Asymptotically Safe Quantum Gravity and Cosmology

(M. Reuter)

The 4 Fundamental Interactions : theoretical status

electromagnetic, weak, strong :

- class.: Yang-Mills theories
- quant.: perturbatively renormalizable QFTh.s

gravity :

- class.: General Relativity, with $S[g_{\mu\nu}] \sim \int d^4x \sqrt{-g}\, R$
- quant.: ???

Quantum field theory of spacetime metric $g_{\mu\nu}(x)$ based upon $\int \sqrt{-g}\, R$ is not renormalizable <u>in perturbation theory</u> :

increasing order in pert. th. ⇒
" number of counter terms ⇒
" " " undetermined parameters

⤳ no / very questionable predictivity at high energies
("effective" rather than "fundamental" theory)

Standard quantization of gravity $\hat{=}$

degrees of freedom carried by:	$g_{\mu\nu}(x)$
bare action:	$\int d^4x \sqrt{-g}\, R$
calculational method:	perturbation theory in G; infinite cutoff limit at the trivial "Gaussian" fixed point

What should be given up in order to arrive at a "fundamental" or "microscopic" quantum theory of gravity?

<u>String Theory</u>: d.o.f., action, calc. meth.

<u>Loop Quantum Gravity</u>: d.o.f., calc. meth.

<u>Asymptotic Safety</u>: calc. meth., action

Asymptotic Safety Approach:

- degrees of freedom carried by $g_{\mu\nu}$
- quantization / renormalization is non-perturbative in an essential way
- bare action Γ_* is not an ad hoc assumption, but a prediction:

  $\Gamma_* \sim \int d^4x \sqrt{-g}\, R$ + "more" is a non-Gaussian fixed point of the (∞-dimensional, non-pert.) Wilsonian renormalization group flow

- fixed point "controls" UV divergences

... provided it exists

Weinberg's "asymptotic safety" conjecture (1979):

Perhaps Quantum Einstein Gravity can be defined nonperturbatively at a non-Gaussian fixed point.

$d = 2 + \varepsilon$: FP known to exist

$d = 4$: progress hampered by lack of appropriate calculational scheme

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$\rightsquigarrow$ Use "effective average action" which seems ideally suited. Wetterich 1993

Effective average action for gravity: M.R. 1996

$$\Gamma_k [g_{\mu\nu}, \cdots]$$

C. Wetterich 1993

The Effective Average Action Γ_k

- Wilson-type (coarse grained) free energy functional

- <u>IR cutoff at k:</u> Γ_k contains the effect of all quantum fluctuations with momenta $p > k$, not (yet) of those with $p < k$.

- modes with $p < k$ suppressed in the path integral by $(\text{mass})^2 = R_k(p^2)$

- $\Gamma_{k\to\infty} = S$, classical (bare) action
  $\Gamma_{k\to 0} = \Gamma$, standard effective action

- Γ_k satisfies <u>exact RG equation</u>; symbolically:

$$" \; k\,\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\,\partial_k R_k\right] \; "$$

- Powerful nonperturbative <u>approximation scheme</u>:
  "truncate" the space of action functionals, project RG flow onto finite dimensional subspace

Construction of Γ_k for Gravity

- starting point : $\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$

- decompose $\gamma_{\mu\nu} = \underbrace{\bar{g}_{\mu\nu}}_{\text{fixed backgrd. metric}} + h_{\mu\nu}$

- add background gauge fixing $S_{gf}[h;\bar{g}]$ + ghost terms

- expand $h_{\mu\nu}$ in $\bar{D}^2$-eigenmodes, and introduce IR cutoff k^2 : only modes with generalized momenta ($\bar{D}^2$-eigenvalues) are integrated out.

- add sources: generating fctl. $W_k[\text{sources};\bar{g}]$

$$\downarrow \text{Legendre transf.}$$

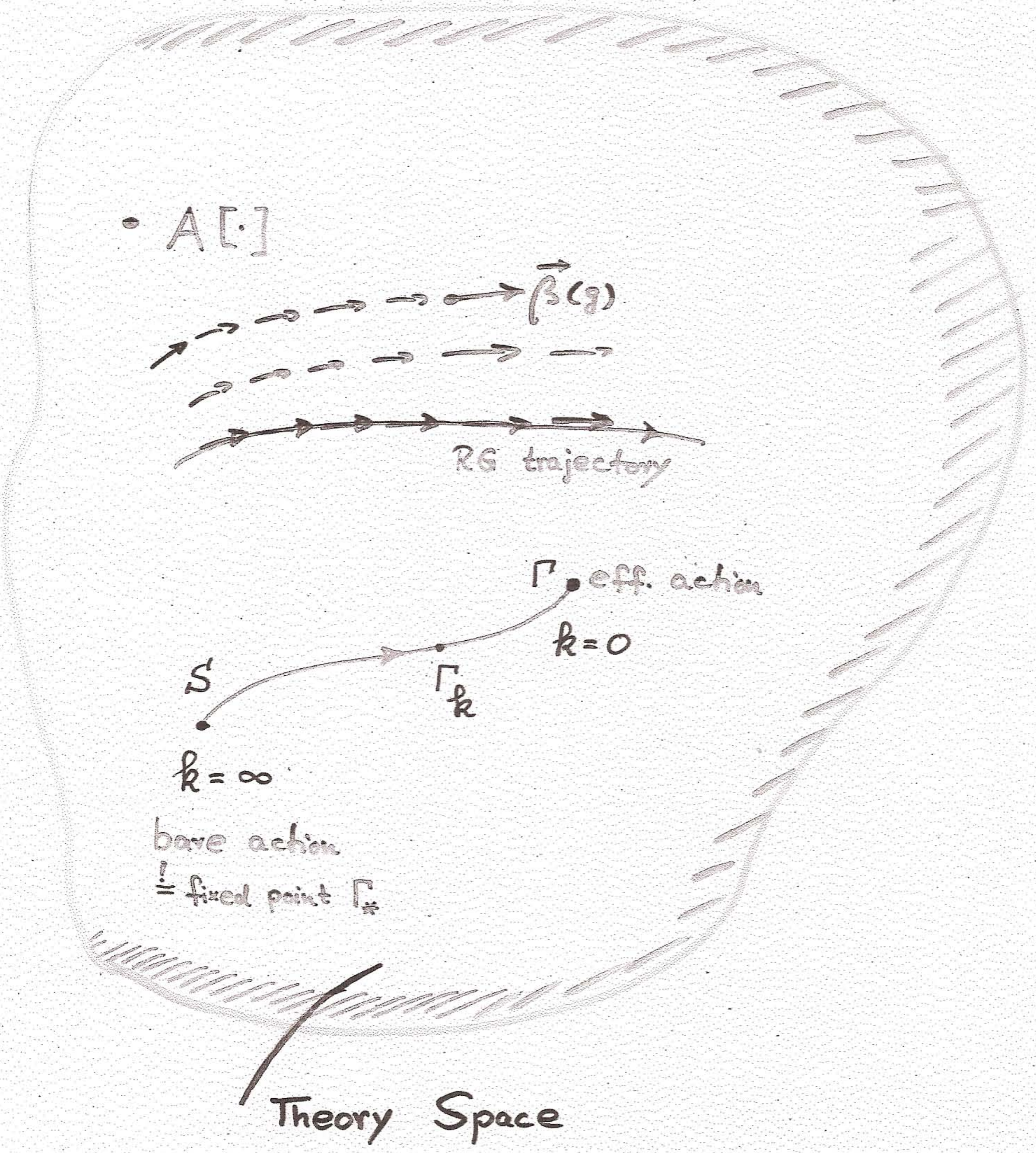
$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle$ $\qquad$ $\Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$

