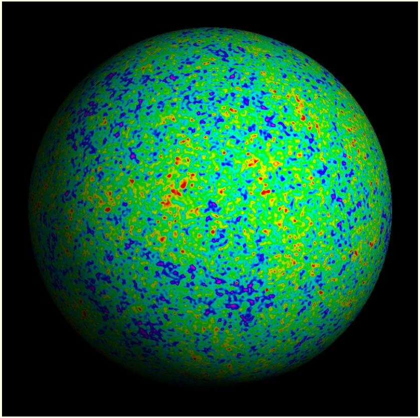
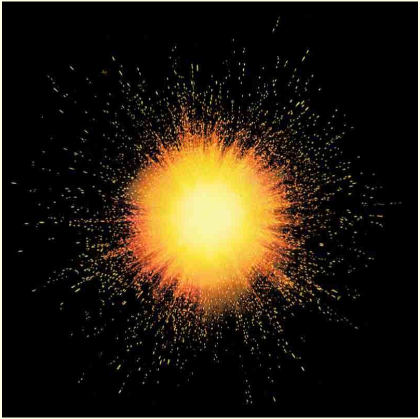
Cosmological Puzzles A: Isotropy of the CMB

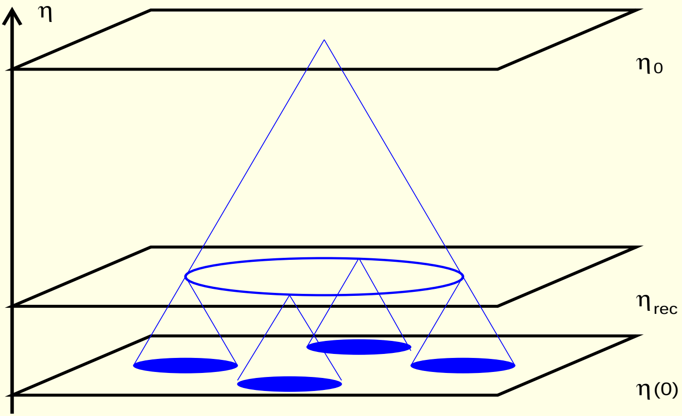
New era: high precision cosmology (COBE, WMAP, PLANCK)



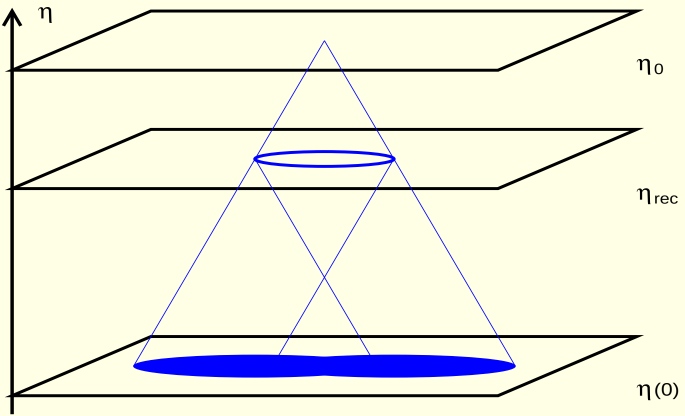
Classical GR: Time had a beginning



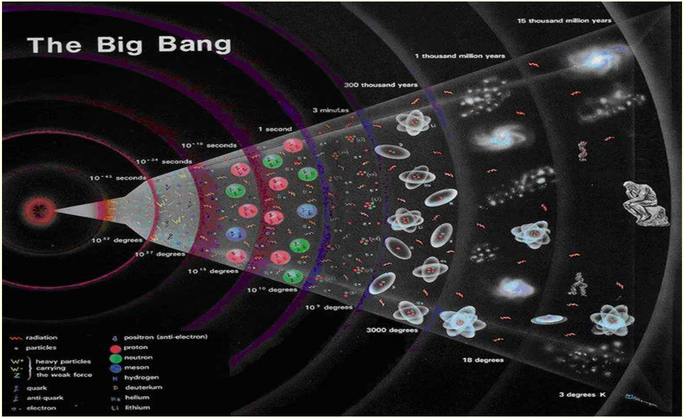
Usual matter: Horizon Problem



Exotic matter (inflaton): solves horizon problem

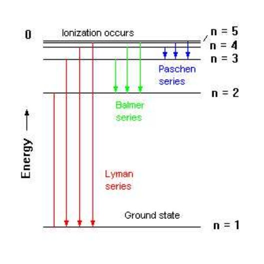


Cosmological standard model



Inflation also explains temperature fluctuations, but:

- Is GR really valid until t_P?
- Maybe quantum mech. absence of singularity



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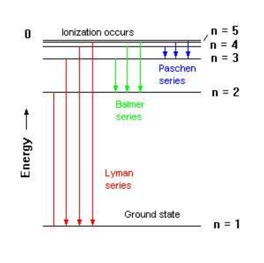

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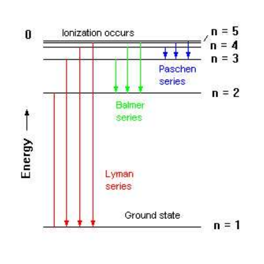

