

Chiral excitations of quantum geometry as elementary particles

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D. Kribs and F. Markopoulou gr-qc/0510052

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S. Bilson-Thompson, F. Markopoulou, LS hep-th/0603022

J. Hackett hep-th/0702198

Bilson-Thompson, Hackett, Kauffman, in preparation

Wan, ls, in preparation

There are things we expect when we construct a quantum theory of gravity:

- make QFT ultraviolet finite**
- explain black hole entropy**
- Newton's law**
- graviton propagator and scattering**
- get GR as a low energy limit**

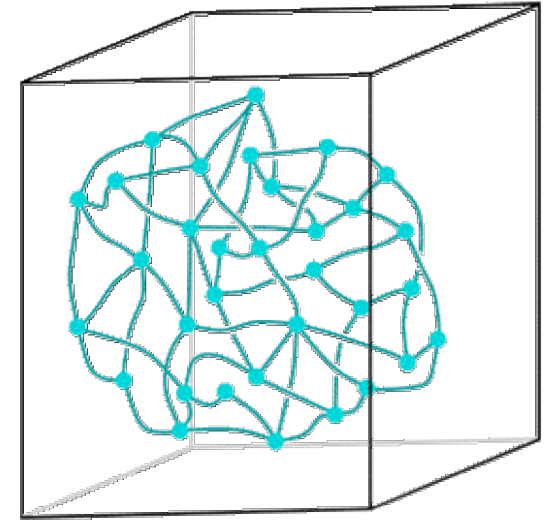
LQG does the first three and there is some evidence it may do the the rest.

But none of this will convince skeptics it is the right theory of nature.

To convince skeptics we need *surprises*-physical phenomena that jump out that were not put in.

A surprise should be:

- **A novel** physical phenomena
- Have definite **consequences for experiments**
- **Unexpected**, was not put in
- **Generic**, cannot easily be taken out.



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We know very little about the physics of LQG and spin foam models, so there is lots of room for surprises!

Have LQG and related models produced any such surprises?

- discreteness of area and volume**
- space-like singularities bounce**
- DSR at least in 2+1 with matter**
- fine structure of Hawking radiation- Ansari**
- disordered locality**
- emergent chiral matter**

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This talk

Some open issues

Kinematical issues for LQG:

- *Are the graphs embedded in a 3 manifold or not?*

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Embedded follows from canonical quantization of GR.

But group field theory and other spin foam models are simplest without embedding.

The geometric operators, area and volume do not measure topology of the embedding

What observables or degrees of freedom are represented by the braiding and knotting of the embeddings?

Kinematical issues for LQG:

How should the graphs be labeled?

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How should the graphs be labeled?

- *$SU(2)$ labels come from canonical quantization of GR.*
- *Lorentz or Poincare in some spin foam models*
- *Perhaps some or all of the group structure is emergent at low energy.
This would simplify the theory.*

Why should the symmetries of the classical limit be acting at Planck scales?

Are there consequences of dynamics that don't depend on details of labeling and amplitudes?

Kinematical issues for LQG:

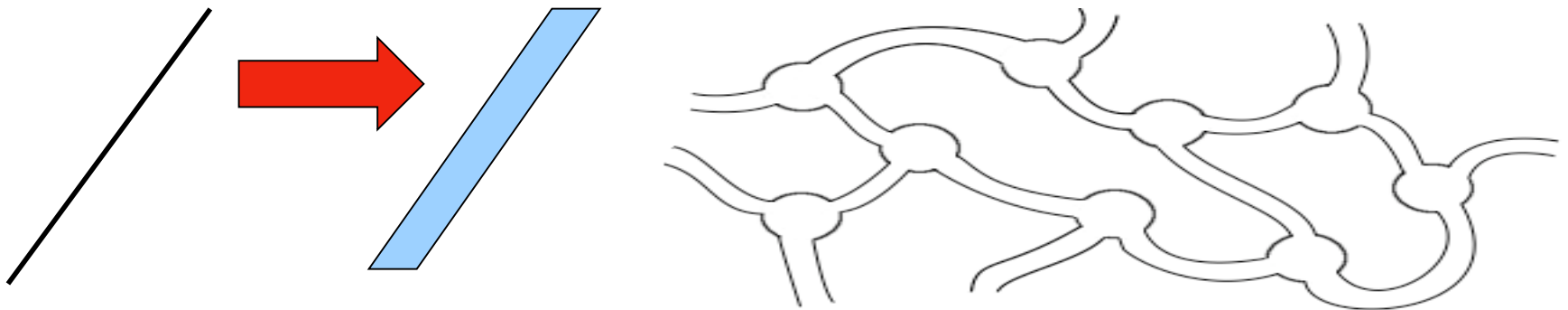
Are the graphs framed or not?

Framing is needed if there is a cosmological constant; because $SU(2)$ is quantum deformed

$$q = e^{2\pi i/k+2}$$

$$k = 6\pi/G\Lambda$$

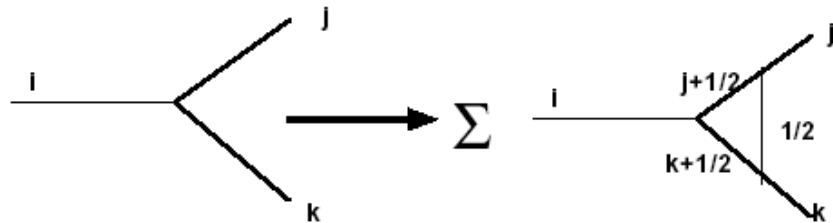
To represent this the spin network graphs must be framed:



Dynamical issues for LQG:

Hamiltonian constraint

only the expansion move



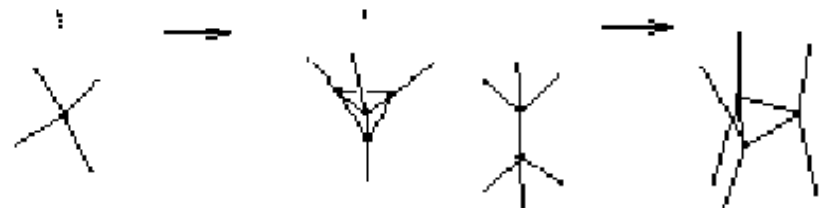
Spin foam models

expansion AND exchange moves

3 valent moves



4 valent moves



Issues about observables

- How do we describe the low energy limit of the theory?
- What does **locality** mean? How do we define local **subsystems** without a background?
- *How do we recognize gravitons and other local excitations in a background independent theory?*

Issues about excitations:

- What protects a photon traveling in Minkowski spacetime from decohering with the noisy vacuum?

ANSWER: *The photon and vacuum are in different irreducible representations of the Poincare group.*

- In quantum GR we expect Poincare symmetry is only emergent at low energies, at shorter scales there are quantum fluctuations of the spacetime geometry not governed by a symmetry,
- *So what keeps the photon from decohering with the spacetime foam?*

Lessons from quantum information theory

(Kribs, Markopoulou)

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hep-th/0604120 gr-qc/0510052

- Define local as a characteristic of excitations of the graph states. To identify them in a background independent way look for *noiseless subsystems*, in the language of quantum information theory.
- Identify the ground state as the state in which these *propagate coherently*, without decoherence.
- This can happen if there is also an *emergent symmetry* which protect the excitations from decoherence. Thus the ground state has symmetries because this is necessary for excitations to persist as pure states.

Hence, photons are in noiseless subsectors which have the symmetries of flat spacetime.

Suppose we find, a set of emergent symmetries which protect some local excitations from decoherence. Those local excitations will be emergent particle degrees of freedom.

Three main results:

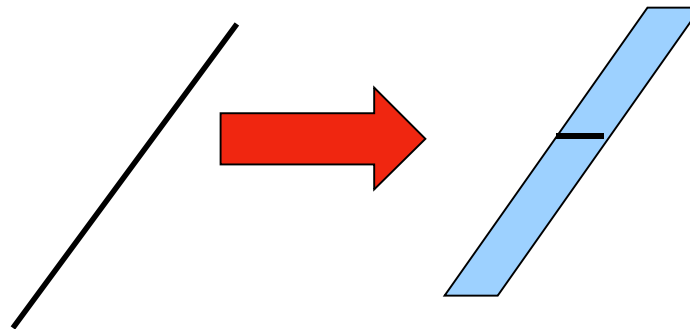
- A large class of causal spinnet theories have noiseless subsystems that can be interpreted as local excitations.*
- There is a class of such models for which the simplest such coherent excitations match the quantum numbers of the fermions of the standard model.*
- Spin foam models with embedded graphs, evolving under dual Pachner moves have locally stable chiral excitations that propagate and, in some cases, interact.*

First example: trivalent framed graphs

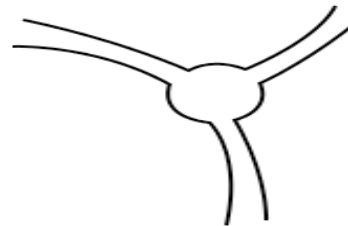
(Bilson-Thompson, Markopoulou, ls)

First example: theories based on framed graphs with trivalent nodes

The edges are framed:

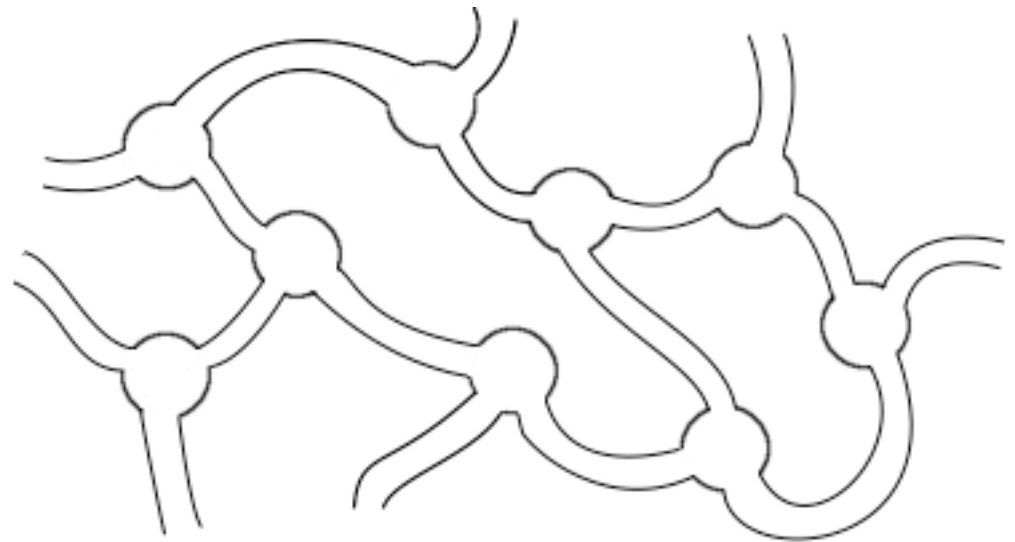


The nodes become trinions:



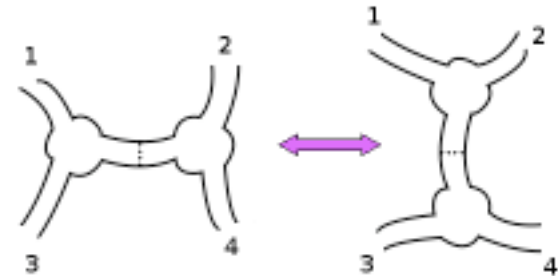
Basis States: *Oriented, twisted ribbon graphs, embedded in S^3 topology, up to topological class.*

Labelings: any quantum group...or none.

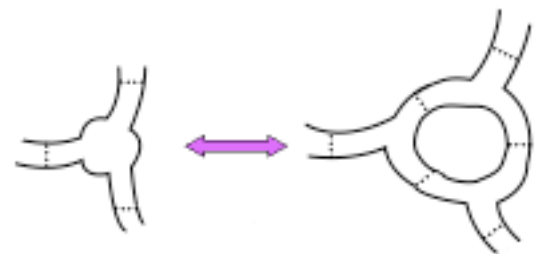


The evolution moves:

Exchange moves:



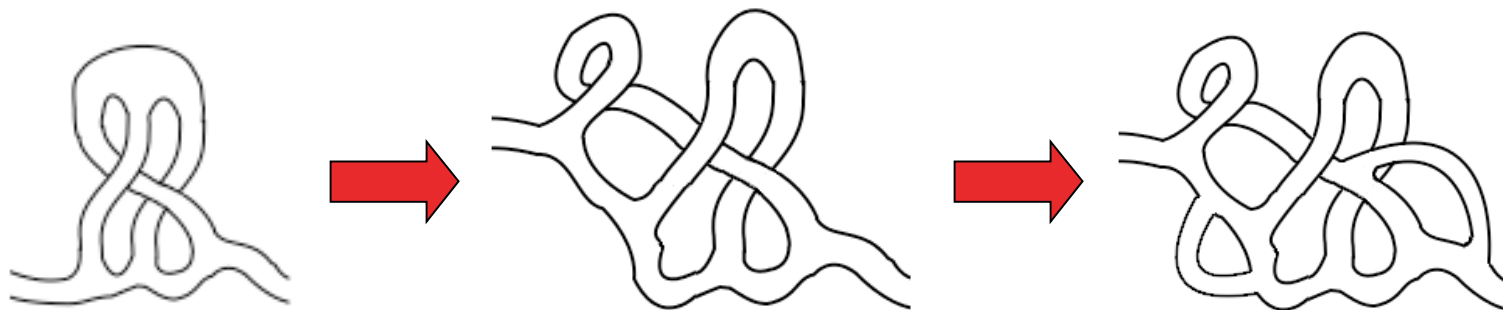
Expansion moves:



The amplitudes: arbitrary functions of the labels

Questions: Are there invariants under the moves?