- derive exact RG equation from path integral:

$$k\frac{\partial}{\partial k}\Gamma_k[g,\bar{g},\cdots] = \mathrm{Tr}(\cdots) \qquad \begin{array}{l}\Gamma_\infty = S\\ \Gamma_0 = \Gamma\end{array}$$

- "Ordinary" diffeomorphism invariant action:

$$\bar{\Gamma}_k[g] = \Gamma_k[g, \bar{g}=g, \text{ghosts}=0]$$



A[·]
$\vec{\beta}(g)$
RG trajectory
Γ eff. action
$k=0$
S
Γ_k
$k=\infty$
bare action
$\overset{!}{=}$ fixed point Γ_*
Theory Space

The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$$\Lambda_k \equiv \bar{\lambda}_k$$

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left\{ R - 2\Lambda_k \right\}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant G_k, dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k, dimensionless: $\lambda(k) = \Lambda_k / k^2$

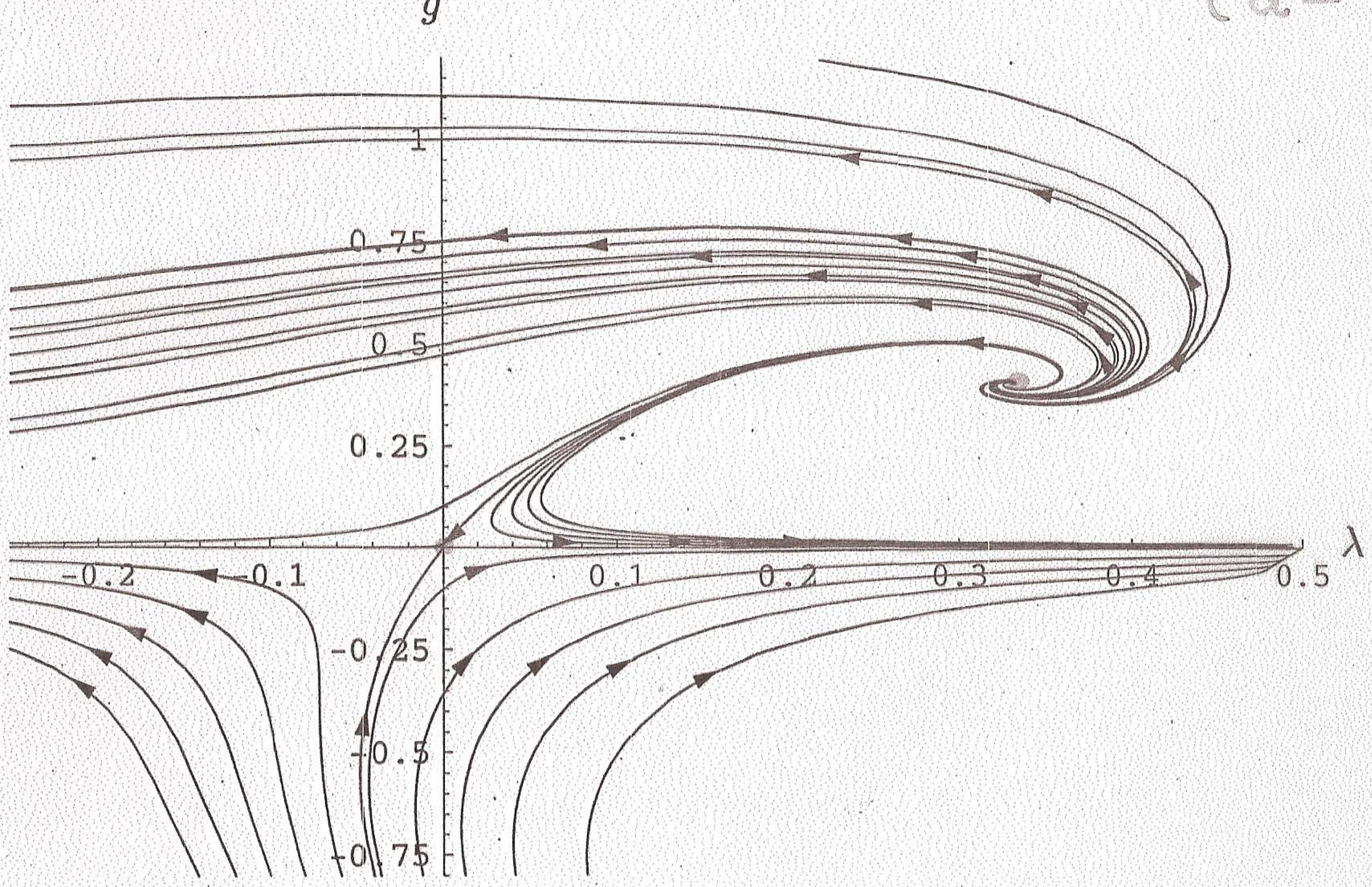
insert ansatz into flow equation, expand

$$\mathrm{Tr}[\cdots] = (\cdots) \int \sqrt{g} + (\cdots) \int \sqrt{g} R + \cdots$$

$$\rightsquigarrow \quad \boxed{\begin{aligned} k\partial_k g(k) &= \beta_g(g,\lambda) \\ k\partial_k \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

RG - Flow in the Einstein - Hilbert Truncation

($d=4$)



M.R., F. Saueressig, hep-th/0110054

Reliability checks (universality, ...),
more general truncations $\Longrightarrow$

The NGFP seems to exist in
the full un-truncated theory
beyond any reasonable doubt.