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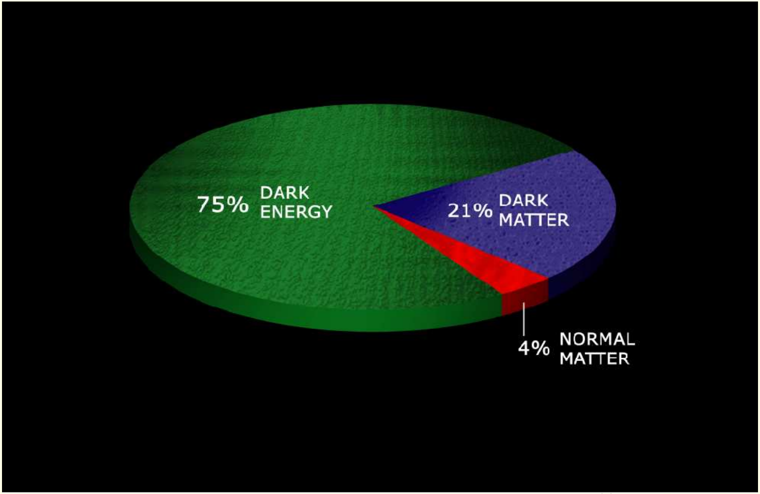
Cosmological Puzzle B: Dark Energy



Problem of cosmological constant:

- Dark energy = vacuum fluct.? E.g. zero point energy

$$<\widehat{H}>_{scalar} - <:\widehat{H}:>_{scalar} = \frac{\hbar}{2}\int_{R^3} d^3x \int_{R^3} d^3k\, |k|$$

- Naive: Quantum gravity Cut – off at $k\ell_P \approx 1$ where $\ell_P^2 = \hbar G$?

$$<\widehat{H}> - <:\widehat{H}:> = \frac{\hbar}{2}\int_{R^3} d^3x\, \ell_P^{-4}$$

- Comparison with cosmological term

$$H_{cosmo} = \frac{\Lambda}{G}\int_{R^3} d^3x \sqrt{|\det(g)|} \quad \Rightarrow \quad \Lambda\ell_P^2 \approx 1$$

- Worst prediction in history of physics:
  Experimentally: $\Lambda\ell_P^2 \approx 10^{-120}$.
- What is dark energy (matter), why Λ so unnaturally tiny?

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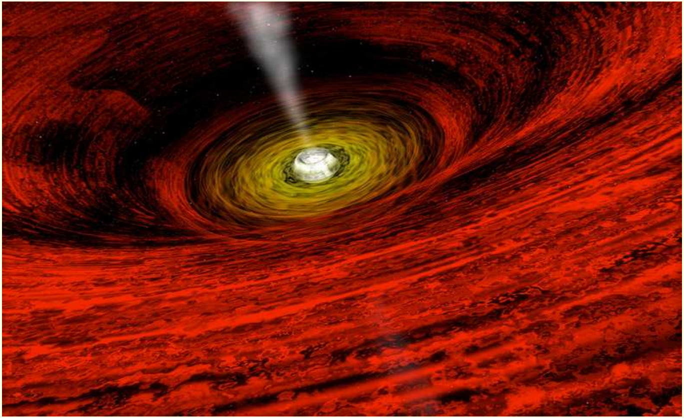
Astrophysical Puzzles: Black Holes



Entropy of Black Holes

- Class. GR (Penrose & Hawking): $\delta \mathrm{Ar}(H) \geq 0 \Rightarrow S \propto \mathrm{Ar}(H)$ (cf. 2nd law).
- QFT on CST (Hawking – effect): Black holes = black radiators $kT = \hbar\omega \approx \hbar c/r$.
- Entropy (Bekenstein): For Schwarzschild solution $r = 2Gm/c^2$ with $S = E/(kT) = mc^2/(kT)$

$$S_{BH} = \frac{\mathrm{Ar}(H)}{4\ell_p^2}$$

- What is the microscopic origin of this entropy?

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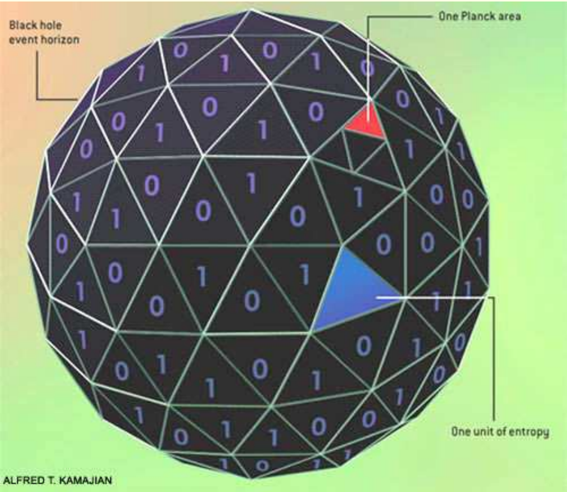
Holographic Explanation of BH – Entropy? ['t Hooft, Susskind 92]



Principle of Background Independence

- It is widely believed that only a full fledged quantum theory of gravity can answer these fundamental questions.
- For more than 70 years physicists are looking for a unified theory of general relativity and quantum mechanics – so far w/o success.
- Why?

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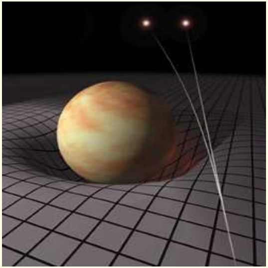

- Einstein's equations

$$R_{\mu\nu}[g] - \frac{1}{2} R[g]\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}[g]$$

- Background independence: Geometry g not prescribed but dynamically determined by matter energy density T.