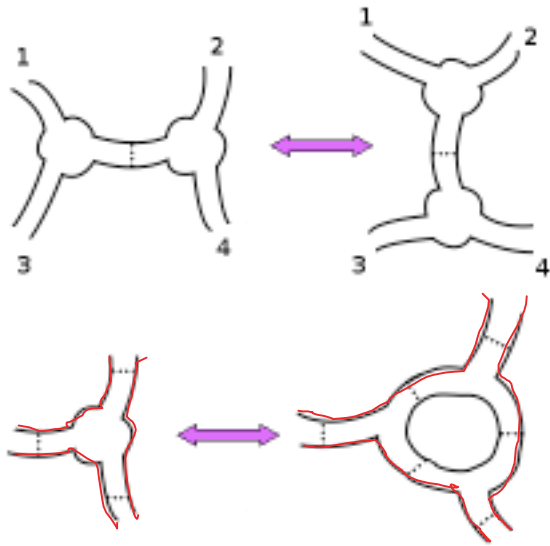
What are the simplest states preserved by the moves?



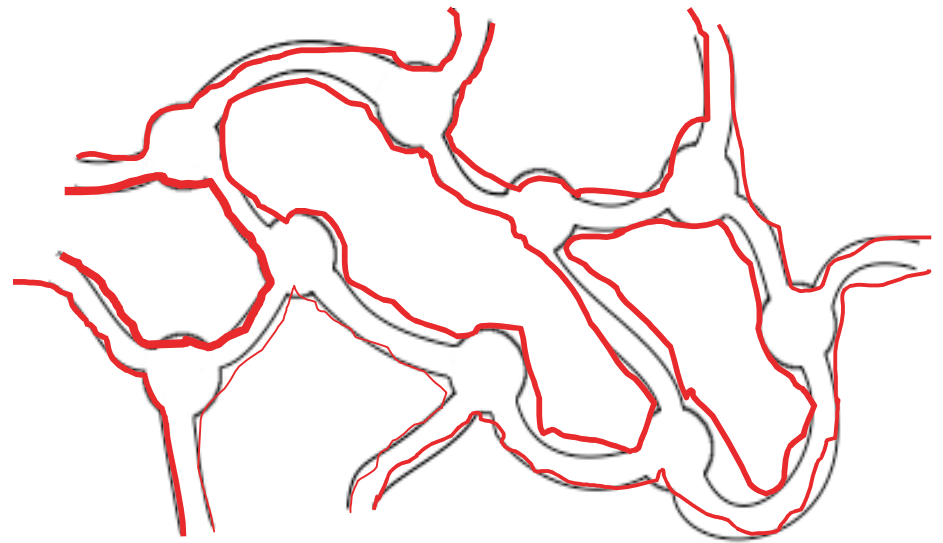
The reduced link of the ribbon is a constant of the motion

Link= topology of embedding of edges of the ribbons

Reduced = remove all unlinked unknotted circles

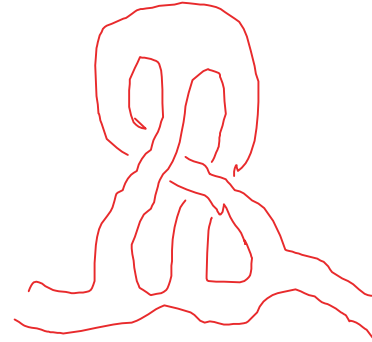
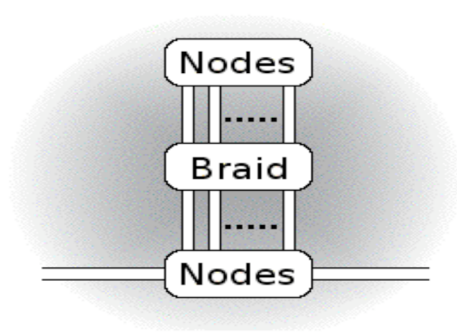


Reduced link



Link

Classification of simplest chiral conserved states:

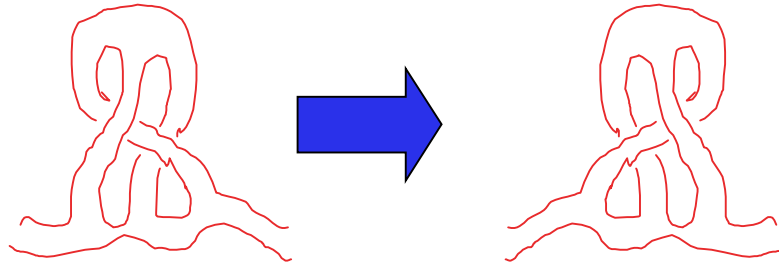


These are three strand ribbon braids.

Conserved quantum numbers

Chirality

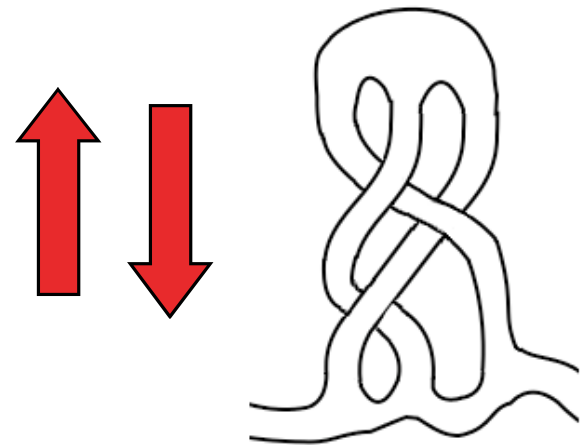
P:



Charge conjugation:

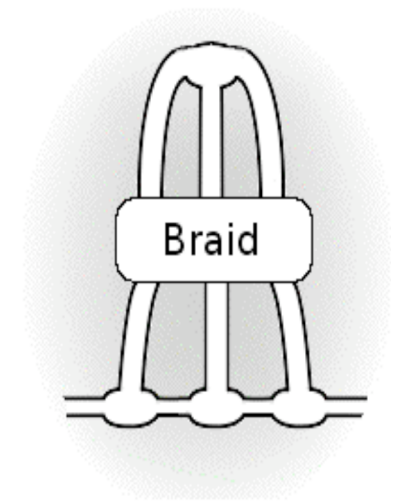
read the braiding up rather than down

(inverse in the braid group algebra)



Braids on three ribbons and preons (Bilson-Thompson)

preon	ribbon
Charge/3	twist
P,C	P,C
triplet	3-strand braid
Position??	Position in braid



In the preon models there is a rule about mixing charges:

No triplet with both positive and negative charges.

This becomes: *No braid with both left and right twists.*

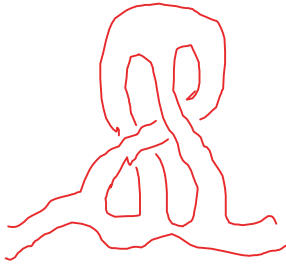
In the future we should find a dynamical justification, for the moment we just assume it.

The preons are not independent degrees of freedom, just elements of quantum geometry. But braided triplets of them are bound by topological conservation laws from quantum geometry.

Two crossing left handed + twist braids:

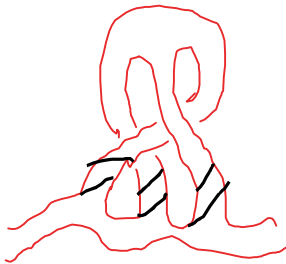
Charge = twist/3

No twists:



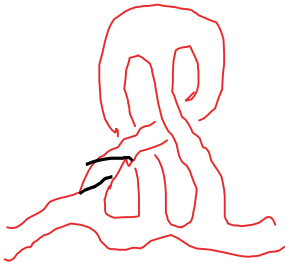
v_L

3 + twists

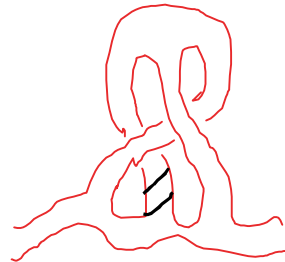


e_L^+

1 + twist



$\overline{d}_L^r$



$\overline{d}_L^b$



$\overline{d}_L^g$

2 + twists



u_L^r



u_L^b

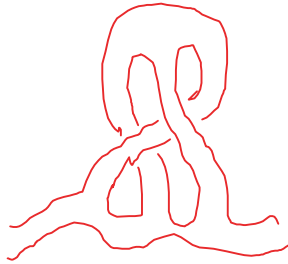


u_L^g

Two crossing left handed + twist braids:

Charge = twist/3

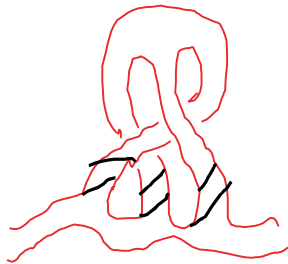
No twists:



ν_L

Including the negative twists (charge) these are exactly the 15 left handed states of the first generation of the standard model.

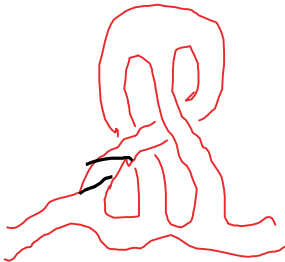
3 + twists



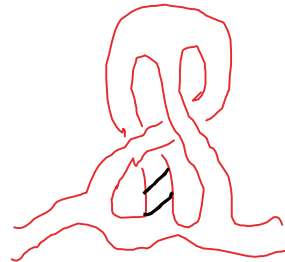
e_L^+

Straightforward to prove them distinct.

1 + twist



$\bar{d}_L^r$



$\bar{d}_L^b$



$\bar{d}_L^g$

2 + twists



u_L^r

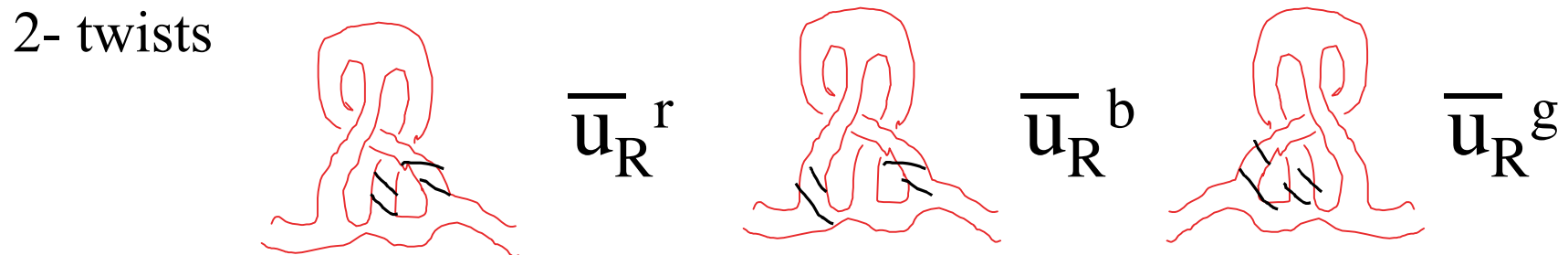
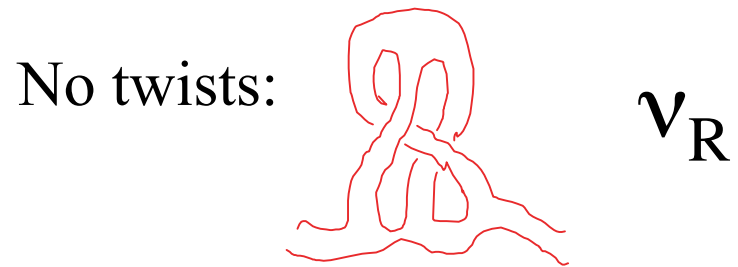


u_L^b

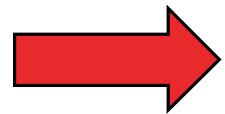


u_L^g

The right handed states come from parity inversion:



Left and right handed states are separately conserved

 **No coupling of left and right states**

 **If they propagate, they will be massless**

Do these states propagate?

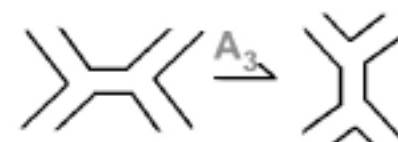
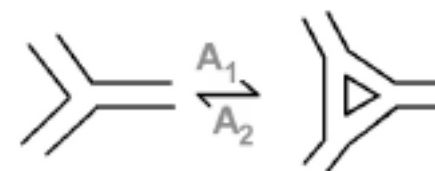
Jonathan Hackett: hep-th/0702198

Do these states propagate?

