Note Title 6/25/2007

Background Independence in String Theory

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Outline:

1. Introduction: role and definition of BI, outline of talk.

2. Background (in)dependence in perturbative ST.

3. Interlude: gauge theory dualities, role of gauge invariance.

4. Non-perturbative strings: duality and holography.

5. Holographic dualities and BI.

6. Conclusions and outlook.

1 Background Independence (BI) has provided guidance in quantum gravity research, including in String Theory. It will probably continue being an important issue.

As we learned more about ST and QFT, especially non-perturbatively, the concept of BI (or more generally gauge invariance) has been transformed.

In particular, we will concentrate on the two concepts of duality and holography. They both alter the way we think about quantum physics, including the question of BI.

Questions to keep in mind:

- What precisely is BI?

- Is it property of the formulation or of the physics?
- What do we need it for?

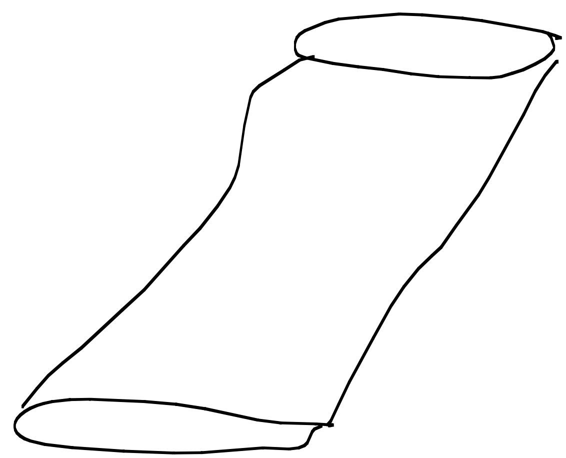
how much of it we need?

how much is too much?
- What are the ways of realizing BI?

We'll see some evolution in range of possibilities.

2 In the beginning ... perturbative strings (1984-1990)

Best way of visualizing ST: it is defined as theory on 1-dimensional extended objects moving in given background.



So we have: $X^\mu(\sigma, \tau) : \Sigma \longrightarrow M$

$\uparrow$ world sheet $\qquad \uparrow$ spacetime

$$\text{action} = \frac{1}{4\pi\alpha'} \int d\sigma\, d\tau\; G_{\mu\nu}(X)\, \partial_\alpha X^\mu\, \partial_\beta X^\nu\, \delta^{\alpha\beta}$$

(Euclidean world-sheet, conformal gauge)

Full story is much more intricate, but this will do for now (bckd. $\neq$ geometry.).

Is it BI?

$G_{\mu\nu}(x)$ is the background spacetime metric, appearing explicitely in the formulation. This therefore sounds a lot like background-dependence ... but what do we mean precisely by that?

In fact we have generalization of GR, formulated in background-field gauge. All physical observables are shown to be diff. invariant: when changing them and $G_{\mu\nu}(x)$ by diff., all physical results are same.

We don't usually call GR (or gauge theory) background-dependent just because we chose a gauge.

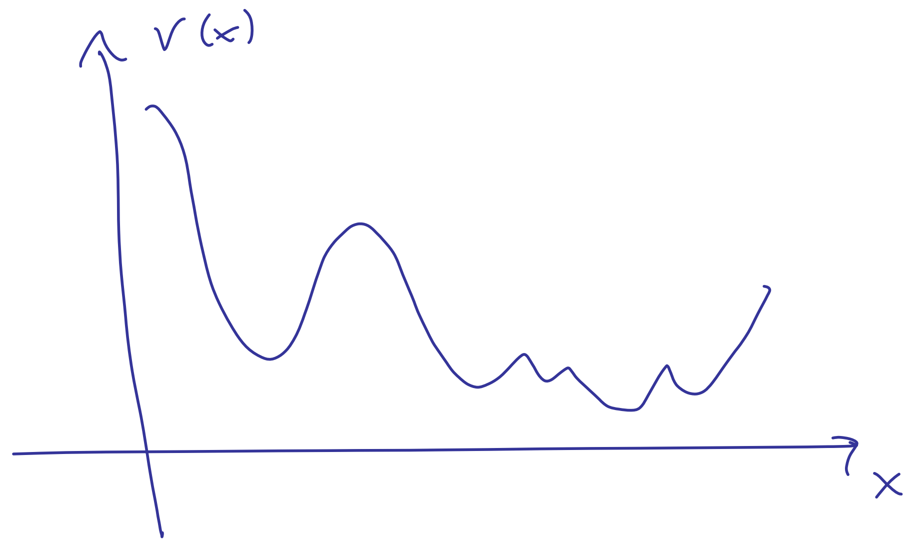
Rather, we say that we have an inconvenient formulation, in which BI is not manifest.