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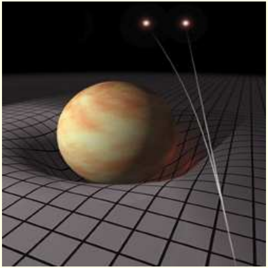

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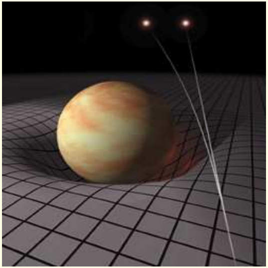

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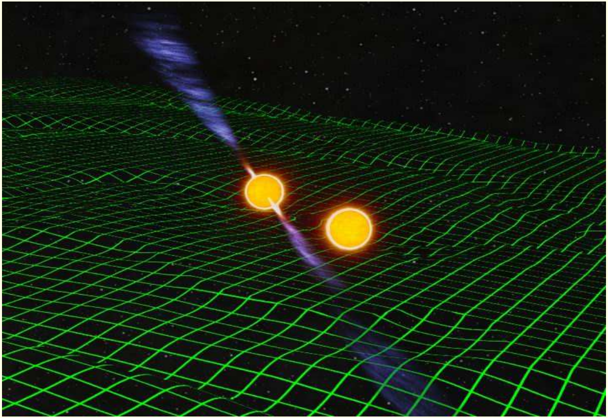
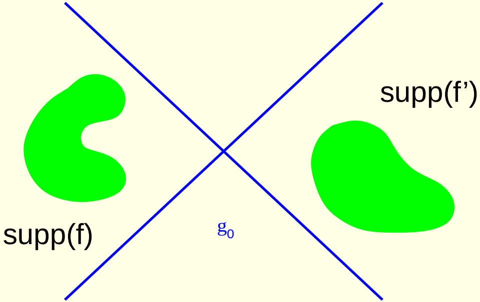
Background independence and backreaction (gravitational waves)



- Usual QFT (on CST) background dependent



- E.g. via causality axiom:
If supp(f), supp(f') spacelike separated wrt g_0 then

$$[\hat{\phi}(f), \hat{\phi}(f')] = 0, \quad \hat{\phi}(f) := \int_M d^4x \;\; f(x) \;\; \hat{\phi}(x)$$

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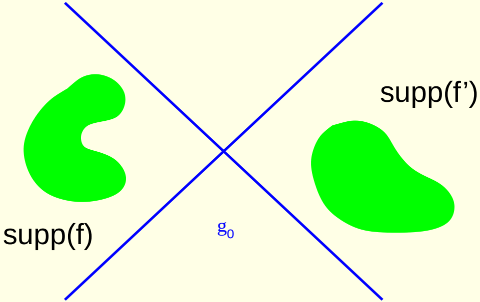

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$$\underset{\text{Background}}{g_0} \Rightarrow \underset{\text{Light Cone}}{(x-y)^2 < 0} \Rightarrow \underset{\text{Algebra}}{\mathfrak{A}}$$

- collapses when g_0 not available.
- ignores gravitational backreaction.
- invalid approx. in extreme cosm. & astrophys. situat.
- Perturbative approach

$$\underset{\text{Total Metric}}{\overset{g}{\uparrow}} = \underset{\text{Background}}{\overset{g_0}{\uparrow}} + \underset{\text{Perturbation (Graviton)}}{\overset{h}{\uparrow}}$$

violates BI, inacceptable due to non – renormalisability, merely effect. graviton QFT over g_0.

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Quantum - Einstein - Equations

- Radical generalisation of principles of QFT on CST.
- QFT on diff. mfd. M rather than QFT on spacetime (M, g_0).
- No Fock spaces but new BI Hilbert spaces.