YES

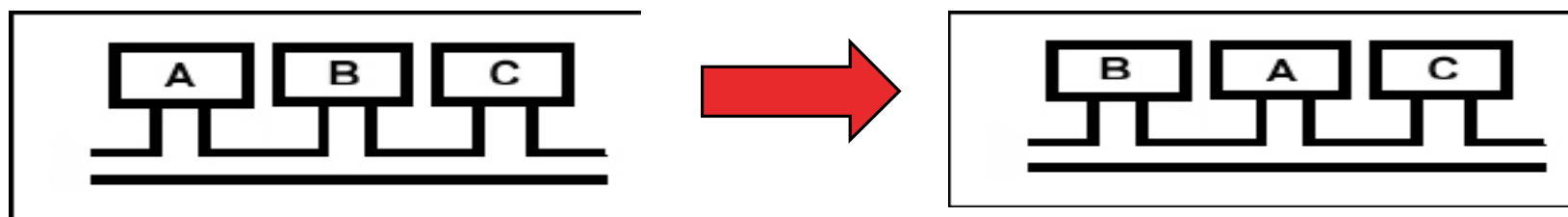
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A braid can evolve to an *isolated structure*
(a subgraph connected to a larger graph with a
single edge):



local moves:

Isolated structures propagate through each other:



Hence, like solitons, they do not interact

Key issues:

Interactions: *How can we get these states to interact without falling apart?*

Generations: *does the model work for higher generations?*

LQG & spin foam: *are there emergent local chiral excitations for spin foam & LQG models?*

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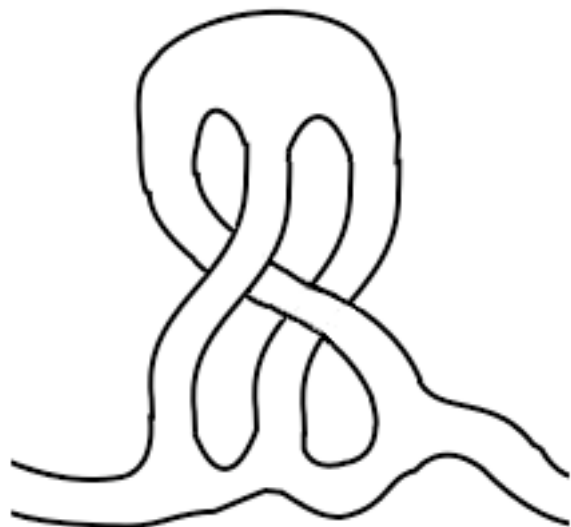
Generations: *does the model work for higher generations?*

LQG & spin foam: *are there emergent local chiral excitations for spin foam & LQG models?*

Each has recently been answered positively.

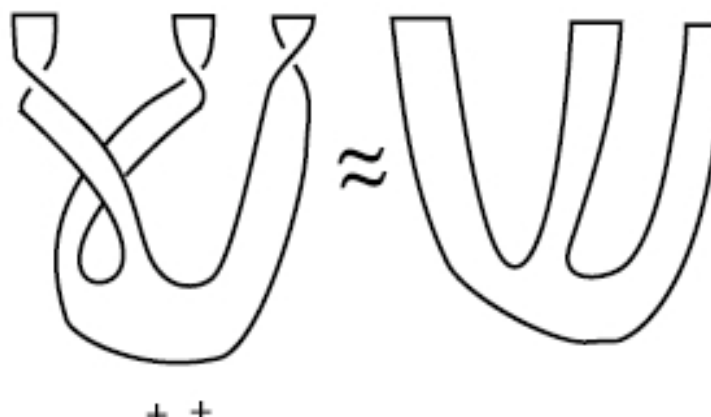
How to get higher generations:

more on capped, trivalent framed braids

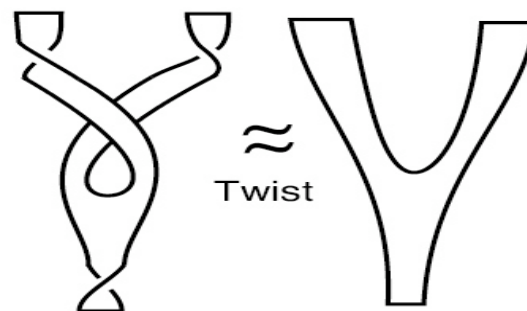


(Bilson-Thompson, Hackett, Kauffman)

Braids are equivalent to half-twists:

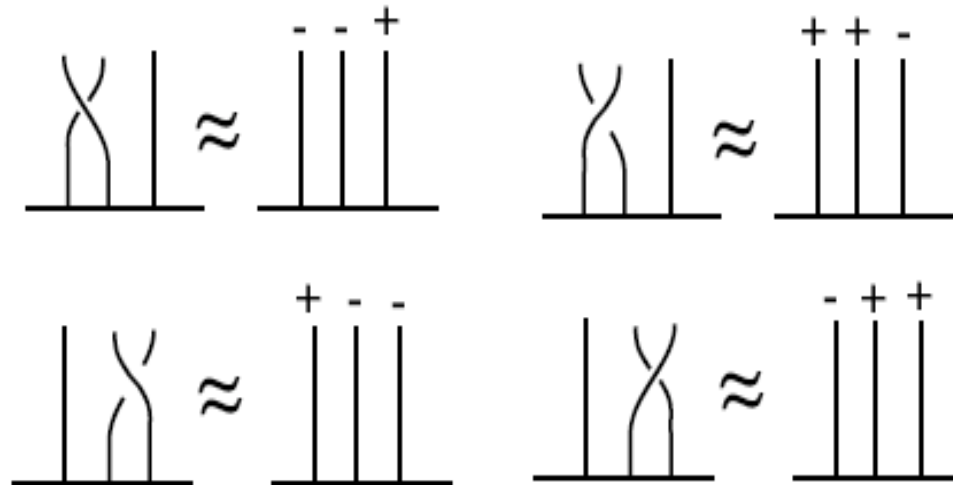


Proof:



We can apply this to eliminate all braids
in favor of half-twists

**braids are
equivalent
to half twists:**



$+=$ half twist

**This gives us topological invariants, which are the three
half-twists after the braids are eliminated:**

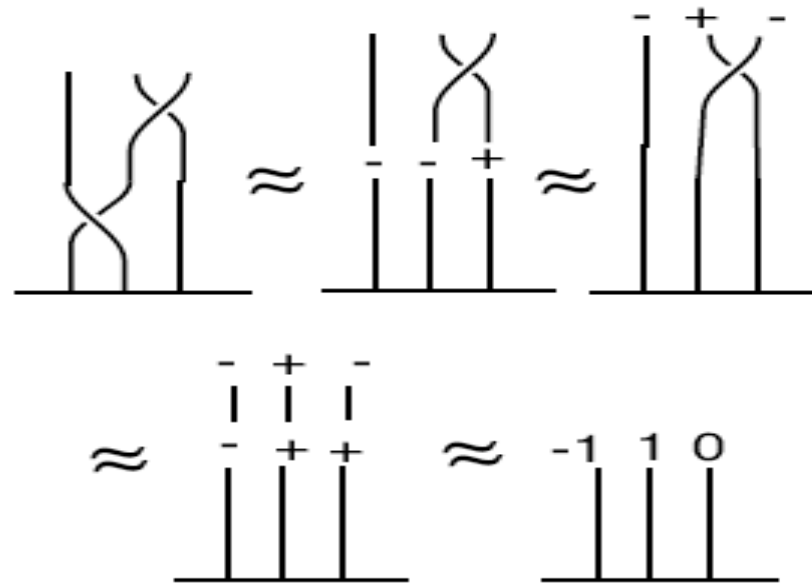
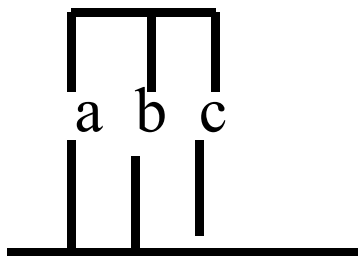
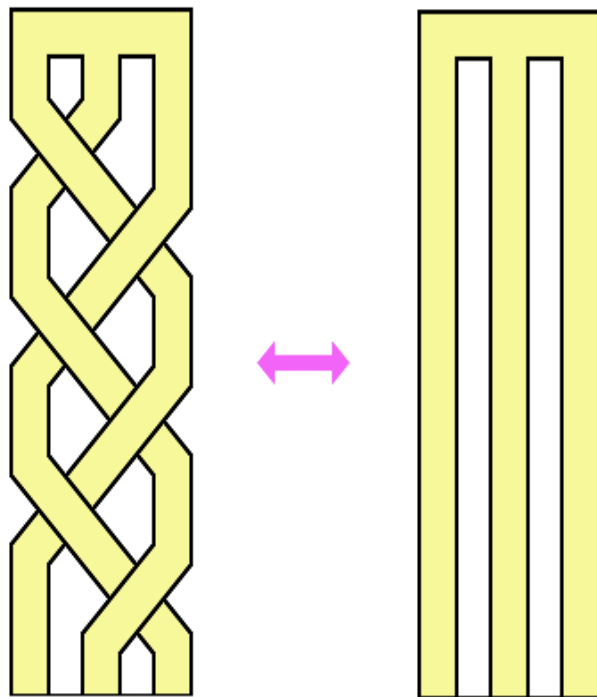


Figure3 - A Product

*Kauffman numbers:
(a,b,c)*

Belt trick identity: $(\sigma_{12} \sigma_{32})^3 = I$



**This reduces the numbers
of distinct capped braids**

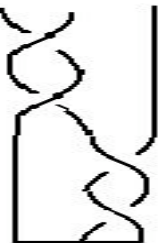
Multigenerational scheme (Hackett):

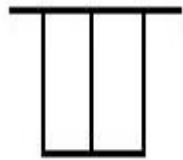
- Capped trivalent ribbon braids
- Based on Kauffman number's (a,b,c)
- Parity: $(a,b,c) \rightarrow (c,b,a)$
- C: $(a,b,c) \rightarrow (-a,-b,-c)$
- Charge = $a+b+c$
- All states distinct
- Interactions assume $(a,b,c)+(d,e,f) \rightarrow (a+d,b+e,c+f)$

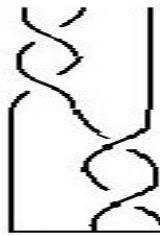
Strategy:

•Pick the weak vector bosons:

Z_L, Z_R, Z_0	$[2, 0, -2], [-2, 0, 2], [0, 0, 0]$
W_L^+, W_R^+, W_0^+	$[3, 1, -1], [-1, 1, 3], [1, 1, 1]$
W_L^-, W_R^-, W_0^-	$[1, -1, -3], [-3, -1, 1], [-1, -1, -1]$

Z_L 
 $(2, 0, -2)$

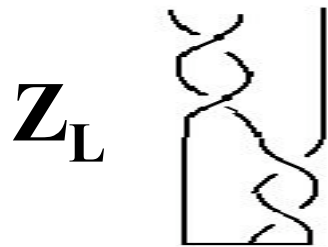
Z_0 
 $(0, 0, 0)$

Z_R 
 $(-2, 0, 2)$

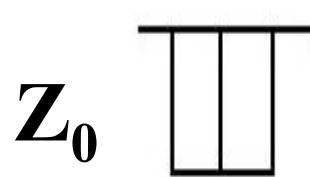
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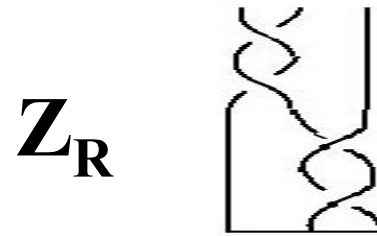
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$(2,0,-2)$

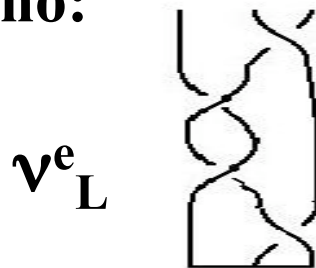


$(0,0,0)$



$(-2,0,2)$

Pick the neutrino:

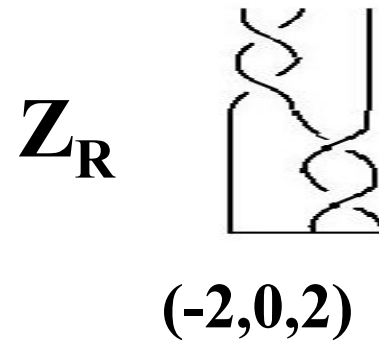
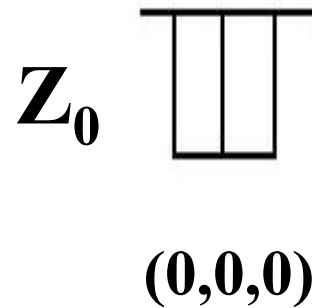
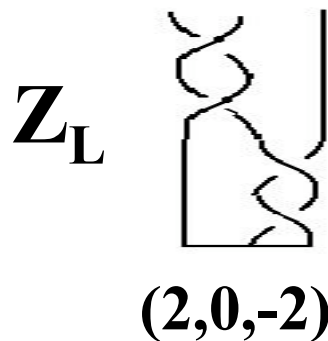