Indeed, in string PT, BI is <u>not</u> manifest. Therefore results which require more than perturbations about one background are difficult to obtain. This is a <u>practical</u> issue, and a serious one.

Analogy:

particle (x) in

potential $V(x)$.



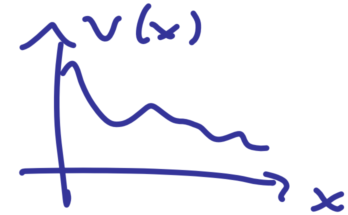
String PT is analogous to expanding around any extremum. Questions that are not easily accessible are, for example, which is the true minimum?

Conclusion: we want formulation of ST which treats all possible configurations equally. Such manifest BI formulation will be extremely useful, if it exists.

This may have been the expectation in that period: we'd ...

... have BI non-perturbative formulation (e.g. String Field Theory) which will make things like vacuum selection easier.

There are already caveats, e.g:



even in the simplest context we don't have selection principle — no reason we live in global minimum, but let's brush that aside for now ...

This was then ... fast forward two decades,

what have we learned?.

Non-Perturbative string & field theory:

We now understand much better the non-perturbative structure of many quantum theories, with or without gravity.

The main new ingredient is that of duality.
It changes radically how we think about things like gauge freedom and General Covariance.

I'll start with ordinary QFT without gravity; later we'll come back to quantum gravity.

3 Cautionary Tale : quantum gauge theory

Take for example $SU(n_c)$ gauge theory with n_f flavors (quarks + anti quarks). Since we like calculating things add $N=1$ SUSY. Assume also for simplicity

$$\frac{1}{3} n_f < n_c < \frac{2}{3} n_f$$

In perturbation theory this is the theory of quarks and gluons. In other words the Hilbert space of the theory is some large Fock space $\mathcal{H}_L$, generated by acting with creation ops. on vacuum, modded out by gauge transf.

$$\mathcal{H}_{phys} = \mathcal{H}_L \,/\, \text{constraints}$$

(Gauss Law).

<u>but</u> ...

- only states in $\mathcal{H}_{phys}$ are physical (Hadrons).

- only operators commuting with constraints are measurable.

What is the role of all this auxiliary structure?

large Fock space $\mathcal{H}_L$, gauge redundancies, constraints ...

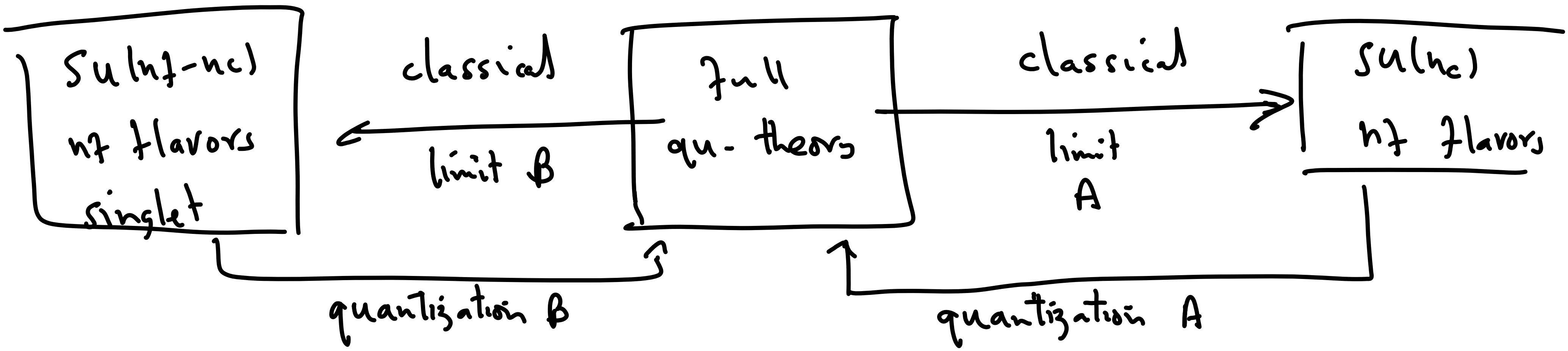
We now suspect that these have to do with perturbation theory only, in other words with specific classical limit of the quantum theory...