Properties of QEG

- Background-independent quantization scheme:

No special <u>metric</u> plays any distinguished role!

<u>The background field method</u>:

a) Fix arbitrary $\bar{g}_{\mu\nu}$

b) Quantize (nonlinear) fluctuations $h_{\mu\nu} \equiv \gamma_{\mu\nu} - \bar{g}_{\mu\nu}$ in the backgrd. of $\bar{g}_{\mu\nu}$

c) Adjust $\bar{g}_{\mu\nu}$ such that $\langle h_{\mu\nu} \rangle = 0$

$$\rightsquigarrow \quad g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu}$$

- Fundamental action $S \approx \Gamma_*$ is a prediction:

No special <u>action</u> plays any distinguished role!

<u>The only input</u>: field contents + symmetries $\hat{=}$ theory space

<u>The output</u>: $\Gamma_* = S_{\text{Einstein-Hilbert}}$ + "more"

Einstein-Hilbert action is often a reliable approximation, but not distinguished conceptually.

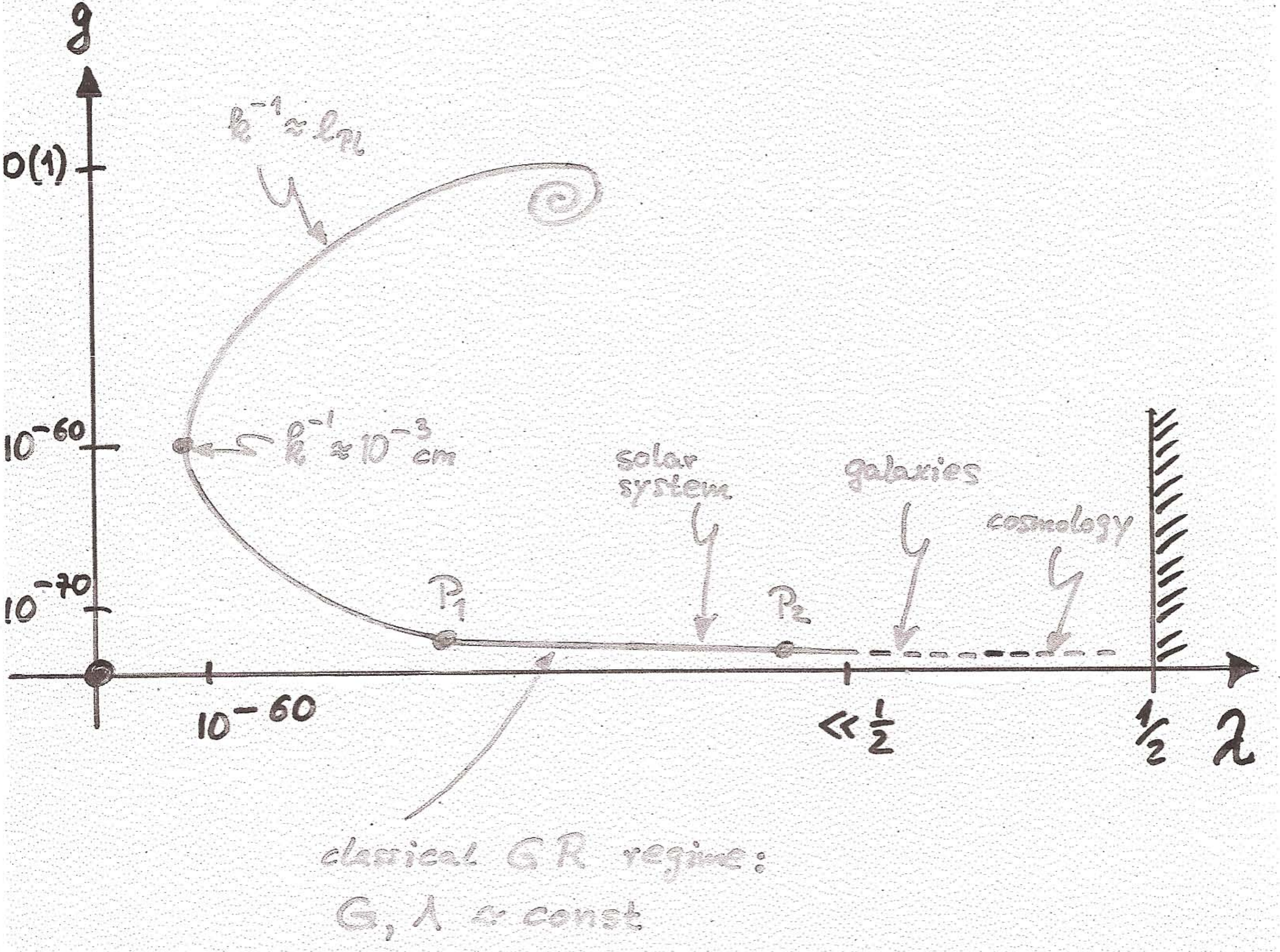
- Combination average action + background method successfully tested in QED and Yang-Mills theory.

- QEG reproduces successes of classical General Relativity: $\exists$ trajectories with long classical regime ($G = \text{const}$, $\Lambda = \text{const}$)

- QEG reproduces results of "QFTh. in curved spacetimes" in the classical regime: Hawking radiation, cosmological particle creation, ...

- Coexistence Asymptotic Safety $\leftrightarrow$ perturbative non-renormalizability well understood; A.S. tested in models (Gross-Neveu, ...)
- Consistent quantization of gravity seems not to require "fine tuning" of matter system, special symmetries (SUSY, etc.), or unification with the other fundamental forces of Nature.

The RG trajectory "realized in Nature"