$(2,-2,0)$

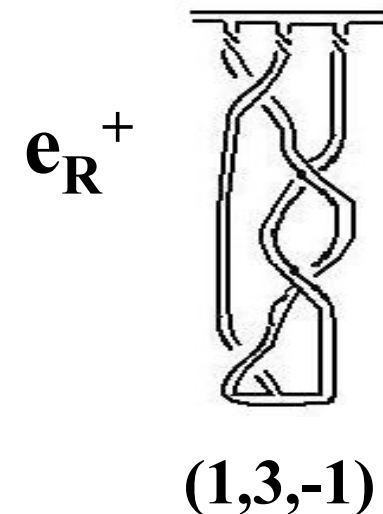
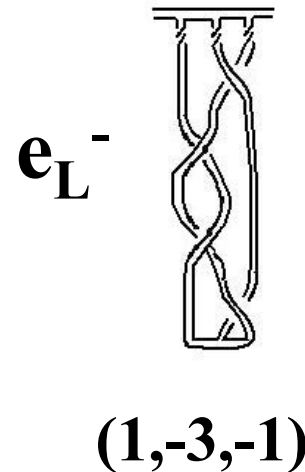
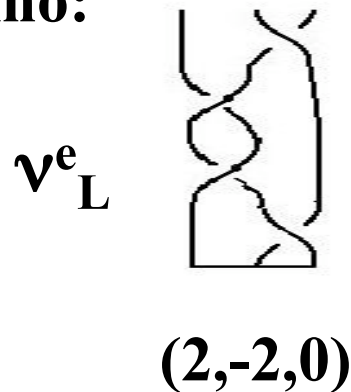
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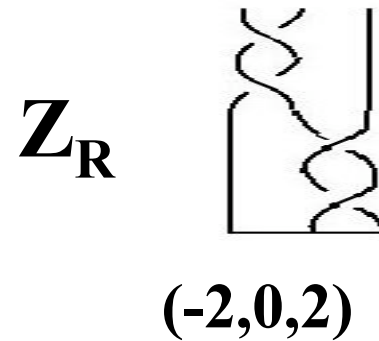
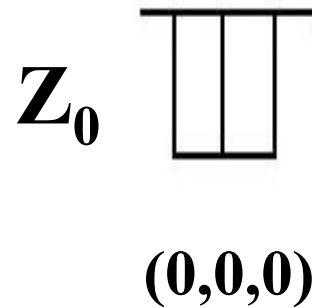
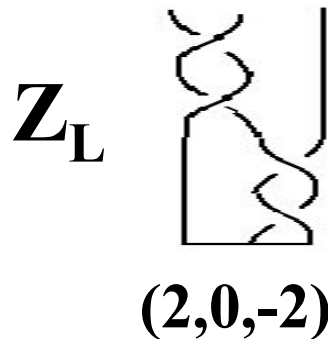


Then the rest of the lepton doublet is determined by the weak interactions.

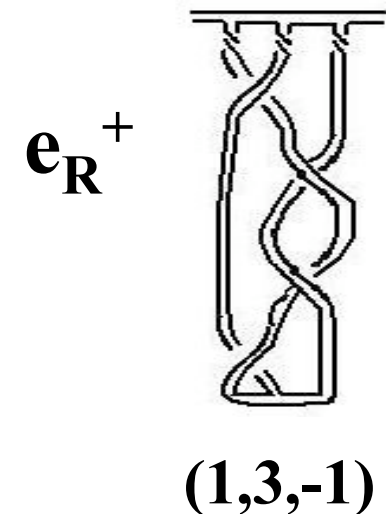
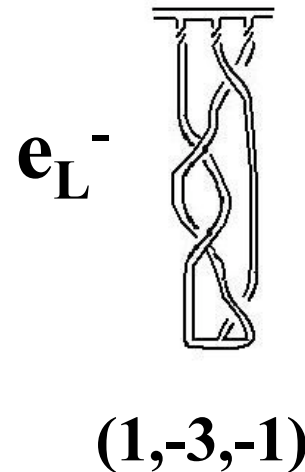
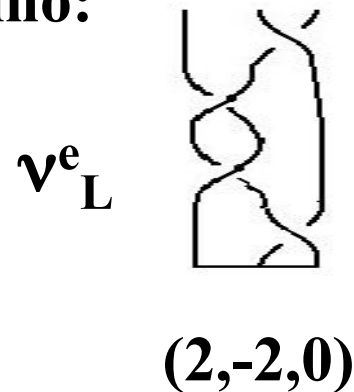
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Z_L, Z_R, Z_0	$[2, 0, -2], [-2, 0, 2], [0, 0, 0]$
W_L^+, W_R^+, W_0^+	$[3, 1, -1], [-1, 1, 3], [1, 1, 1]$
W_L^-, W_R^-, W_0^-	$[1, -1, -3], [-3, -1, 1], [-1, -1, -1]$



Pick the neutrino:

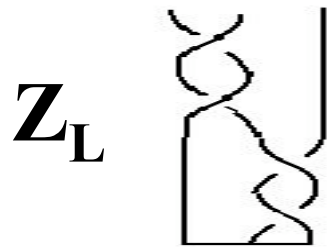


Make the quarks by subtracting charges from the electrons.

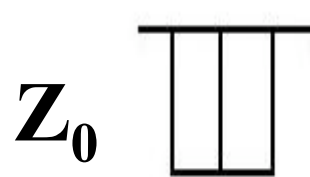
Strategy:

•Pick the weak vector bosons:

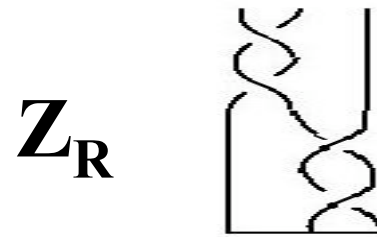
Z_L, Z_R, Z_0	$[2, 0, -2], [-2, 0, 2], [0, 0, 0]$
W_L^+, W_R^+, W_0^+	$[3, 1, -1], [-1, 1, 3], [1, 1, 1]$
W_L^-, W_R^-, W_0^-	$[1, -1, -3], [-3, -1, 1], [-1, -1, -1]$



$(2,0,-2)$



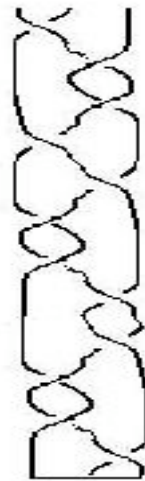
$(0,0,0)$



$(-2,0,2)$

Pick the neutrino:

ν_L^τ

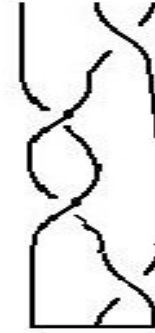


$(6,-8,2)$

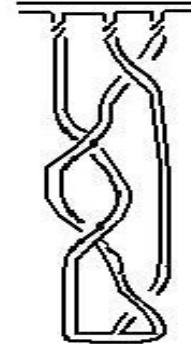
Multigenerational scheme (Hackett):

Particle	Invariant
Z_L, Z_R, Z_0	$[2, 0, -2], [-2, 0, 2], [0, 0, 0]$
W_L^+, W_R^+, W_0^+	$[3, 1, -1], [-1, 1, 3], [1, 1, 1]$
W_L^-, W_R^-, W_0^-	$[1, -1, -3], [-3, -1, 1], [-1, -1, -1]$
$\nu_L^{(e)}, \nu_R^{(e)}, \bar{\nu}_L^{(e)}, \bar{\nu}_R^{(e)}$	$[2, -2, 0], [0, -2, 2], [-2, 2, 0], [0, 2, -2]$
$e_L^-, e_R^-, e_L^+, e_R^+$	$[1, -3, -1], [-1, -3, 1], [-1, 3, 1], [1, 3, -1]$
$d_{L,r}, d_{L,g}, d_{L,b}$	$[1, -2, 0], [2, -3, 0], [2, -2, -1]$
$d_{R,r}, d_{R,g}, d_{R,b}$	$[-1, -2, 2], [0, -3, 2], [0, -2, 1]$
$\bar{d}_{L,r}, \bar{d}_{L,g}, \bar{d}_{L,b}$	$[-1, 2, 0], [-2, 3, 0], [-2, 2, 1]$
$\bar{d}_{R,r}, \bar{d}_{R,g}, \bar{d}_{R,b}$	$[1, 2, -2], [0, 3, -2], [0, 2, -1]$
$u_{L,r}, u_{L,g}, u_{L,b}$	$[2, -1, 1], [3, -2, 1], [3, -1, 0]$
$u_{R,r}, u_{R,g}, u_{R,b}$	$[0, -1, 3], [1, -2, 3], [1, -1, 2]$
$\bar{u}_{L,r}, \bar{u}_{L,g}, \bar{u}_{L,b}$	$[-2, 1, -1], [-3, 2, -1], [-3, 1, 0]$
$\bar{u}_{R,r}, \bar{u}_{R,g}, \bar{u}_{R,b}$	$[0, 1, -3], [-1, 2, -3], [-1, 1, -2]$
$\nu_L^{(\mu)}, \nu_R^{(\mu)}, \bar{\nu}_L^{(\mu)}, \bar{\nu}_R^{(\mu)}$	$[4, -6, 2], [2, -6, 4], [-4, 6, -2], [-2, 6, -4]$
$\mu_L, \mu_R, \bar{\mu}_L, \bar{\mu}_R$	$[3, -7, 1], [1, -7, 3], [-3, 7, -1], [-1, 7, -3]$
$c_{L,r}, c_{L,g}, c_{L,b}$	$[4, -5, 3], [5, -6, 3], [5, -5, 2]$
$c_{R,r}, c_{R,g}, c_{R,b}$	$[2, -5, 5], [3, -6, 5], [3, -5, 4]$
$\bar{c}_{L,r}, \bar{c}_{L,g}, \bar{c}_{L,b}$	$[-4, 5, -3], [-5, 6, -3], [-5, 5, -2]$
$\bar{c}_{R,r}, \bar{c}_{R,g}, \bar{c}_{R,b}$	$[-2, 5, -5], [-3, 6, -5], [-3, 5, -4]$
$s_{L,r}, s_{L,g}, s_{L,b}$	$[5, -6, 2], [4, -7, 2], [4, -6, 1]$
$s_{R,r}, s_{R,g}, s_{R,b}$	$[1, -6, 4], [2, -7, 4], [2, -6, 3]$
$\bar{s}_{L,r}, \bar{s}_{L,g}, \bar{s}_{L,b}$	$[-3, 6, -2], [-4, 7, -2], [-4, 6, -1]$
$\bar{s}_{R,r}, \bar{s}_{R,g}, \bar{s}_{R,b}$	$[-1, 6, -4], [-2, 7, -4], [-2, 6, -3]$

ν_L^e



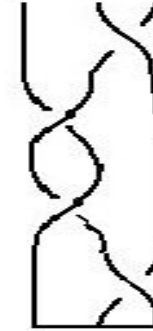
e_L^-



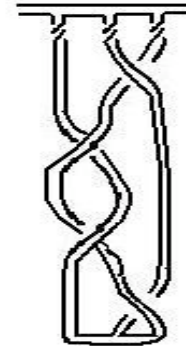
Multigenerational scheme (Hackett):