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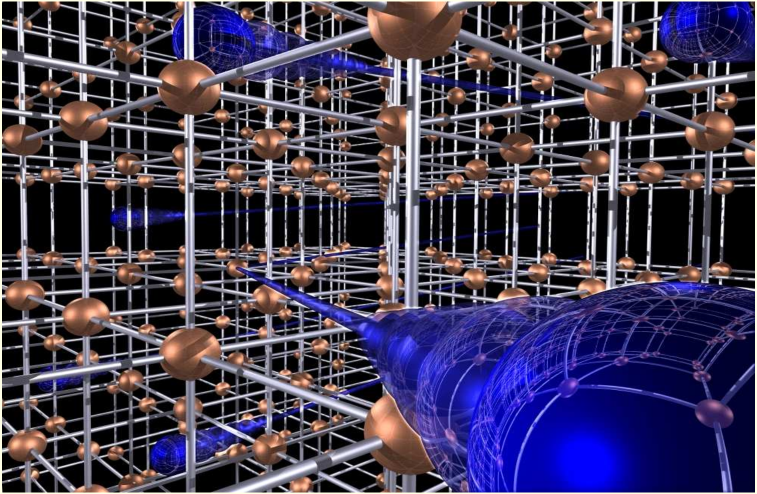
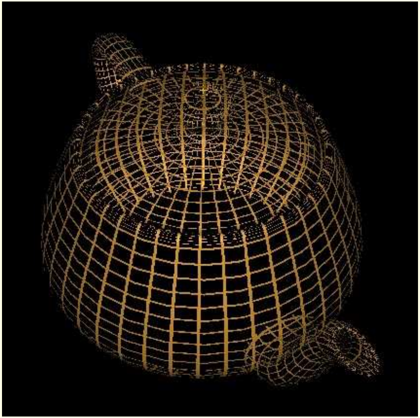
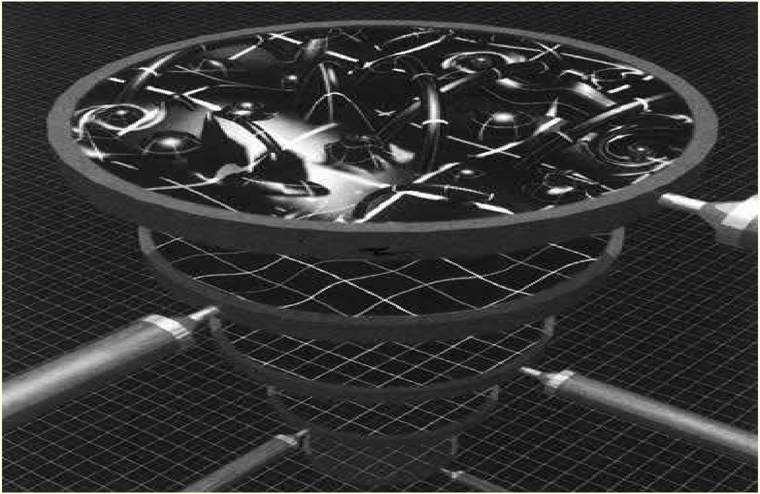
Ordinary QFT: Matter propagates on rigid spacetime



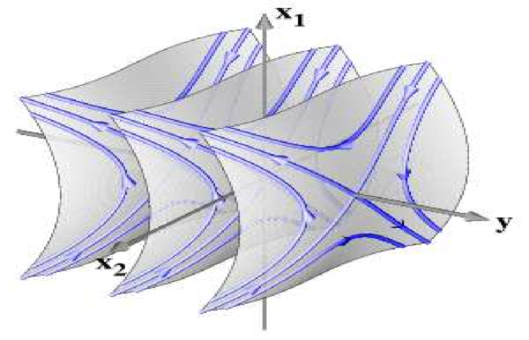
New QFT: Matter can exist only where geometry is excited





Classical Formulation

Canonical formulation: $M \cong R \times \sigma$



- Palatini action with (on shell) topological term

$$S = \frac{1}{8\pi G} \int_M \Omega_{IJ} \wedge [\epsilon^{IJKL} e_K \wedge e_L + \frac{1}{\beta} e^I \wedge e^J]$$

- Legendre transformation reveals canonical pair

$$(\omega_a^{IJ}, E^a_{IJ} = \frac{1}{2}\epsilon^{abc}[\epsilon_{IJKL} e_b^K e_c^L + \frac{1}{\beta} e_{bI} e_{cJ}])$$

- plus constraints, in particular simplicity constraints,

$$S^{ab} = \epsilon^{IJKL} E^a_{IJ} E^b_{KL} = 0$$

- spatial diffeo, Hamiltonian and Spin(1,3) Gauß constraints

$$G_{IJ} = \partial_a E^a_{IJ} - 2\omega_{a[I}{}^K E^a_{J]K} = 0$$

- 2nd class system! Solving 2nd class constr. yields canonical pair

$$(A_a^{jk} = \Gamma_a^{jk} + \beta K_{ab} e^{bl}\, \epsilon_{jkl},\; E^a_{jk} = |\det(e)| \epsilon^{abc} e_{bj} e_{ck})$$

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- 2nd class system! Solving 2nd class constr. yields canonical pair

$$(A_a^{jk} = \Gamma_a^{jk} + \beta K_{ab} e^{bl}\, \epsilon_{jkl},\ E_{jk}^a = |\det(e)| \epsilon^{abc} e_{bj} e_{ck})$$

- where $\Gamma =$ spin connection of triad and K= extrinsic curvature associated to $q_{ab} = e_a^j e_b^j$

- Palatini action with (on shell) topological term

$$S = \frac{1}{8\pi G}\int_M \Omega_{IJ} \wedge [\epsilon^{IJKL} e_K \wedge e_L + \frac{1}{\beta} e^I \wedge e^J]$$

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Full set of first class constraints

- SU(2) Gauß constraint

$$C_j = \partial_a E^a_j + A_{aj}{}^k E^a_k = 0$$

- spatial diffeomorphism constraint

$$C_a = \mathrm{Tr}\left(F_{ab} E^b\right) = 0$$

- Hamiltonian constraint

$$C = \frac{\mathrm{Tr}\left(F_{ab} E^a E^b\right)}{\sqrt{|\det(E)|}} + \ldots$$

- Convenient to summarise them in Master Constraint

$$M = \int_\sigma d^3x \, \frac{[C^2 + q^{ab} C_a C_b + \delta^{jk} C_j C_k]}{\sqrt{|\det(E)|}}$$

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- Master Constraint $\mathbf{M} = 0$ (Constraint Hypersurface)
- Gauge invariant (Dirac) observables

$$\{O, \{O, \mathbf{M}\}\}_{\mathbf{M}=0} = 0$$

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  1. No canonical Hamiltonian in generally covariant theories
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Observables & Physical Hamiltonian