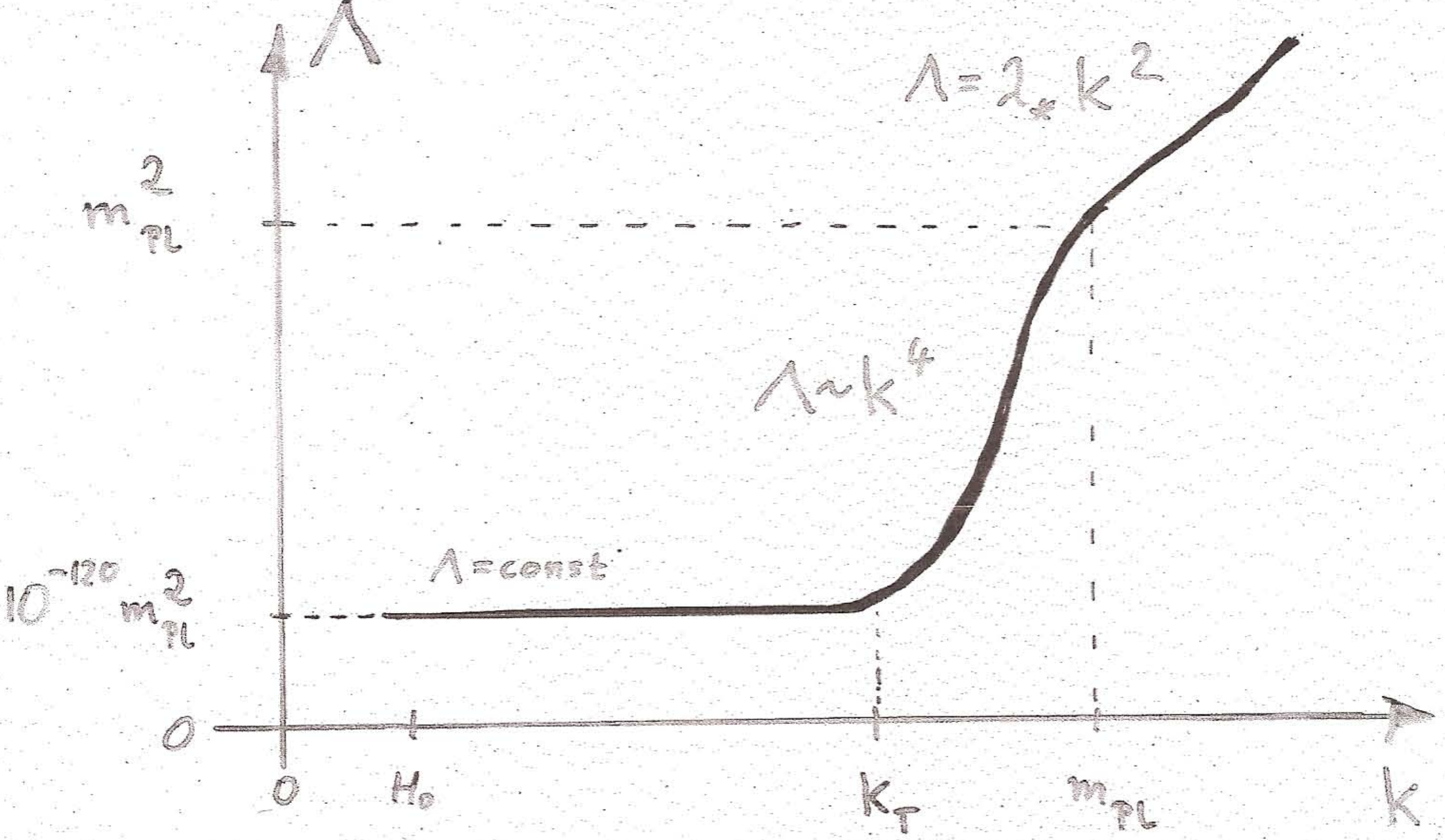


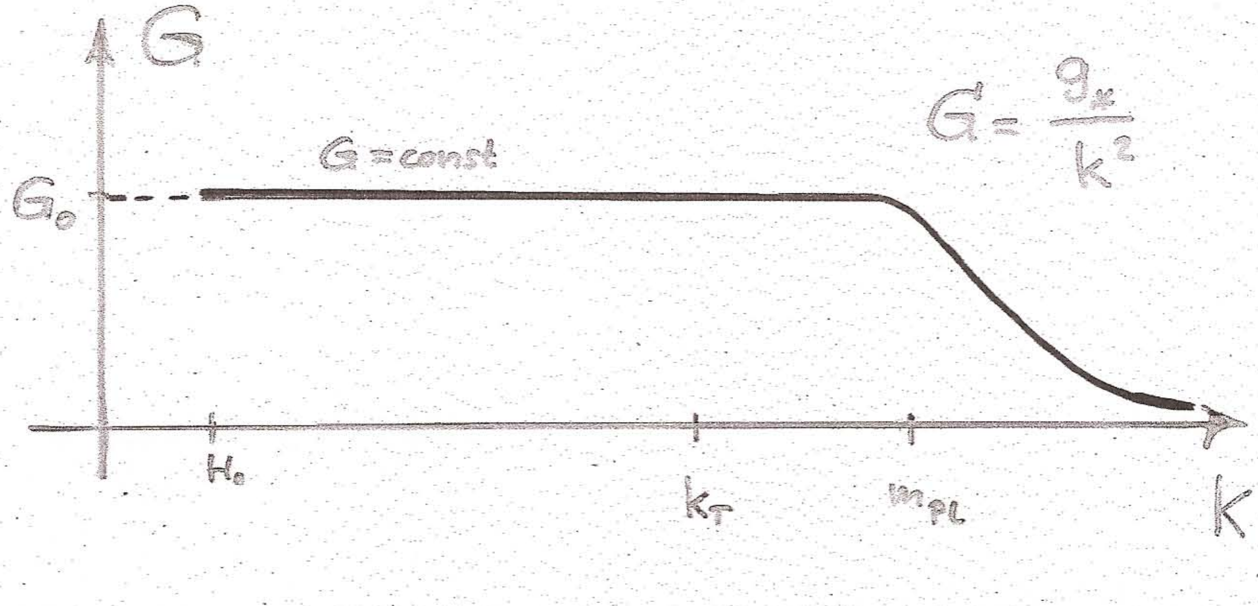
"Today" in cosmology:

$$\lambda_{cosmo} \equiv \frac{\Lambda_{cosmo}}{k^2_{cosmo}} = O(1) \;!!!$$

$\Lambda_{cosmo} \approx H_0^2$

$k^2_{cosmo} \approx H_0^2$





Definition of Planck scale: $m_{Pl} \equiv \ell_{Pl} \equiv G_0^{-\frac{1}{2}}$

Are there observable / observed physical phenomena related to the RG-running implied by QEG ?

Candidates in cosmology:

a) <u>Entropy carried by cosmological matter</u> (CMBR photons, ...)

$$\left[S_{CMBR} / \text{Hubble volume}\right]_{\text{today}} \approx 10^{88} \gg 1$$

most plausible initial value $= O(1)$!