Particle	Invariant
Z_L, Z_R, Z_0	$[2, 0, -2], [-2, 0, 2], [0, 0, 0]$
W_L^+, W_R^+, W_0^+	$[3, 1, -1], [-1, 1, 3], [1, 1, 1]$
W_L^-, W_R^-, W_0^-	$[1, -1, -3], [-3, -1, 1], [-1, -1, -1]$
$\nu_L^{(e)}, \nu_R^{(e)}, \bar{\nu}_L^{(e)}, \bar{\nu}_R^{(e)}$	$[2, -2, 0], [0, -2, 2], [-2, 2, 0], [0, 2, -2]$
$e_L^-, e_R^-, e_L^+, e_R^+$	$[1, -3, -1], [-1, -3, 1], [-1, 3, 1], [1, 3, -1]$
$d_{L,r}, d_{L,g}, d_{L,b}$	$[1, -2, 0], [2, -3, 0], [2, -2, -1]$
$d_{R,r}, d_{R,g}, d_{R,b}$	$[-1, -2, 2], [0, -3, 2], [0, -2, 1]$
$\bar{d}_{L,r}, \bar{d}_{L,g}, \bar{d}_{L,b}$	$[-1, 2, 0], [-2, 3, 0], [-2, 2, 1]$
$\bar{d}_{R,r}, \bar{d}_{R,g}, \bar{d}_{R,b}$	$[1, 2, -2], [0, 3, -2], [0, 2, -1]$
$u_{L,r}, u_{L,g}, u_{L,b}$	$[2, -1, 1], [3, -2, 1], [3, -1, 0]$
$u_{R,r}, u_{R,g}, u_{R,b}$	$[0, -1, 3], [1, -2, 3], [1, -1, 2]$
$\bar{u}_{L,r}, \bar{u}_{L,g}, \bar{u}_{L,b}$	$[-2, 1, -1], [-3, 2, -1], [-3, 1, 0]$
$\bar{u}_{R,r}, \bar{u}_{R,g}, \bar{u}_{R,b}$	$[0, 1, -3], [-1, 2, -3], [-1, 1, -2]$
$\nu_L^{(\mu)}, \nu_R^{(\mu)}, \bar{\nu}_L^{(\mu)}, \bar{\nu}_R^{(\mu)}$	$[4, -6, 2], [2, -6, 4], [-4, 6, -2], [-2, 6, -4]$
$\mu_L, \mu_R, \bar{\mu}_L, \bar{\mu}_R$	$[3, -7, 1], [1, -7, 3], [-3, 7, -1], [-1, 7, -3]$
$c_{L,r}, c_{L,g}, c_{L,b}$	$[4, -5, 3], [5, -6, 3], [5, -5, 2]$
$c_{R,r}, c_{R,g}, c_{R,b}$	$[2, -5, 5], [3, -6, 5], [3, -5, 4]$
$\bar{c}_{L,r}, \bar{c}_{L,g}, \bar{c}_{L,b}$	$[-4, 5, -3], [-5, 6, -3], [-5, 5, -2]$
$\bar{c}_{R,r}, \bar{c}_{R,g}, \bar{c}_{R,b}$	$[-2, 5, -5], [-3, 6, -5], [-3, 5, -4]$
$s_{L,r}, s_{L,g}, s_{L,b}$	$[5, -6, 2], [4, -7, 2], [4, -6, 1]$
$s_{R,r}, s_{R,g}, s_{R,b}$	$[1, -6, 4], [2, -7, 4], [2, -6, 3]$
$\bar{s}_{L,r}, \bar{s}_{L,g}, \bar{s}_{L,b}$	$[-3, 6, -2], [-4, 7, -2], [-4, 6, -1]$
$\bar{s}_{R,r}, \bar{s}_{R,g}, \bar{s}_{R,b}$	$[-1, 6, -4], [-2, 7, -4], [-2, 6, -3]$

ν_L^e



e_L^-



**there are exotic states:
fractionally charged
states which are neither
color triplets nor weak
doublets**

Do these interact?

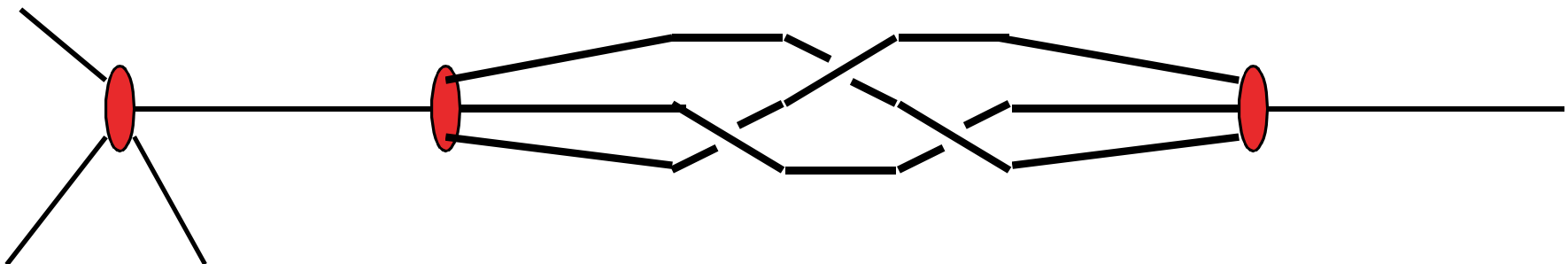
No, they travel right through each other, like solitons.

Can we get them to interact, naturally?

We have to add interactions....

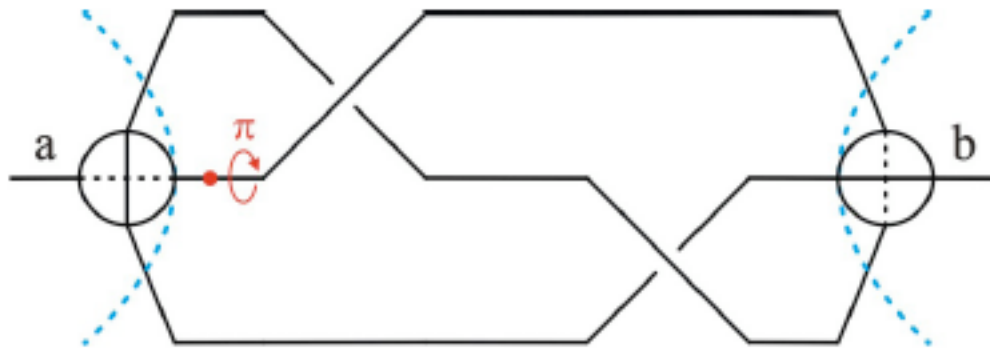
**Non-trivial, many proposed interactions
de-stabalize the braids.**

**So far one idea works: go to four valent graphs and
dual Pachner moves (Markopoulou, Wan & ls)**



Four valent graphs, framed and unframed

We consider three strand braids formed by connecting three edges from two four valent nodes:

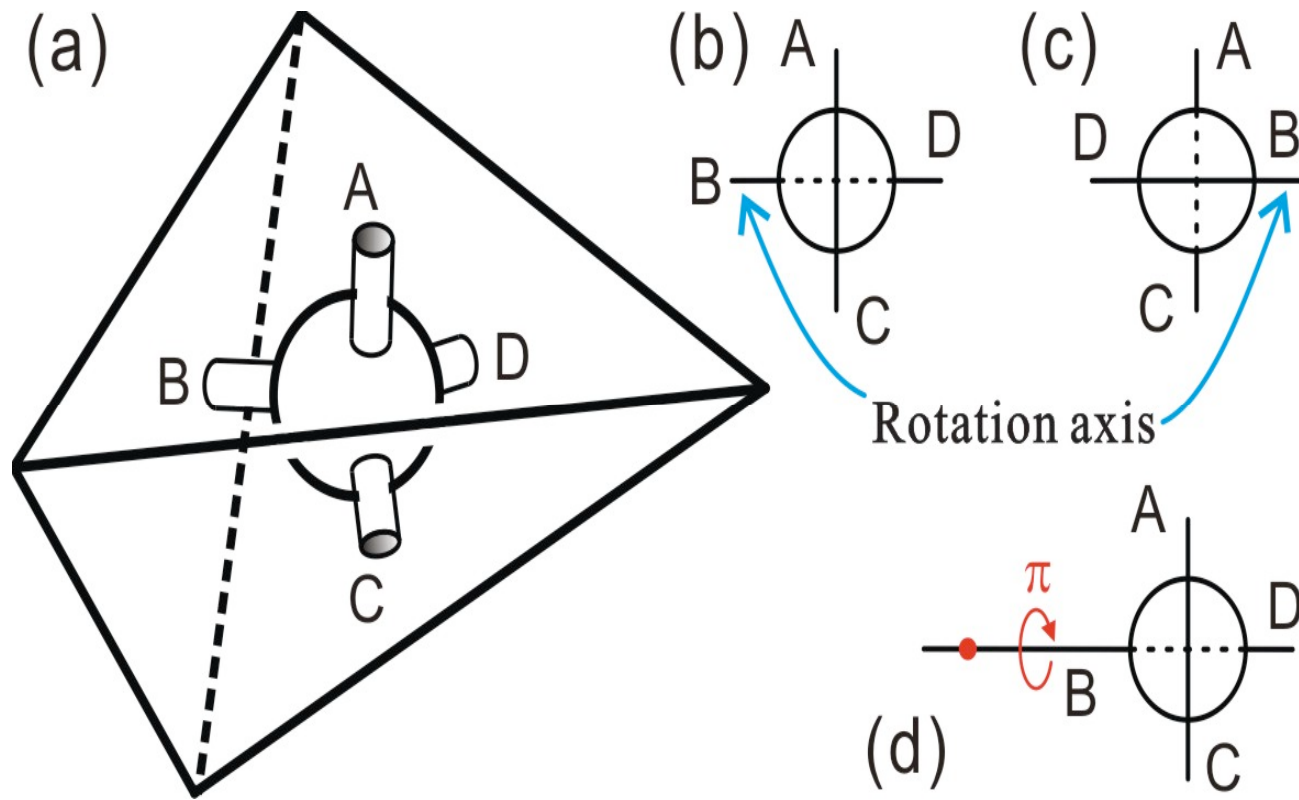


Evolution is via dual Pachner moves:

- Braids are stable when moves are only allowed on sets of nodes that are dual to triangulations of trivial balls in $\mathbb{R}^3$ (Markopoulou)
- Remaining dual pachner moves naturally give interactions between some braids (Wan)

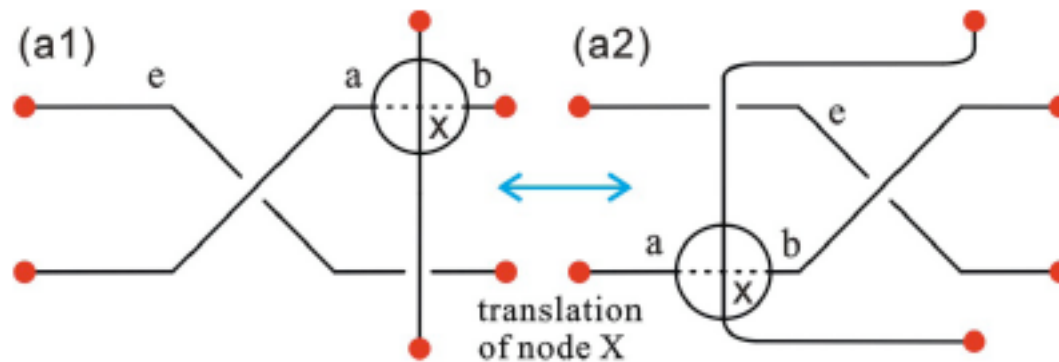
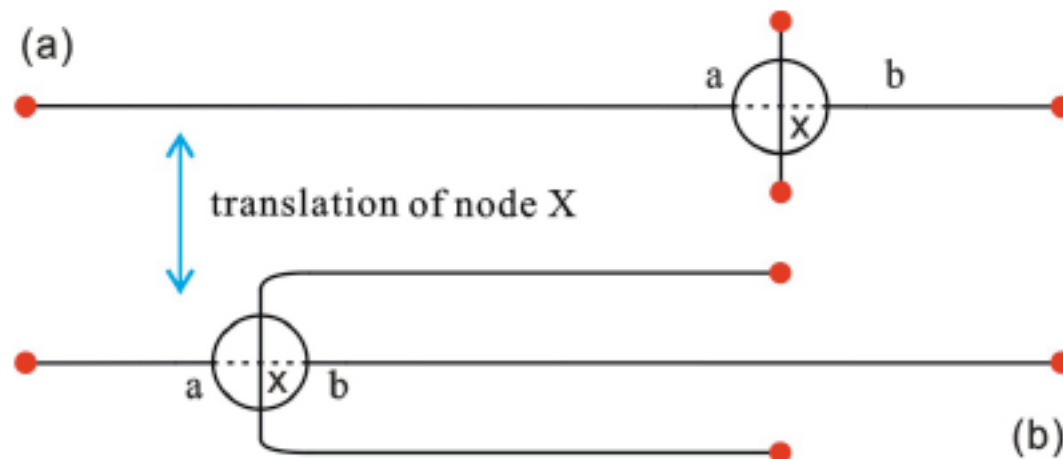
4 valent nodes

Four equivalent edges, dual to non-degenerate tetrahedron



Equivalence moves for graph projections:

- Reidemeister moves: translations:



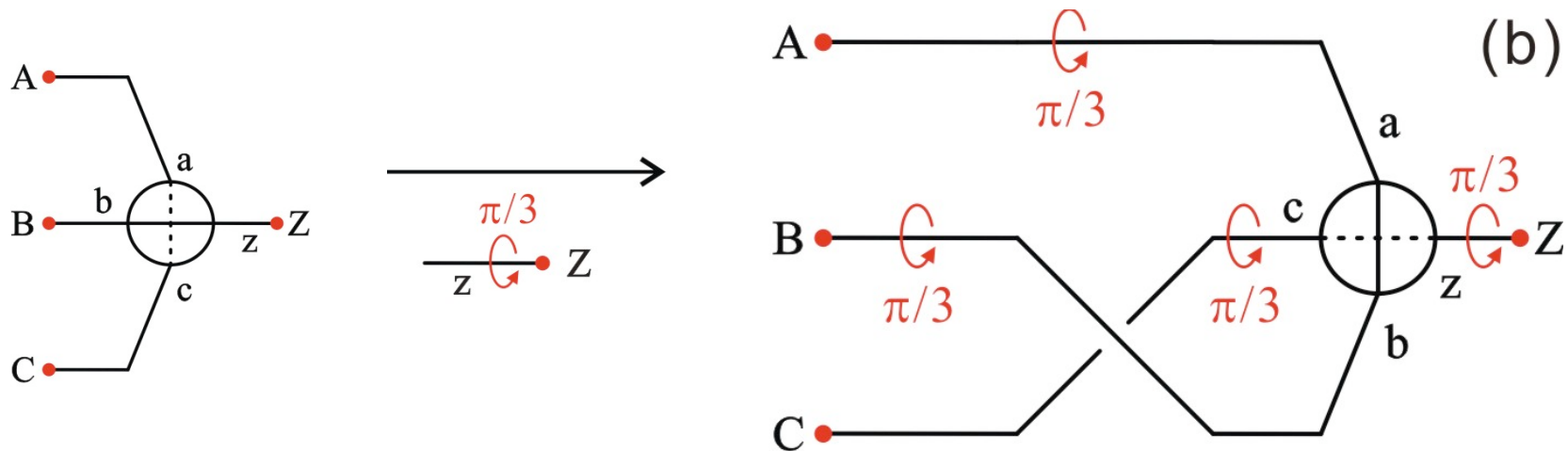
Graph embeddings are represented by projections, but many projections represent the same diffeo class of an embedded graph. Thus, there are equivalence classes of projections related by a finite set of moves.