Relational Ansatz [Rovelli 92 –], [Dittrich 04 –]

- Suitable scalar matter ϕ: Brown – Kuchař deparam. [Brown,Kuchař 92],[T.T. 06]

$$\mathbf{M} = 0 \quad \Leftrightarrow \quad \pi(x) + H(x) = 0$$

- Physical Hamiltonian (dep. only on non scalar d.o.f.)

$$\mathbf{H} = \int_\sigma d^3x \; H(x)$$

- Dirac observables (f: spat. diff. inv.)

$$O_f(\tau) := \sum_{n=0}^{\infty} \frac{1}{n!} \{H_\tau, f\}_{(n)}, \quad H_\tau = \int d^3x \; (\tau - \phi) H$$

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Quantum Kinematics

Lattice – inspired canon. q'ion [Gambini,Trias et al 81], [Jacobson,Rovelli,Smolin 88]

- Magnet. dof.: Holonomy (Wilson – Loop)

$$A(e) = \mathcal{P} \exp(\int_e A)$$

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$$E_j(S) = \int_S \frac{1}{2} E^{akl} \epsilon_{jkl} \epsilon_{abc} dx^b \wedge dx^c$$

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$$\{E_j(S), A(e)\} = G\, A(e_1)\, \tau_j\, A(e_2); \quad e = e_1 \circ e_2,\; e_1 \cap e_2 = e \cap S$$

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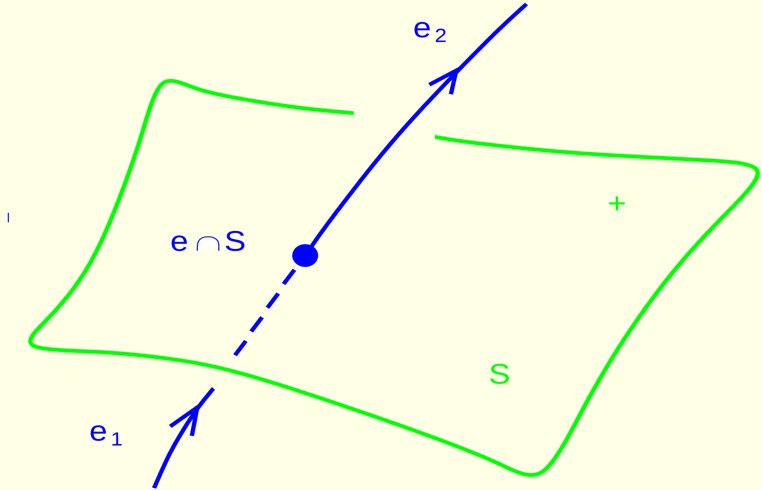
e 2
e ∩ S
+
S
e 1

- HS Reps.: In QFT no Stone – von Neumann Theorem!!!

Theorem [Ashtekar,Isham,Lewandowski 92-93], [Sahlmann 02], [L., Okolow,S.,T.T. 03-05], [Fleischhack 04]

Diff(σ) inv. states on hol. – flux algebra $\mathfrak{A}$ unique.

- wave functions

$$\psi(A) = \psi_\gamma(A(e_1),..,A(e_N)), \quad \psi_\gamma : \ SU(2)^N \to C$$

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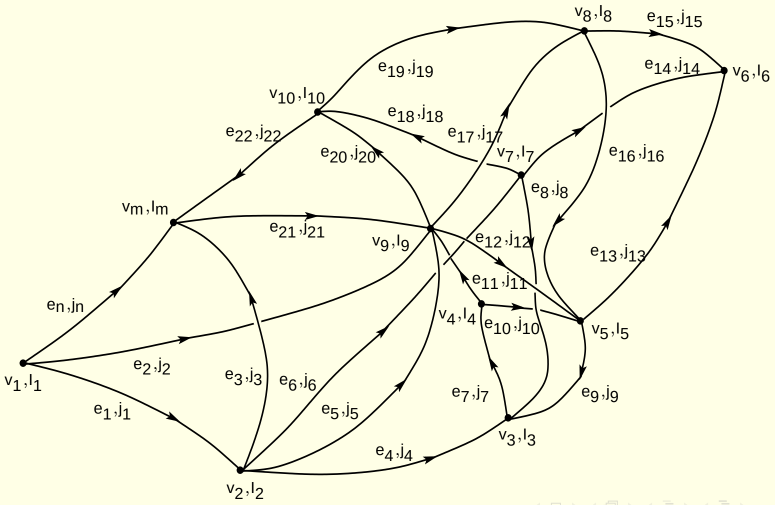
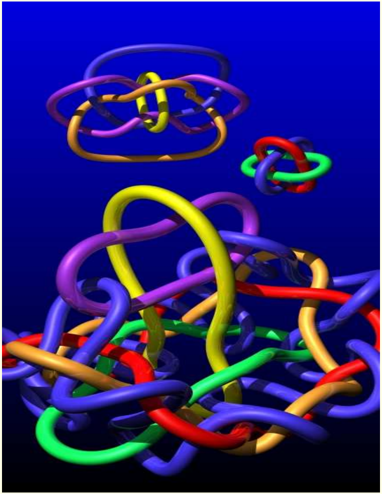
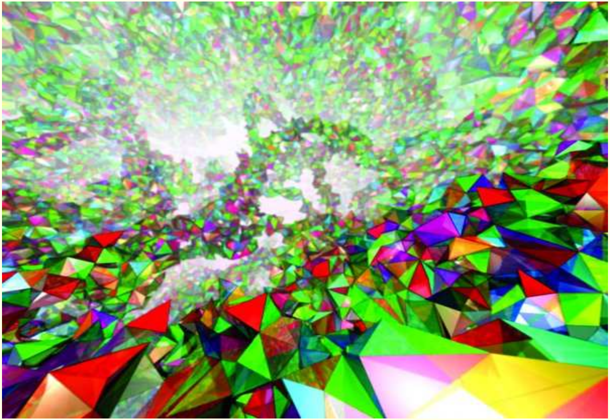
Spin Network Basis $T_{\gamma,j,l}$ $\sim$ H_j Hermite Polynomials