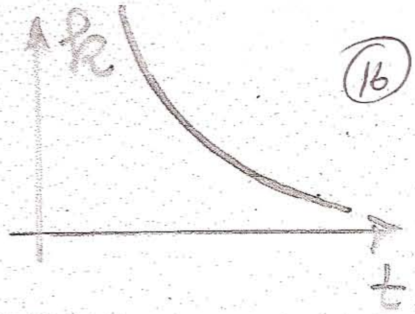
b) <u>Automatic inflation in the NGFP regime</u>

no inflaton needed,
no reheating necessary;
$\Lambda(k)$ large: cosmolog. const. drives inflation
$\Lambda(k)$ small: inflation stops automatically

c) <u>Generation of primordial density perturbations</u>

spectrum scale free:
big bang = "critical phenomenon" governed by the NGFP

For monotonic cosmological cutoff identification $k = k(t)$



and  below the NGFP regime:

$$\boxed{\Lambda(t) \equiv \Lambda\big(k = k(t)\big) \text{ is a positive and decreasing function of time.}}$$

$\Longrightarrow$

$$\boxed{\text{Energy transfer into the matter system ("heating up").}}$$

Example: de Sitter plus test particle

$$ds^2 = -\left[1 + 2\phi_N(r)\right] dt^2 + \left[1 + 2\phi_N(r)\right]^{-1} dr^2 + r^2 d\Omega^2$$

$$\phi_N(r) = -\frac{1}{6}\Lambda\, r^2$$

$\Lambda > 0$:

Newtonian interpretation:

Decreasing Λ increases the test particle's potential energy!

RG-Improved Cosmology

Spatially flat RW geometry:

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)\right]$$

$$T_\mu{}^\nu = \mathrm{diag}\left[-\rho(t), p(t), p(t), p(t)\right]$$

The scale factor is determined by

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda(t)\, g_{\mu\nu} + 8\pi G(t)\, T_{\mu\nu}}$$

where $\Lambda(t) \equiv \Lambda(k = k(t))$, $G(t) \equiv G(k = k(t))$

"Improved" Einstein's eq. $\Longleftrightarrow$

modified Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G(t)\, \rho + \frac{1}{3}\Lambda(t)$$

modified conservation law $D^\mu\left[-\Lambda(t) g_{\mu\nu} + 8\pi G(t) T_{\mu\nu}\right] = 0$:

$$\underbrace{\dot{\rho} + 3H(\rho + p)}_{\sim D^\mu T_{\mu 0}} = -\frac{\dot{\Lambda} + 8\pi \rho \dot{G}}{8\pi G}$$

Describes energy exchange between matter and fields Λ, G ("vacuum").

A special case:

Separate conservation of matter and vacuum energy

$$\begin{cases} \dot{\rho} + 3H(\rho + p) = 0 & \text{, i.e. } D^{\mu} T_{\mu\nu} = 0 \\ \dot{\Lambda} + 8\pi\rho\,\dot{G} = 0 & \text{"consistency condition"} \end{cases}$$

A. Bonanno, M.R. (2002)

M.R., F. Saueressig (2005)

General case:

No separate conservation

A. Bonanno, M.R.

hep-th/0706.0174

Entropy Generation

modified conservation law $\Longrightarrow$

$$\frac{d}{dt}(\rho a^3) + p \frac{d}{dt}(a^3) = \widetilde{\mathcal{P}}(t)$$

with $$\widetilde{\mathcal{P}}(t) \equiv -\left(\frac{\dot{\Lambda} + 8\pi\rho\dot{G}}{8\pi G}\right) a^3$$

Cosmological fluid within unit comoving volume:

proper volume: $V = a^3$

contains energy $U = \rho a^3$

and entropy $S = s a^3$. $\Longrightarrow$

$$\boxed{\frac{dU}{dt} + p\frac{dV}{dt} = \widetilde{\mathcal{P}}}$$

$dU + p\,dV = T\,dS \Rightarrow$ $\boxed{\widetilde{\mathcal{P}} = T\frac{dS}{dt}}$

Classical FRW: $\widetilde{\mathcal{P}} = 0$, $\frac{d}{dt}S = 0$, adiabat. expans.

Improved cosmology: Comoving entropy changes as a consequence of the RG effects

$$\boxed{\frac{d}{dt}S \equiv \frac{d}{dt}(s a^3) = \mathcal{P}} \text{ with } \boxed{\mathcal{P} \equiv \frac{\widetilde{\mathcal{P}}}{T}}$$

Rate of entropy production:

$$\mathcal{P}(t) = -\left(\frac{\dot{\Lambda} + 8\pi\rho\dot{G}}{8\pi G}\right) a^3 \cdot \frac{1}{T}$$

Need <u>non-equilibrium</u> relationship $T \leftrightarrow p, \rho$!

The Postulate:

Entropy generation disturbs equilibrium as little as possible, expansion is "almost adiabatic".

→ Lima (1997)

Concrete assumption about the non-equilibrium thermodynamics of the cosmolog. fluid:

> The matter system consists of
>
> $n_{eff} \equiv n_b + \frac{7}{8} n_f$ species of massless d.o.f.,
>
> all at the same temperature T, with
>
> equation of state $p = \frac{1}{3}\rho$ and
>
> $$\rho = \rho(T) = \kappa^4 T^4 \,, \quad \kappa^4 \equiv \frac{\pi^2}{30} n_{eff} \,.$$
>
> No assumption is made about $s = s(T)$.

... as in equilibrium !

The postulate implies:

$$\mathcal{P} \equiv \frac{1}{T}\,\tilde{\mathcal{P}} = \frac{1}{T}\left[\frac{d}{dt}(\rho a^3) + p\,\frac{d}{dt}(a^3)\right]$$

$T = \rho^{1/4}/\varkappa \qquad p = \rho/3$

$\vdots$

$$= \frac{d}{dt}\Big[\underbrace{\tfrac{4}{3}\,\varkappa\, a^3 \rho^{3/4}}_{= S = s a^3}\Big]$$

integrate: $\boxed{S(t) = \frac{4}{3}\varkappa\, a^3 \rho^{3/4} + S_c}$

the relevant solutions have $S(t=0) = S_c$

$$\boxed{s(t) = \frac{2\pi^2}{45}\, n_{\text{eff}}\, T(t)^3 + \frac{S_c}{a^3}}$$

For $S_c = 0$ exactly the proper entropy density of radiation in equilibrium! $\Rightarrow$

> The "heat transfer" into the matter system caused by the RG running can account for the entire entropy carried by the massless fields today.

In particular S_{CMBR} / Hubble vol. $\approx 10^{88}$ can envolve from S(big bang) = 0 without any standard dissipative process.

Solutions to the improved Einstein eq.

(1) Solve RG eqs. for EH-truncation with realistic parameter values $g_T = 10^{-60}$, $k_T = 10^{-30}\, m_{Pl}$

(2) Solve imp. Einstein eq. with $k(t) \sim H(t)$ and $p = w\, \rho$.