Equivalence moves for graph projections:

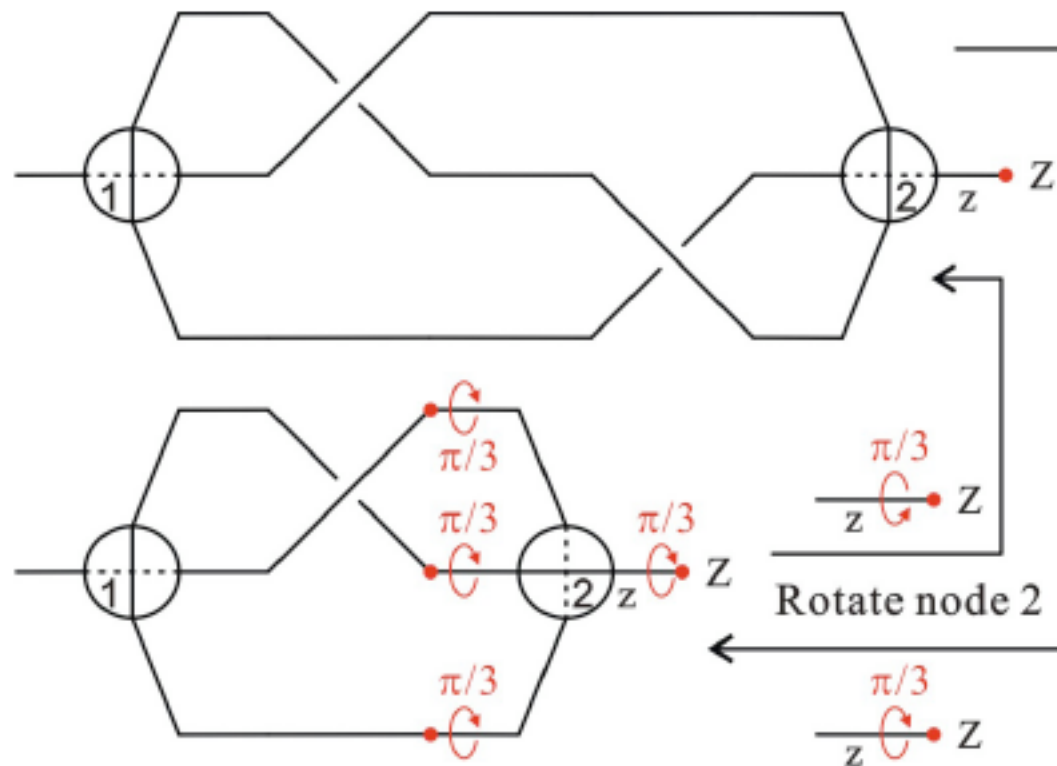
- Reidemeister moves
- Rotations of nodes

These rotations relate equivalent projections of nodes and their edges

$\pi/3$ rotations:



A braid projection may be **reduced** to one with fewer crossings by rotating one of the end-nodes:



A braid is **left irreducible** if no rotation of the left node reduces it to a braid with fewer crossings. (Same for right.)

A braid is **irreducible** if it is both left and right irreducible.

A braid is completely reducible if it can be reduced to a trivial braid (with twists) by a combination of rotations of the left and right node.

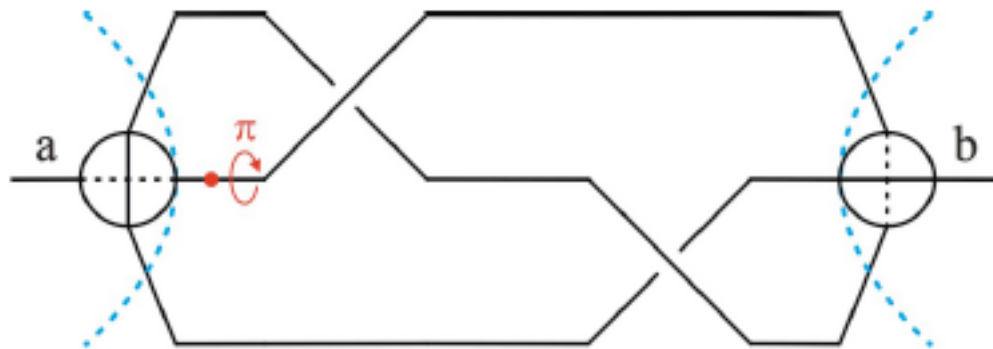
- *The irreducible braids can be classified.*
- *There are irreducible braids for N crossings for every $N > 0$.*

Evolution moves

Dual Pachner moves:

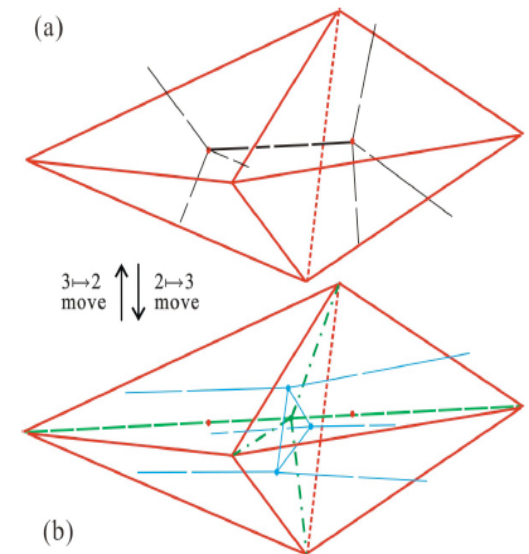
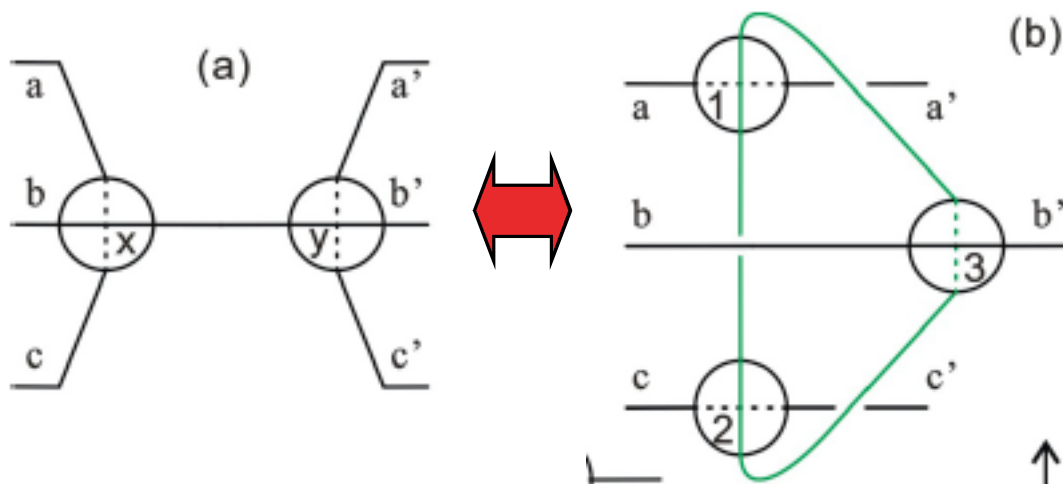
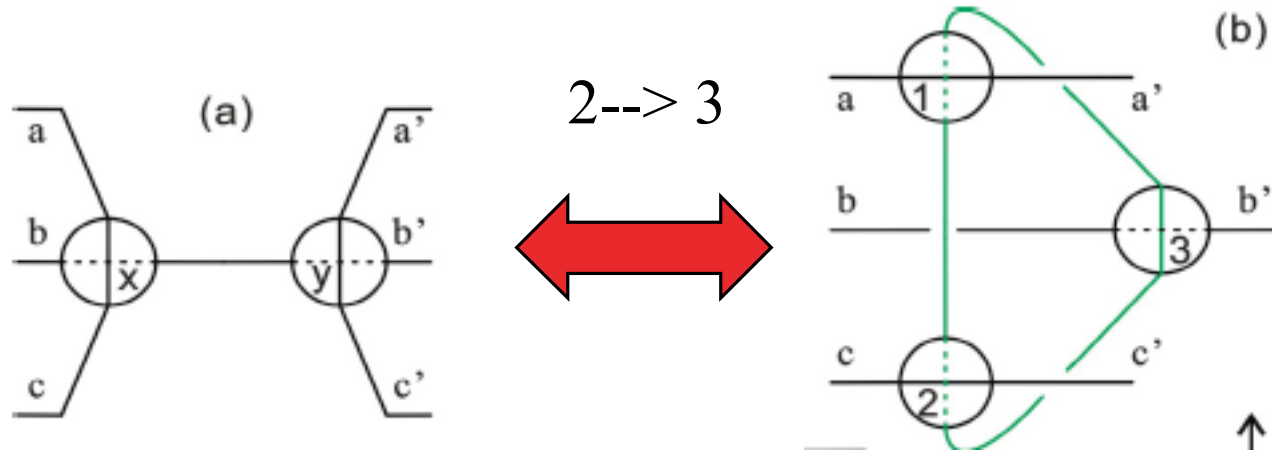
- Basic principle: When a subgraph with 1 to 4 nodes is dual to a triangulation of a disk in $\mathbb{R}^3$ which allows a Pachner move, the corresponding dual Pachner move is allowed.

 There are stable braids

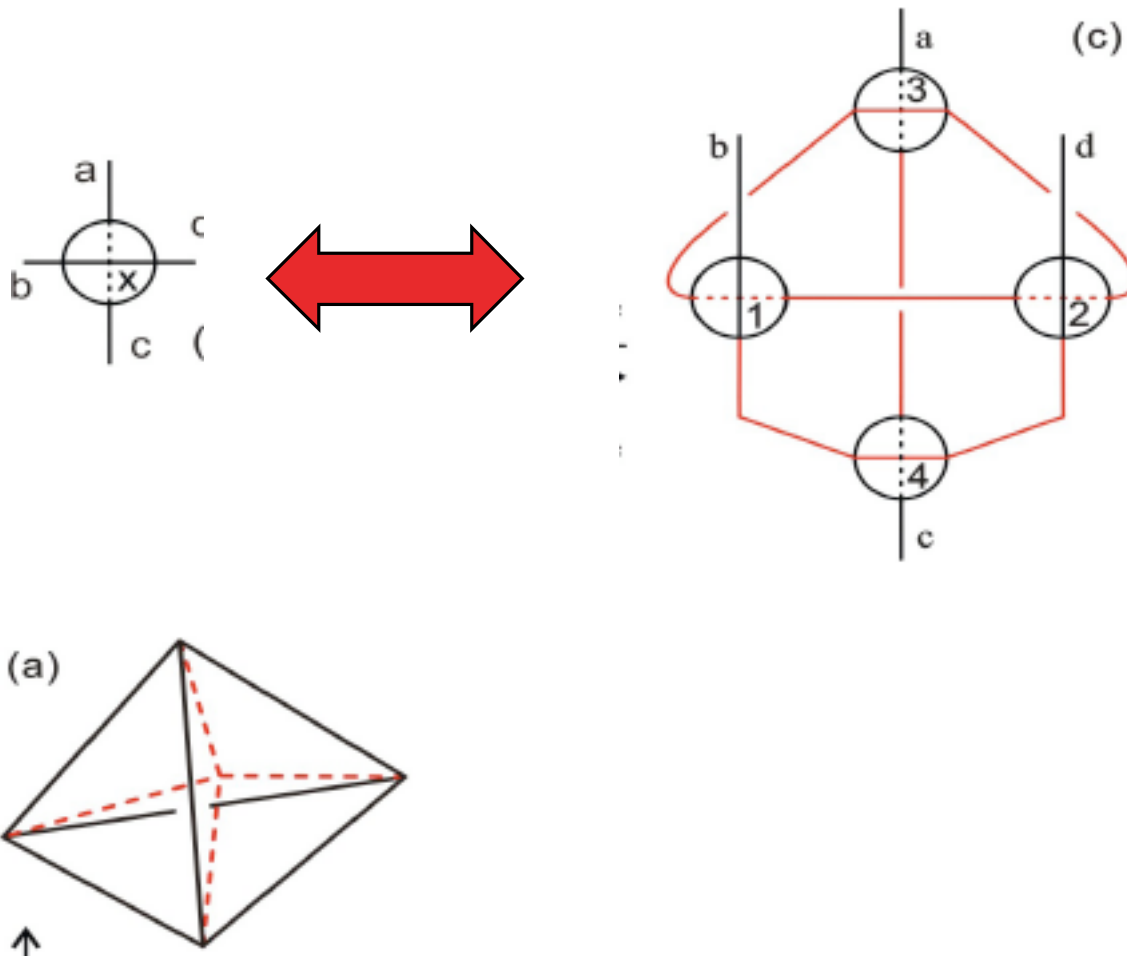


i.e. the $2 \rightarrow 3$ move is not allowed on these two nodes.