Colour Coding of Spin Quantum Numbers on graph edges ...



... or on Faces of Dual Cell Complex (Triangulation)



Animation:

http://www.einstein-online.info/de/vertiefung/Spinnetzwerke/index.html.

Kinematical spatial geometry operators

Area – Operator [Rovelli & Smolin 94], [Ashtekar & Lewandowski 95]

- Class. Area functional for 2 – mfd. S

$$\mathrm{Ar}(S) = \int_S \sqrt{\mathrm{Tr}\left(\left[E^a \epsilon_{abc} dx^b \wedge dx^c\right]^2\right)}$$

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- Spectrum explicitly known, purely discrete. Eigenstates = spin – network – states $T_{\gamma,j,I}$
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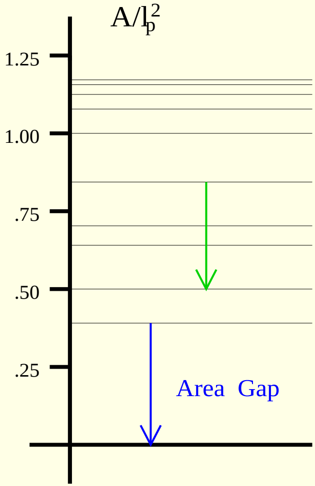
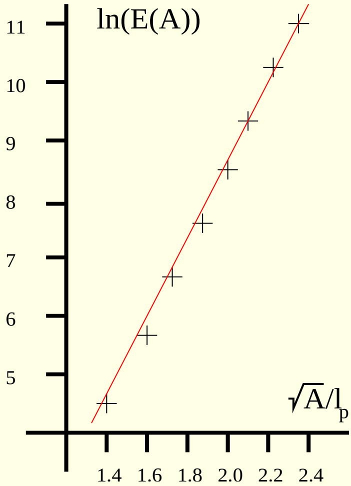

A/l_p^2
1.25
1.00
.75
.50
.25
Area Gap


ln(E(A))
11
10
9
8
7
6
5
√A/l_p
1.4
1.6
1.8
2.0
2.2
2.4

Kinematical Coherent States

Non – pert. approach, no pert. theory, rather different tool:

[T.T. 00], [T.T., Winkler 00 – 02], [Varadarajan 02], [Ashtekar & Lewandowski 02]

- $\exists$ minimal uncertainty states f. $\mathfrak{A}$, i.e.

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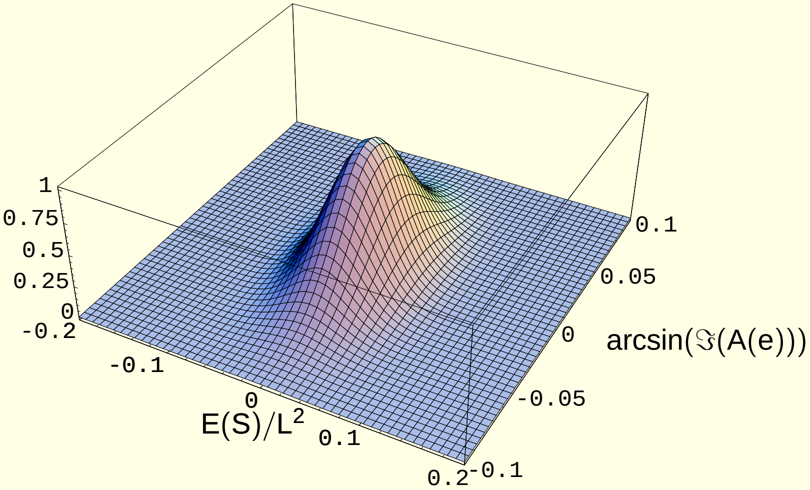
$$< \psi_{A_0,E_0}, \widehat{A(e)} \psi_{A_0,E_0} >= A_0(e), \quad < \psi_{A_0,E_0}, \widehat{E_j(S)} \psi_{A_0,E_0} >= E_{j0}(S)$$