This is due to the phenomena of <u>duality</u>.

It turns out there's another gauge theory, which upon quantization yields precisely same (low energy) physics. This is $SU(n_f - n_c)$ + n_f flavors + singlet.

[This is low energy duality, examples of full duality also exist].

schematic picture:



This is an example, out of many, of duality. Same physics can be formulated in different variables, each more convenient in a particular classical limit.

The existence of dualities was known since the 1970s, but was extended and applied to more realistic models in 90s.

This forces one to distinguish between physical properties, and mathematical properties of particular set of variables, inherently tied to particular set of variables.

So, how about gauge invariance?

look at the $SU(n_c)$ gauge redundancy: in both formulations all physical objects are gauge invariant. This however is acheived in two fundam. different ways:

On one side variables transform non-trivially, one needs to impose Gauss law constraints.

On other side all variables are already gauge inv.

Summary:

The same quantum theory can have many classical limits, or in other words obtained by "quantization" of different classical theories.

The auxiliary mathematical objects one uses may vary between formulations, physical quantities will be the same. in particular:

Gauge symmetries appear to be closely associated with a specific classical limit, <u>not</u> of fundamental importance. In dual formulation they are realized <u>trivially</u>: everything is already gauge invariant.

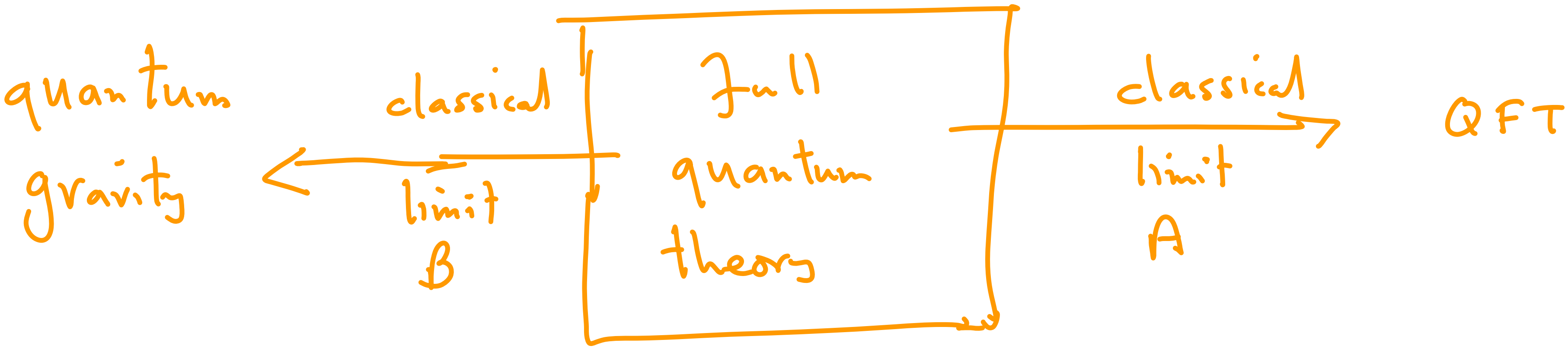
(but why are they so ubiquitous? more later)

Now back to our feature presentation.

Non-Perturbative String Theory:

We saw that gauge-gauge dualities suggest that in different formulations of quantum gauge theory, gauge freedom can be realized differently: by Gauss law constraints, or much more trivially, if everything in sight is already gauge invariant.

Ditto for diffeomorphism invariance.



we have now many examples of dualities where one of the sides is gravitational, the other is not.

For reasons discussed below these are called <u>holographic</u> dualities

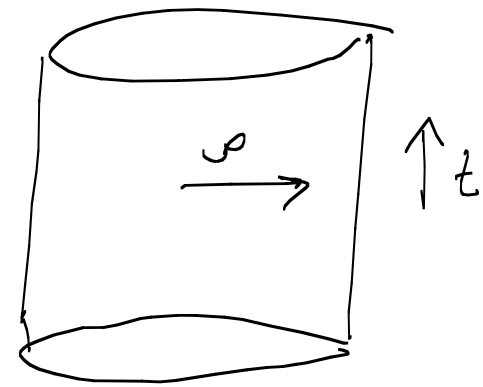
I'll discuss only AdS/CFT, but other examples are matrix models, LST, BFSS, and more general gauge-gravity dualities with less symmetry and in various dimensions.