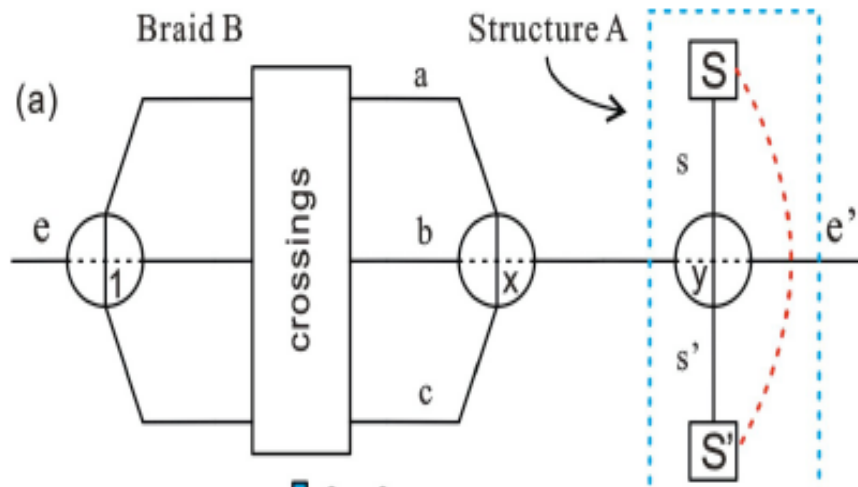
Allowed dual Pachner moves: 2 to 3 move



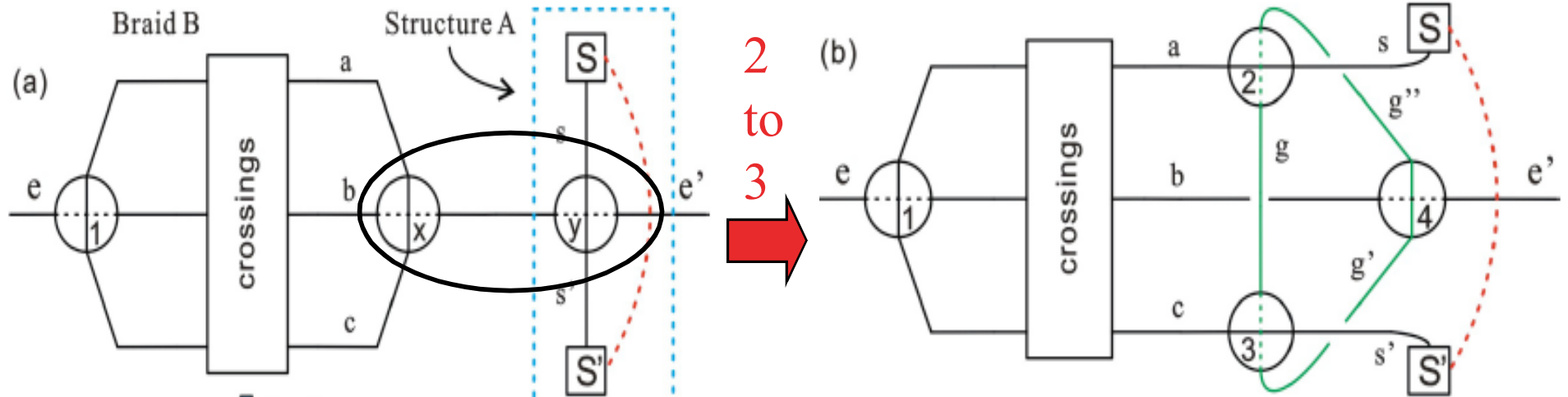
Allowed dual Pachner moves: 1 to 4 move



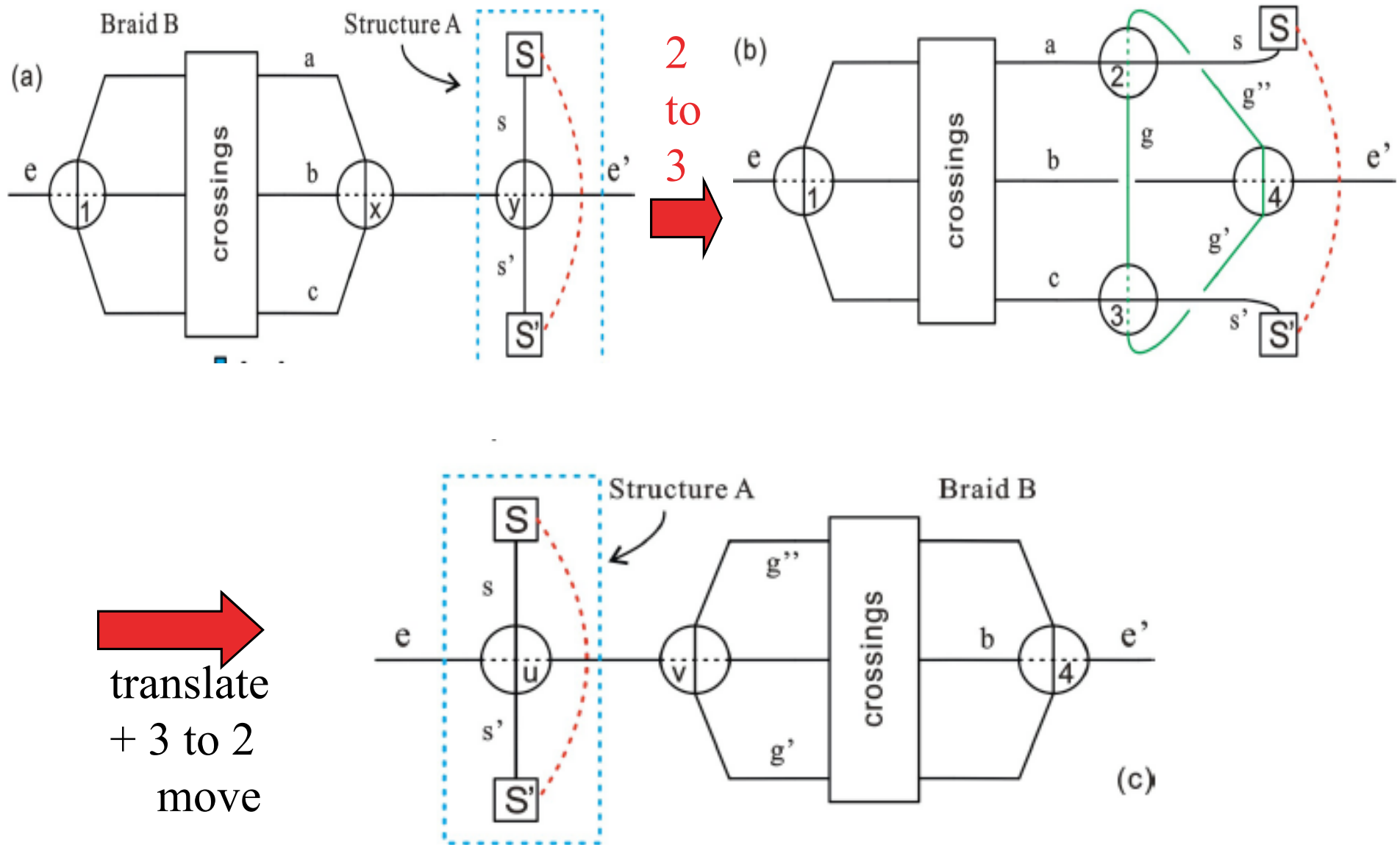
Right propagation, basic schema:



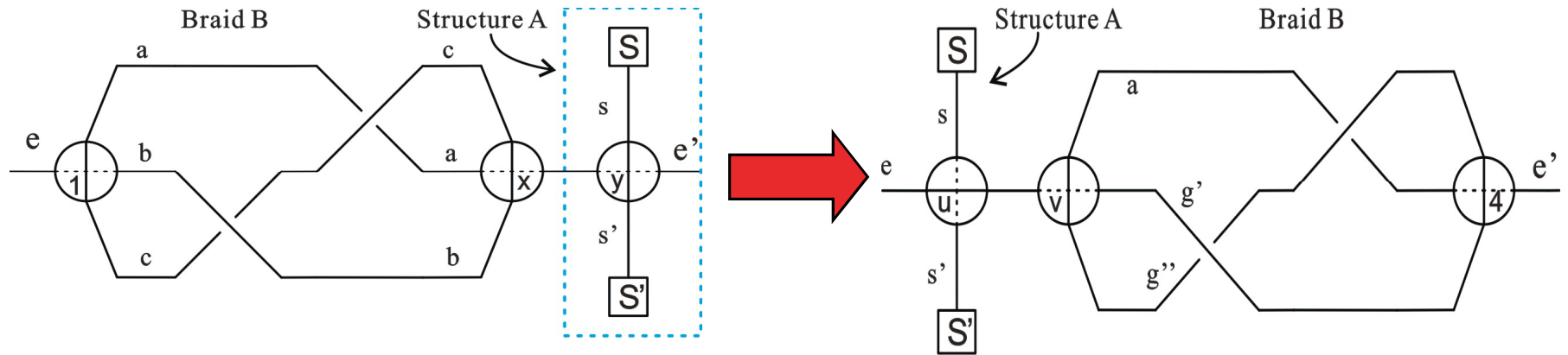
Right propagation, basic schema:



Right propagation, basic schema:



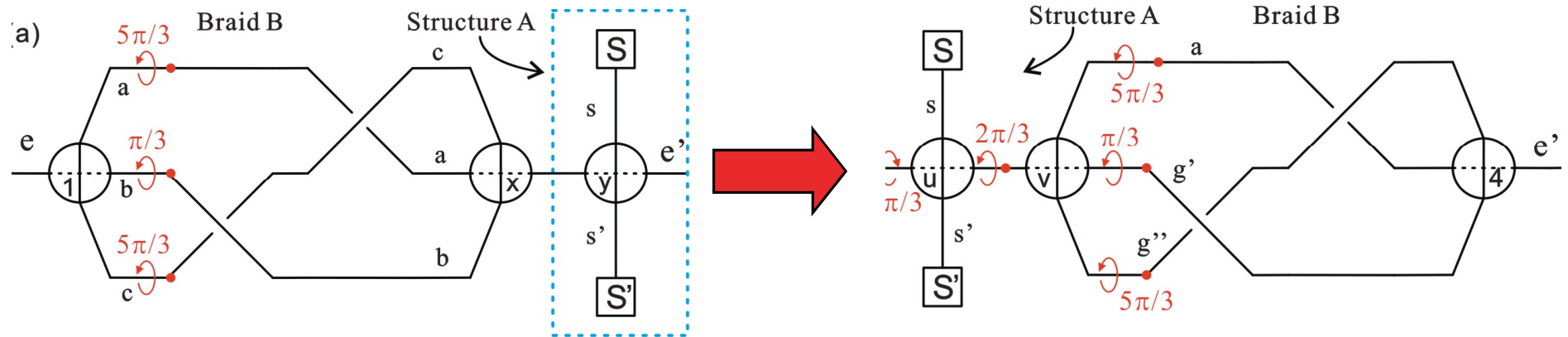
Example: right propagation, no framing:



*This graph propagates only to the right, hence
propagation is chiral!*

*Hence in LQG and spin foam models there are locally stable
braids states which propagate.*

Propagation with twists:



**This requires allowing 3 $\rightarrow$ 2 move with internal twists.
Amplitudes can depend on twists.**

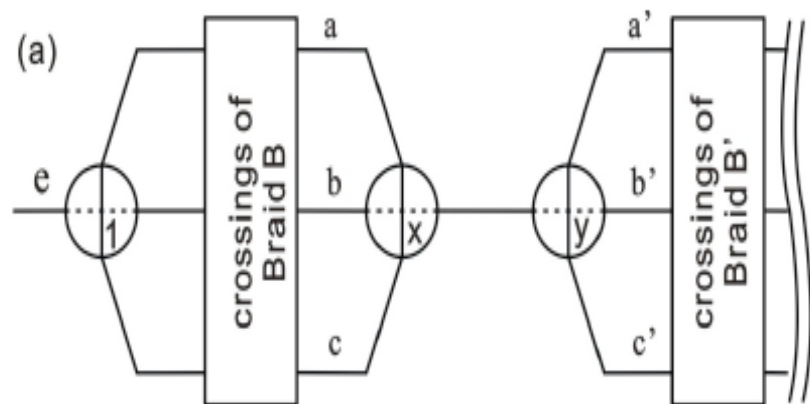
Note that twists are preserved, in this example!!