  2. and

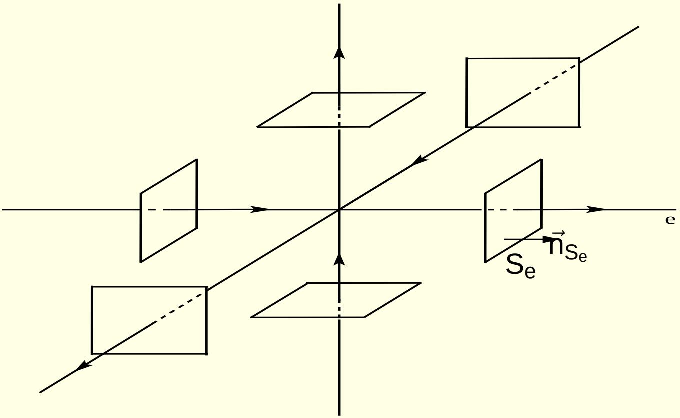
$$< \widehat{\Delta A(e)} > \; < \widehat{\Delta E_j(S)}) >= \frac{1}{2} | < [\widehat{A(e)}, \widehat{E_j(S)})] > |$$

Coherent States



Master Constraint: Definition

For simplicity: cubic graph and dual cell complex



Difference: BD & BI theories

- Yang – Mills on (R^4, η) [Kogut & Susskind 74]

$$H = \frac{\hbar}{2\, g^2\, \epsilon} \sum_{v \in V(\gamma)} \sum_{a=1}^{3} \mathrm{Tr}\left(E(S_v^a)^2 + [A(\alpha_v^a) - A(\alpha_v^a)^{-1}]^2 \right)$$

- Gravity on $R \times \sigma$ [T.T. 96 – 05, Giesel & T.T. 06]

$$\mathbf{M} = \frac{\hbar}{\ell_P^4} \sum_{v \in V(\gamma)} \sum_{j=0}^{3} \left| \sum_{a=1}^{3} \mathrm{Tr}\left(\tau_j\, A(\alpha_v^a)\, A(e_v^a)\, [A(e_v^a)^{-1}, V_v^{1/2}] \right) \right|^2$$

- Volume operator

$$V_v = \sqrt{|\epsilon_{abc} \mathrm{Tr}\left(E(S_v^a)\, E(S_v^b)\, E(S_v^c) \right)|}$$

- Lattice spacing ϵ disappears, replaced by ℓ_P
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Master Constraint: Solution

Physical Hilbert Space

- Dirac's basic idea $\mathbf{M}\Psi = 0$
- General solution $\Psi = \delta(\mathbf{M})\psi, \quad \psi \in \mathcal{H}_{\text{kin}}$ in general ill defined
- Must equip solution space with new inner product. Heuristically:

$$<\Psi, \Psi'>_{\text{phys}} := \frac{<\delta(\mathbf{M})\,\psi, \delta(\mathbf{M})\,\psi'>}{<\delta(\mathbf{M})\,\psi_0, \delta(\mathbf{M})\,\psi_0>} = \frac{\delta(0)}{\delta(0)}\,\frac{<\psi, \delta(\mathbf{M})\,\psi'>}{<\psi_0, \delta(\mathbf{M})\,\psi_0>}$$

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$$<\Psi, \Psi'>_{\text{phys}} = c\ <\psi, \delta(\mathbf{M})\,\psi'>$$

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Semiclass. Limit of Q. Dynamics testable only with kinemat. coh. states

Theorem [Giesel & T.T. 06] For any (A_0, E_0)

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Corollary

i. Quantum Master Constraint correctly implemented
ii. If $\mathbf{M}(A_0, E_0) = 0$ obtain approx. phys. states

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Spin Foam Models

Heuristic starting point [Reisenberger & Rovelli 96]

- Instead of using single M use ∞ number of Ham. Constraints C(x)
- Formal solutions $\Psi = \delta[C]\ \psi := \prod_x\ \delta(C(x))\ \psi$
- Formal physical inner product (functional integral)

$$< \Psi, \Psi' >_{\text{phys}} = < \psi,\ \delta[C]\ \psi' > = \int [dN]\ < \psi,\ \exp(iC(N))\ \psi' >_{\text{kin}}$$

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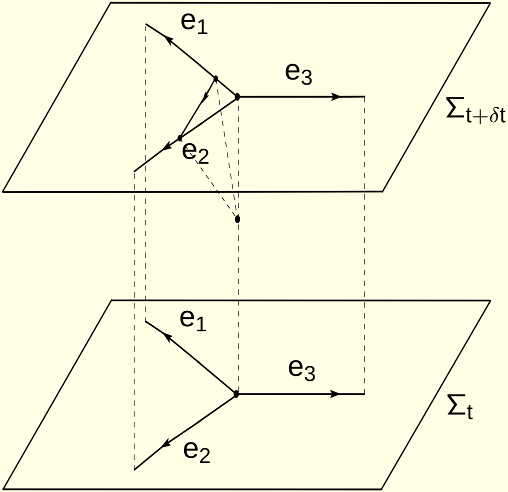
e1
e3
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Σt+δt
e1
e3
e2
Σt

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Spin Foam Models

Heuristic starting point [Reisenberger & Rovelli 96]

- Instead of using single M use ∞ number of Ham. Constraints C(x)
- Formal solutions $\Psi = \delta[C]\,\psi := \prod_x \delta(C(x))\,\psi$
- Formal physical inner product (functional integral)

$$< \Psi, \Psi' >_{\text{phys}} = < \psi,\ \delta[C]\,\psi' > = \int [dN]\ < \psi,\ \exp(iC(N))\,\psi' >_{\text{kin}}$$