Example: NGFP regime $\begin{cases} \Lambda(k) = \lambda_* k^2 \\ G(k) = g_*/k^2 \end{cases}$

1-parameter family of solutions labeled by $\Omega_\Lambda^* \in (0,1)$:

$$\boxed{\begin{aligned} a(t) &\sim t^\alpha, \quad \alpha = \frac{2}{(3+3w)(1-\Omega_\Lambda^*)} \\ \rho(t) &= \frac{2\,\Omega_\Lambda^*}{9\pi\, g_* \lambda_* (1+w)^4 (1-\Omega_\Lambda^*)^3} \cdot \frac{1}{t^4} \\ G(t) &= \frac{3\, g_* \lambda_* (1+w)^2 (1-\Omega_\Lambda^*)^2}{4\,\Omega_\Lambda^*} \cdot t^2 \\ \Lambda(t) &= \frac{4\,\Omega_\Lambda^*}{3(1+w)^2(1-\Omega_\Lambda^*)^2} \cdot \frac{1}{t^2} \end{aligned}}$$

$\rightsquigarrow \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}} = \text{const} = \Omega_\Lambda^*, \; \Omega_M = 1 - \Omega_\Lambda$

Properties of the NGFP solutions:

$\Omega_\Lambda^* \in (\frac{1}{2}, 1)$:

$$\boxed{a(t) \sim t^\alpha, \quad \alpha > 1}$$

- accelerated expansion, "power law inflation"
- Λ-dominated: $\Omega_\Lambda > \Omega_M = 1 - \Omega_\Lambda$
- $\mathcal{P} > 0$, $\mathcal{S}(t \to 0) = \mathcal{S}_c \quad (= 0)$
- no particle horizon
- $\alpha \to \infty$ for $\Omega_\Lambda^* \nearrow 1$: $\approx$ de Sitter

$\Omega_\Lambda^* = \frac{1}{2}$:

$$\boxed{a(t) \sim t}$$

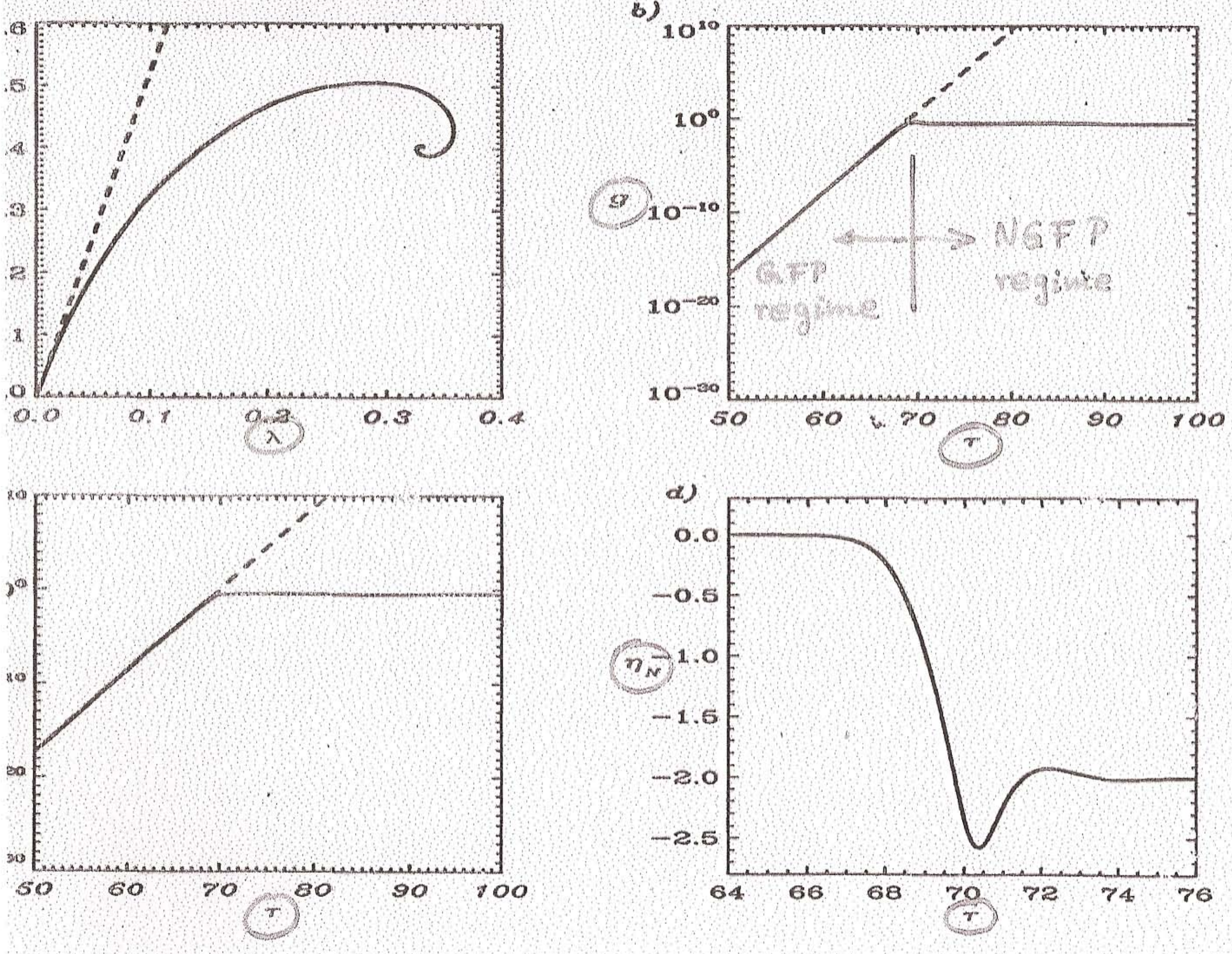
- $\Omega_M = \Omega_\Lambda$
- $\mathcal{P} = 0$; $D^\mu T_{\mu\nu} = 0$, $\dot{\Lambda} + 8\pi \rho \dot{G} = 0$
- no particle horizon

$\Omega_\Lambda^* \in (0, \frac{1}{2})$:

$$\boxed{a(t) \sim t^\alpha, \quad \alpha < 1}$$

- decelerated expansion
- matter dominated: $\Omega_M > \Omega_\Lambda$
- $P < 0$, entropy decreases,
  $S(t \to 0) = +\infty$