AdS/CFT in a (small) nutshell

Let us study quantum gravity with $\Lambda < 0$. No other restriction on metric; this is not "quantization of ST on AdS space". We do fix Λ, later we'll worry if this is a serious restriction.

Far away from all sources (asymptotically) the metric approaches that of pure AdS space.

$$ds^2 = -ch^2(\rho)\, dt^2 + d\rho^2 + sh^2(\rho)\, d\Omega^2$$



AdS space has a boundary, conformal to $S^3 \times \mathbb{R}_t$. The model is not complete before we specify boundary conditions.

Schematically for every field

$$\Phi(\rho, S^3, t) \xrightarrow[\rho \to \infty]{} (\ldots)\, J(S^3, t)$$

(Important subtelties to do with infinities, well-understood)

What are the diff. invariant observables?

The only ones that are known is the partition function (or more generally correlation functions) as function of b.c $Z(J)$

Comments:

- So far nothing is specific to ST, Indeed, could be interesting to compare notes ...
- Boundary conditions can break all symm., be time-dependent etc.

- We fix the condition at $\rho \to \infty$, but in the bulk absolutely <u>anything</u> can happen. Λ can be as small as we'd like, so for local processes this restriction is very mild. For global issues (e.g. cosmology) the restriction is no so mild...

AdS / CFT:

If your QG theory is string theory, turns out another way of calculating $Z(J)$, which packages all the diff. invariant observables. This in terms of gauge theory

$$Z(J) = \int \mathcal{D}A \; \underbrace{\ldots}_{\text{partners}} \; e^{-S(A,\ldots)} \, e^{-\int J \theta}$$

$$S(A) = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \ldots \right)$$

J are now <u>sources</u> coupled to gauge inv. operators, there's one for each bulk (string) field. For example the bdy value of dilaton sources $\mathrm{tr}\,(F_{\mu\nu} F^{\mu\nu})$.

(basic requirement for <u>duality</u>, not determined by symmetries).

The gauge theory is defined on $S^3 \times \mathbb{R}$, the conformal bdy: it is co-dimension 1; <u>holographic</u> duality

Comments:

- One of many instances of duality with grav. theory; the dual theory is always non-gravitational, defined on lower dimensional surface, vaguely associated with spacetime asymptotic region. Holography.

- Lots of evidence, does not rely on SUSY or conf. inv.; goes well beyond SUGRA limit, uses specific features of ST (0-branes, soft scattering at high energy).

Recent <u>example</u>: Maldacena + Alday, gluon scattering at strong coupling; example of check using specific structures of ST.

Back to BI:

We have, in the gauge theory, partially BI formulation in that we are commited to fixing Λ (it maps to N of $SU(N)$ gauge theory). Otherwise, anything can happen: we can have any state of QG with this asymptotic b.c.

Is this enough? maybe, maybe not.

(Banks, hep-th 0011255)

(on isolated vacua 4 BI)

But this is <u>not</u> the most interesting question to ask, rather...

How is that partial BI realized?

In one side of the duality, the "labor intensive" realization: quantize gravity imposing diff. inv. on states, carefully not using any particular background. This is hard!

In another side, the gauge theory, diff. invariance is realized trivially, everything is already diff. inv.

It is then easy quantizing w/o using bulk bckd.

Conclusion and Outlook:

Our experience with non-perturbative quantum systems suggest modified role for gauge freedom.

$$\begin{array}{c}\text{theory B}\\ \text{with gauge}\\ \text{invariance B}\end{array} \xleftarrow[\text{limit B}]{\text{classical}} \boxed{\begin{array}{c}\text{full}\\ \text{theory}\end{array}} \xrightarrow[\text{limit A}]{\text{classical}} \begin{array}{c}\text{theory A}\\ \text{with gauge}\\ \text{invariance A}\end{array}$$

This includes General Covariance

There are two ways to acheive gauge invariance: the "labor-intensive" way, or working directly in terms of gauge-invariant "dual" variables.

Working in dual variables, it is much easier to acheive BI, at least partially, since bulk spacetime is not part of the formulation.

but... why is gauge invariance so abiguitous?
(or diff.)

probably has to do with locality; indeed, this is the property most obscured by working in dual variables; e.g in AdS/CFT.

This suggests a role for GC in the quantum theory, guiding us towards the classical limit of QG, where gravity is local. Perhaps a point of contact...

So: what is LQG in $\Lambda < 0$?

how does it acheive (even partial) BI?

how is it related to the holographic dual?