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Quantum Black Hole Physics

Careful analysis: isolated horizons

[Ashtekar,Baez,Corichi,Dreyer,Engle,Fairhurst,Krasnov, Krishnan,Lewandowski,Pawlowski,Willis 97–]

- Isolated horizon = local generalisation of event horizon
- Precise defi nition leads to σ with interior H$\cong$ S^2 boundary and modifi cations of classical canonical analysis
- Most important consequences for us:
  - Interior bdry conditions make Ar(H) a Dirac observable
  - The gauge invariant surface information consists of ordered N – tuples of intersection points $(p_1, .., p_N)$, $N = 0, 1, 2, ...$ up to diffeomorphisms together with the degeneracies of corresponding (total) spin confi gurations $(j_1, .., j_N)$ on intersecting edges

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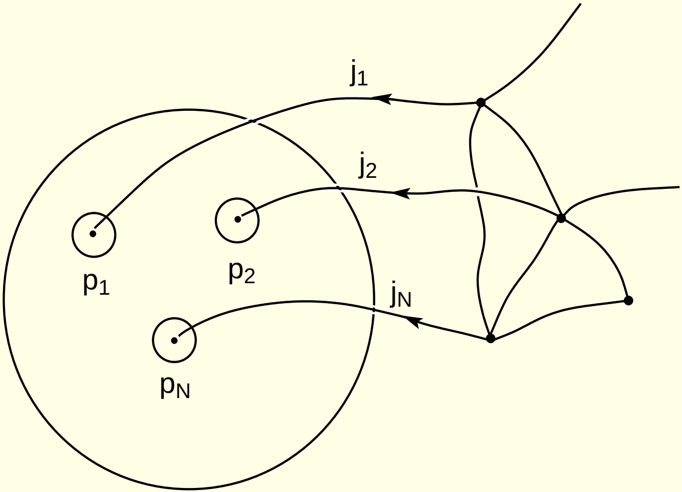
j1
j2
jN
p1
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Entropy – Calculation

[Krasnov 95], [Ashtekar,Baez,Krasnov 97], [Meissner 04], [Domagala,Lewandowski 04], [Ghosh,Mitra 06]

- Given Ar_0, count number $\mathrm{N}(\mathrm{Ar}(\mathrm{H}))$ of SNW Eigenstates T_λ of $\widehat{\mathrm{Ar}(\mathrm{H})}$ with EV $\lambda \in [\mathrm{Ar}_0 - \ell_P^2, \mathrm{Ar}_0 + \ell_P^2]$.
- $\mathrm{spec}(\widehat{\mathrm{Ar}}(\mathrm{H})) = \beta\ell_P^2 \sum_{k=1}^N \sqrt{j_k(j_k+1)}$
- In microcanon. ensemble we find

$$S(\mathrm{Ar}_0) = \ln(N(\mathrm{Ar}_0)) = \frac{\mathrm{Ar}_0}{4\hbar G} + O(\ln(\frac{\mathrm{Ar}_0}{\ell_P^2}))$$

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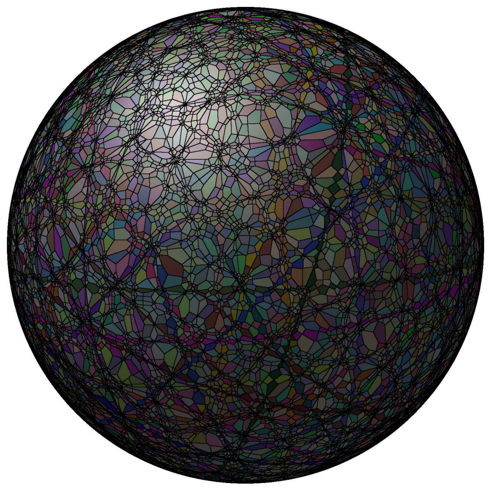
Warning: All spins contribute, no bit picture!



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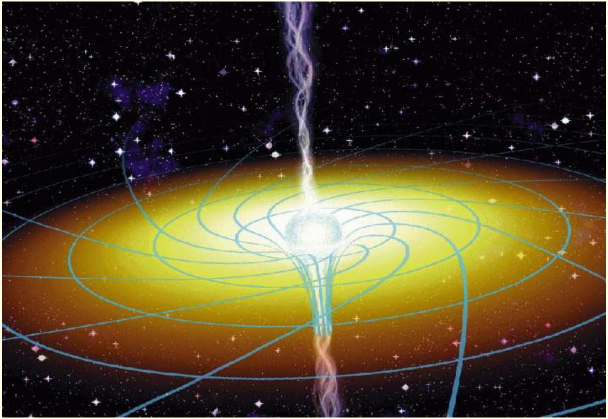
Singularity avoidance

Singularities: Divergence of Riemann Tensor $R^{(4)}_{\mu\nu\rho\sigma}$



Idea:

- Gauss – Codacci

$$\int_M d^4X \sqrt{|\det(g)|}\, R^{(4)} = \int_R dt \int_\sigma d^3x\, N \frac{\mathrm{Tr}\left(F_{ab}E^aE^b\right)}{\sqrt{|\det(E)|}} =: \int_R dt\, R(N)$$

- Consider coh. st. on $\mathcal{H}$ concentrated on class. sing. trajectory $t \mapsto (A_0(t), E_0(t))$.
- Calculate

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- Result [Brunnemann, T.T. 05]:

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