The RG-trajectory with realistic parameter values



$$g_T = 10^{-60}$$

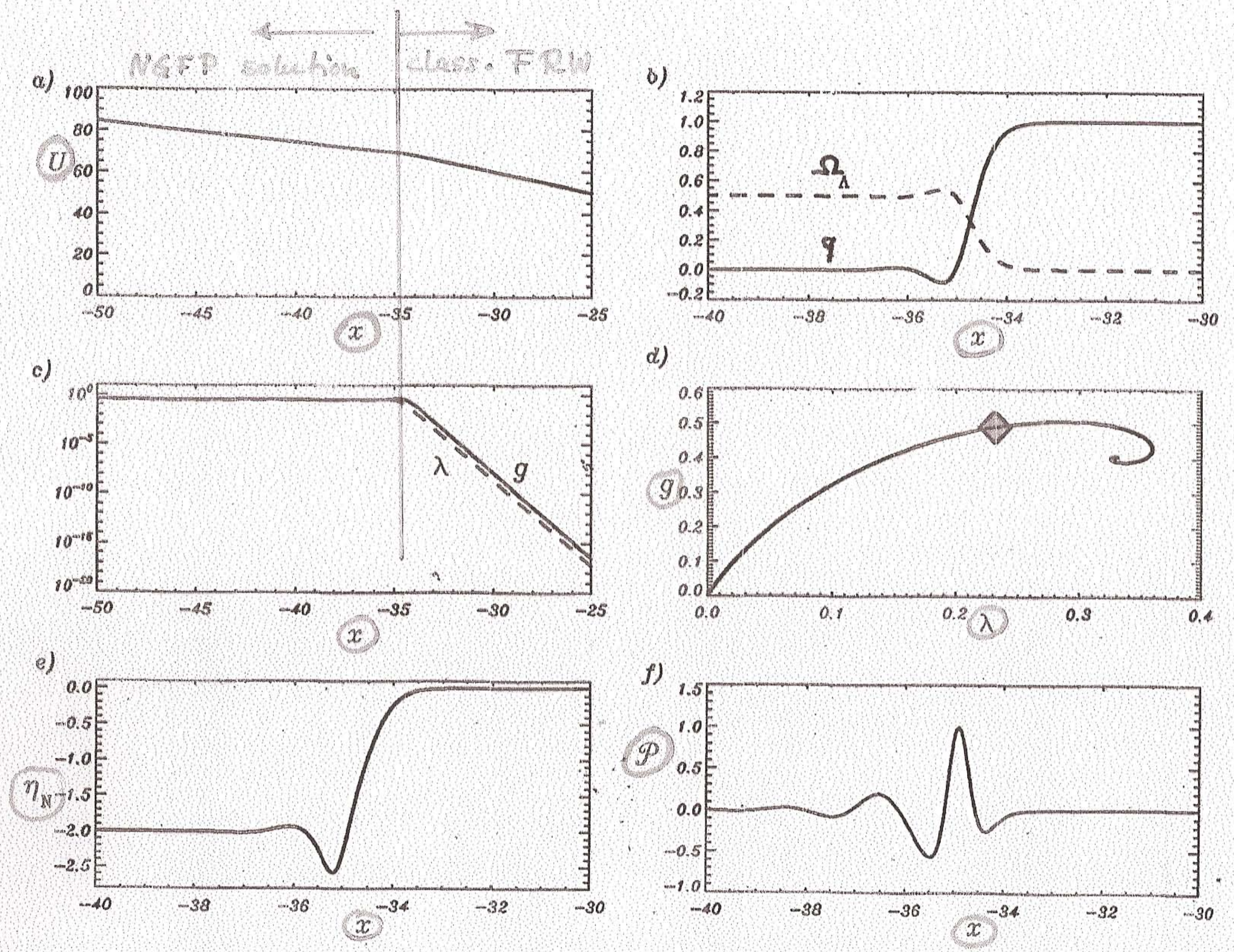
$$k_T = 10^{-30}\, m_{Pl}$$

$$\tau \equiv \ln\left(\frac{k}{k_T}\right)$$

Complete Cosmology: $\boxed{\Omega_\Lambda^* = \frac{1}{2}}$ $(\alpha = 1)$

$U = \ln \frac{H}{H_T}$

$x = \ln \frac{a}{a_T}$

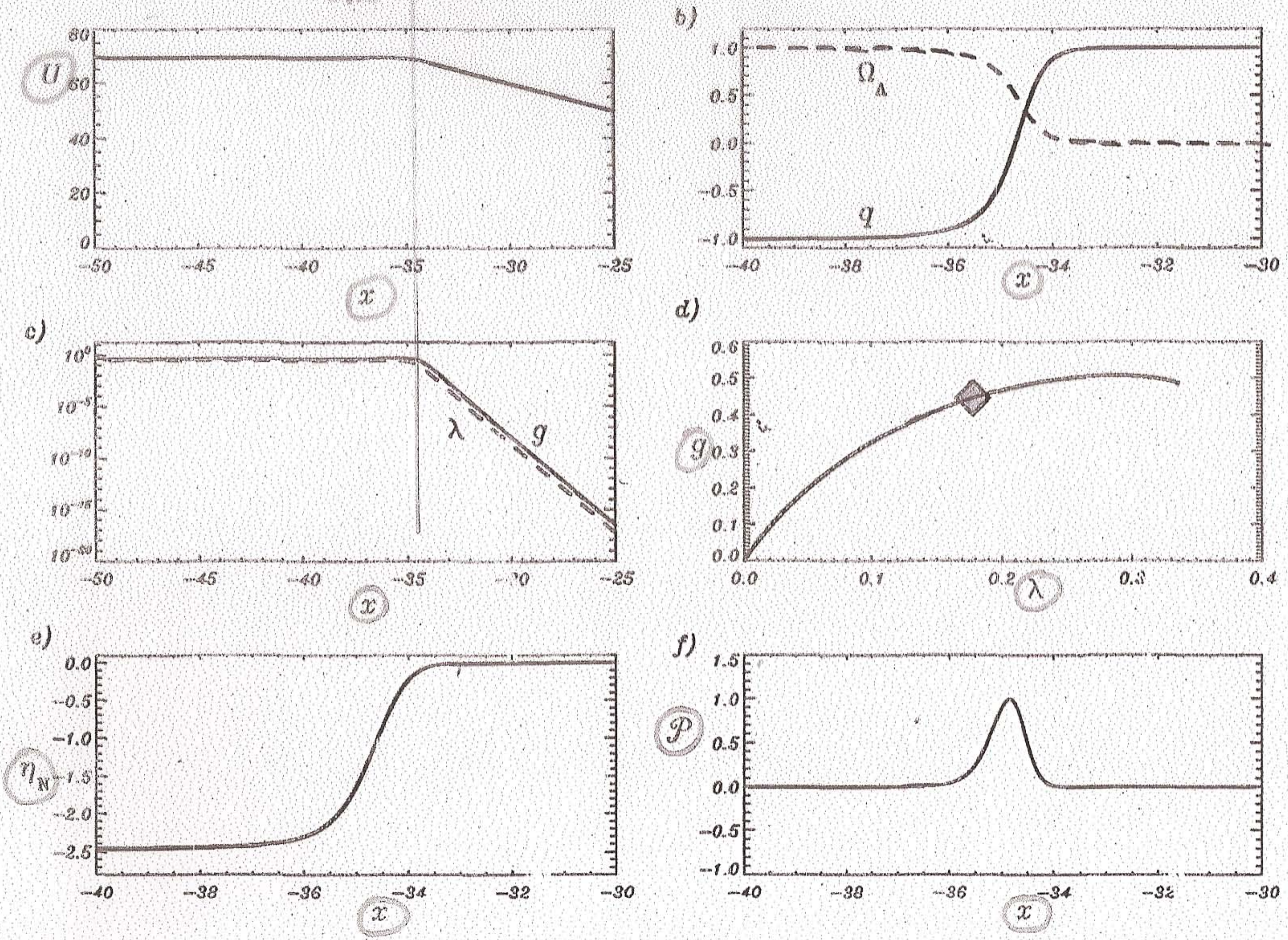


Complete Cosmology : $\boxed{\Omega_\Lambda^* = 0.98}$ $(\alpha = 25)$

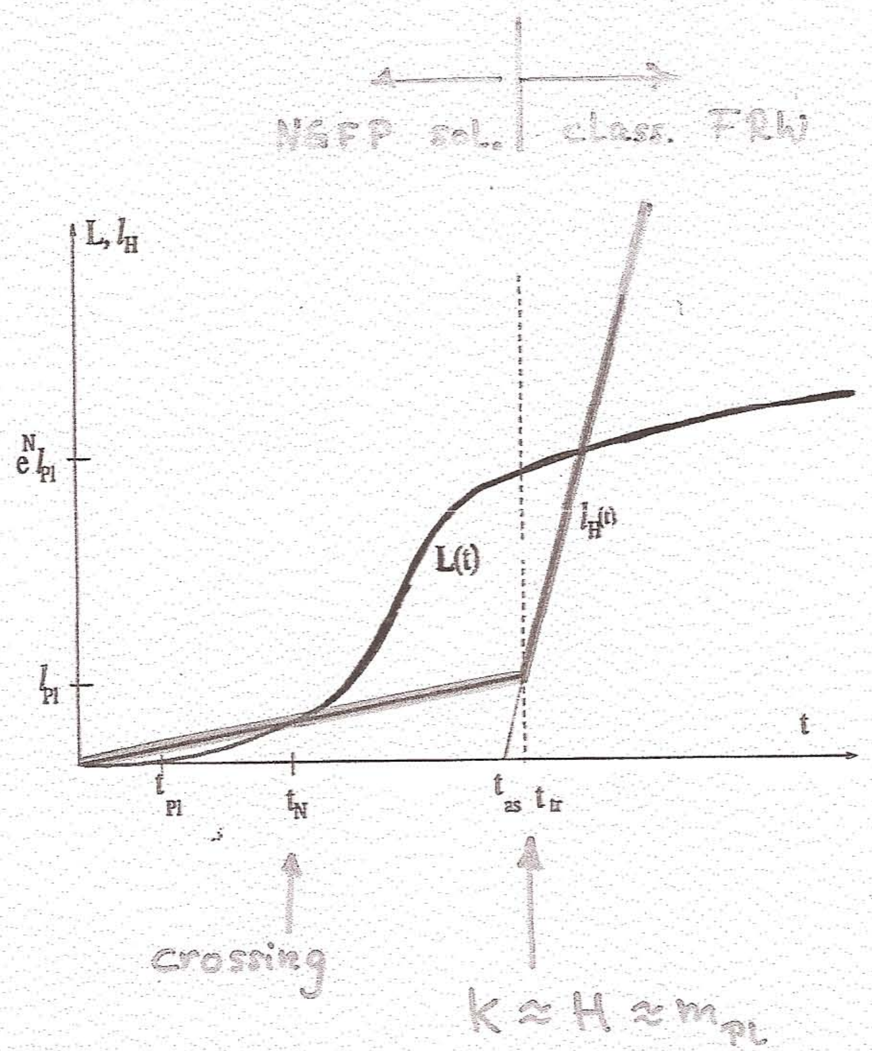
NGFP sol. $\approx dS$ | class. FRW with $\Omega_\Lambda^{FRW} \approx 0$

$U \equiv \ln \frac{H}{H_T}$

$x \equiv \ln \frac{a}{a_T}$



$\alpha \gg 1$:



Hubble radius: $\boxed{\ell_H(t) \equiv \frac{1}{H(t)}}$

Proper length of structure with comoving length Δx:

$$\boxed{L(t) = \Delta x \cdot a(t)}$$

example: $\Omega_\Lambda^* = 0.98$, $\alpha = 25$

$$t_{tr} = 25\, t_{Pl}$$

$$N = 60 \rightsquigarrow t_{60} = 2.05\, t_{Pl}$$
$$= t_{tr}/12.2$$

Generating Primordial Density Perturbations

Decoherence Scenario:

Classical fluctuations $\delta\rho(\vec{x})$ originate from quantum fluctuations of the curvature, $\delta R^{\cdot\cdot}_{\cdots}(\vec{x})$:

$$\langle\langle \delta\rho(\vec{x})\,\delta\rho(0)\rangle\rangle$$

$$\propto \langle \delta R^{\cdot\cdot}_{\cdots}(\vec{x})\,\delta R^{\cdot\cdot}_{\cdots}(0)\rangle \propto \frac{1}{|\vec{x}|^4}$$

dictated by properties of the UV-FP ($\eta_* = -2$)

for $a|\vec{x}| \ll l_H$

$\Rightarrow$ $\delta\rho$ - power spectrum for sub-Hubble scale modes is scale invariant!

($n = 1$, "Harrison - Zeldovich")

These modes cross the Hubble radius and become "super-Hubble" in the NGFP cosmologies with $\alpha > 1$.
they can act as seed for structure formation.

Conclusion

Cosmology is a natural laboratory for confronting the predictions of asymptotically safe QEG with observations.

Candidates for observable / observed phenomena possibly explained by QEG:

- entropy of matter
- primordial density perturbations
  - NGFP regime is a "critical phenomenon"
  - Λ-driven inflation
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