Propagation is chiral, some braids propagate only to the right. Their parity conjugates propagate only to the left.

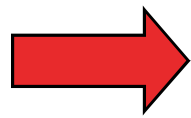
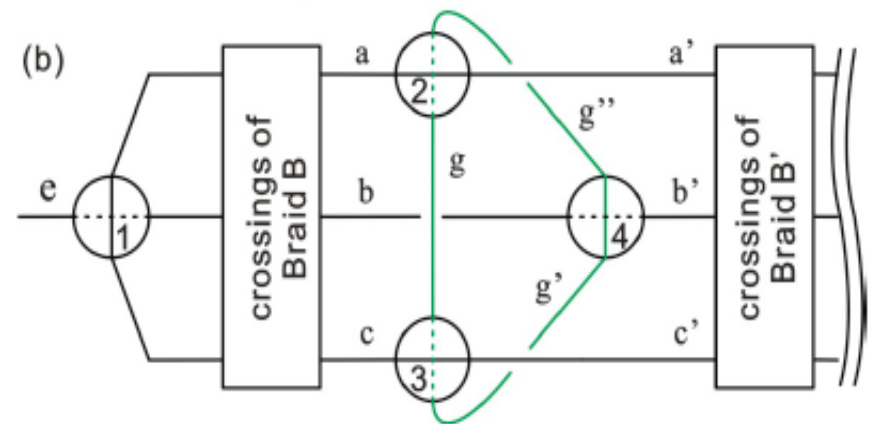
To propagate to the right, a braid must be right-reducible, in a canonical form, with untwisted external edges.

Irreducible braids do not propagate.

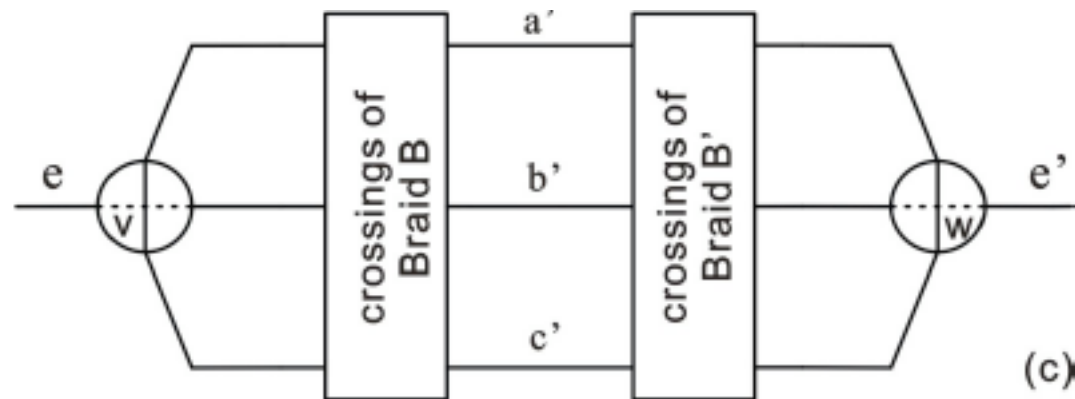
Right interaction, basic schema:



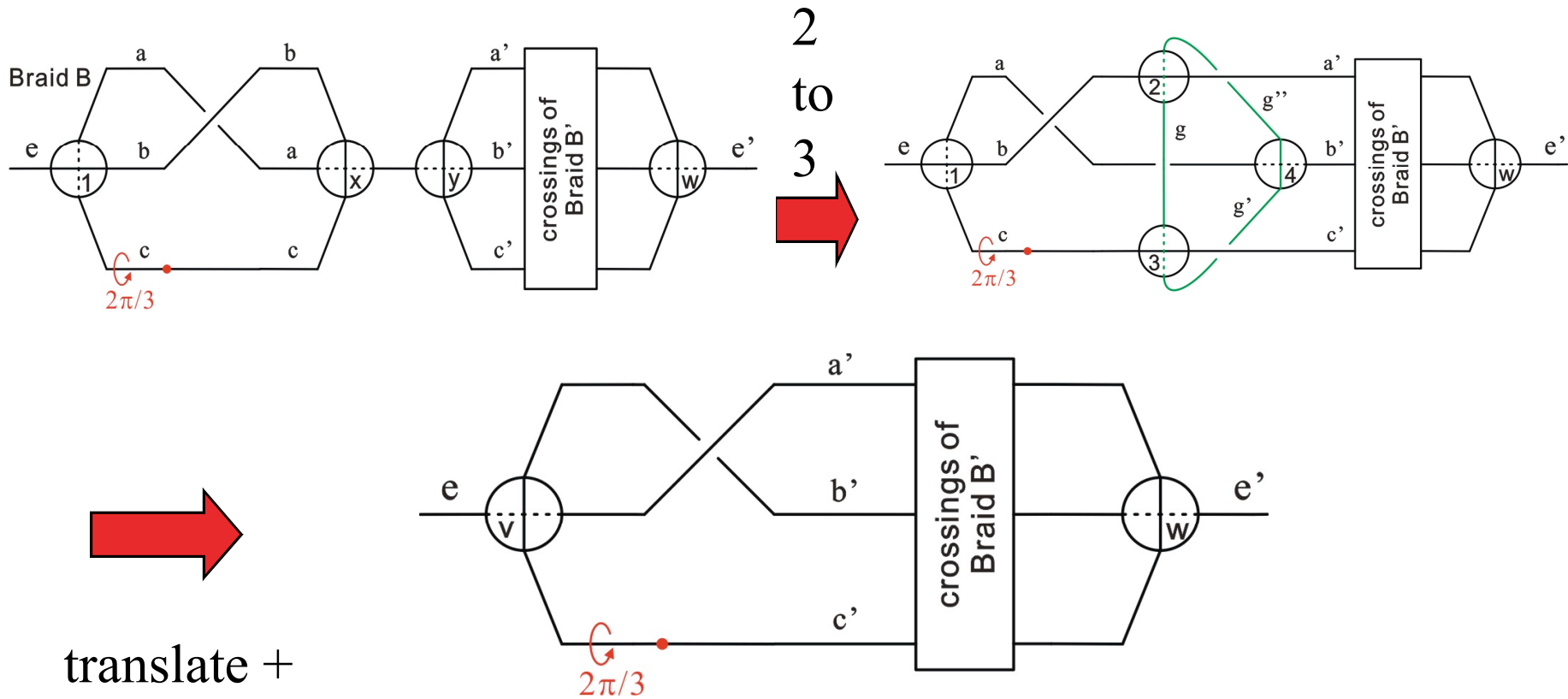
2
to
3



translate +
4 to 1 move



Right interaction, example:



translate +
4 to 1 move

- *The twist is preserved.*
- *This braid also propagates to the right preserving the twist*

Interacting is also chiral.

Right interacting implies right propagating

Now in progress:

- Are these excitations fermions?

They are chiral but could be spinors or chiral vectors.

Edges can be anyonic in 3d

We seek an inverse quantum Hall effect

- Masses? Chiral symmetry breaking?
- Momentum eigenstates constructed by superposing translations on regular lattice.
- Conservation laws in 4-valent case.
- Many other questions are still open...

Conclusions: (All with standard dual Pachner moves)

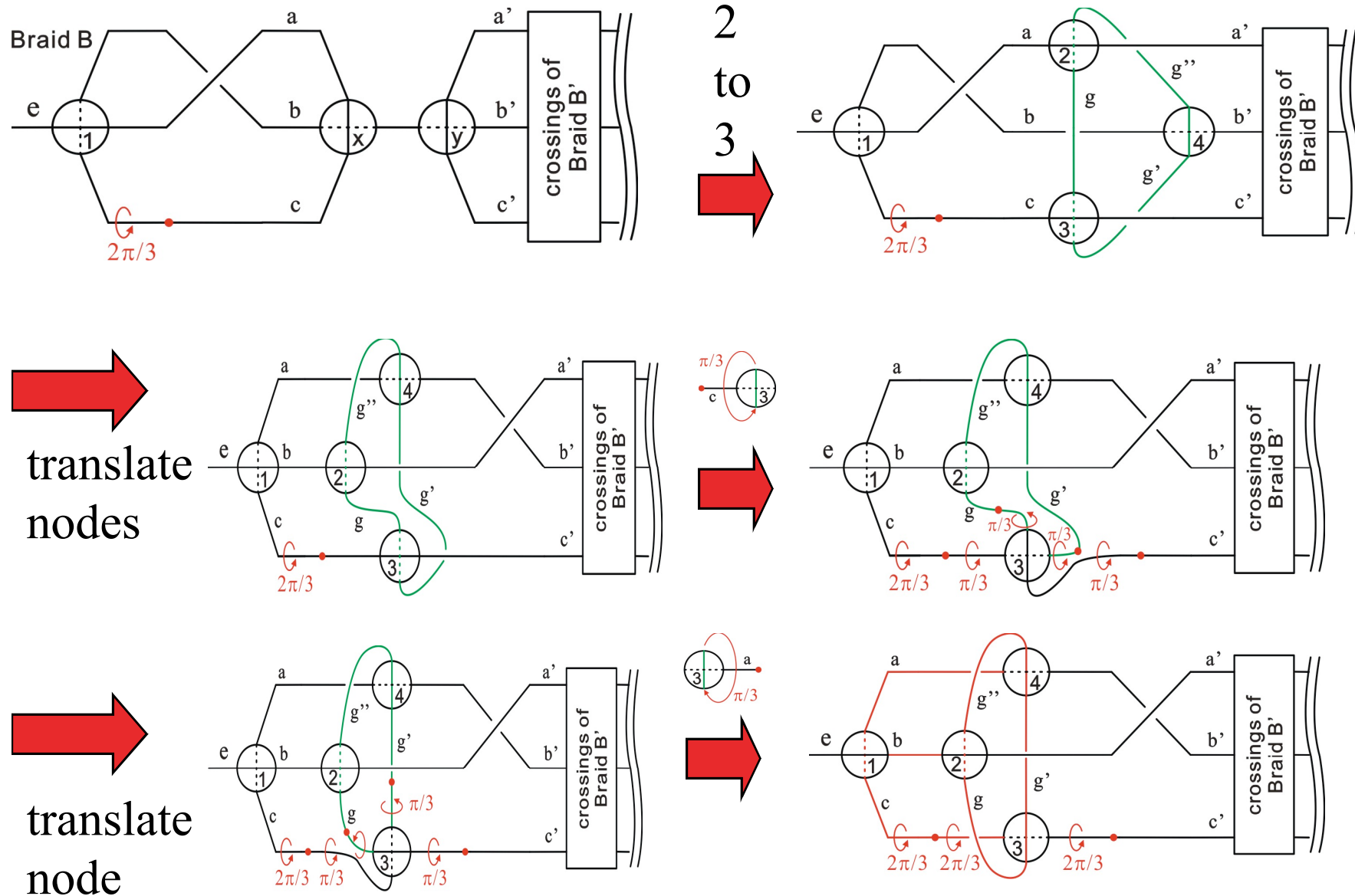
3 valent case:

Braids are absolutely conserved, no interactions
Capped braids propagate along edges of ribbons
Proposal for multi generation model, all states distinct
Correspondence to preon model, but has exotic states

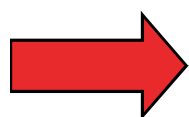
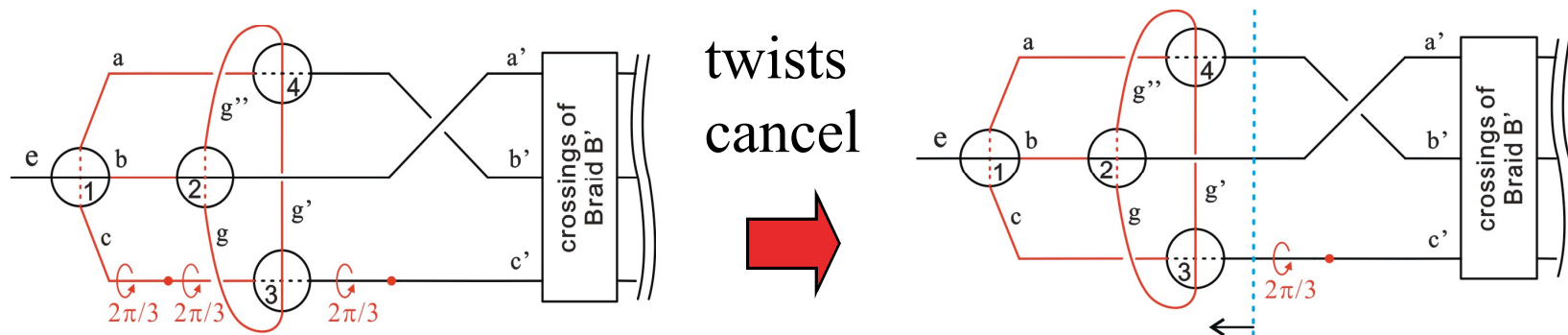
4-valent case: (with standard dual Pachner moves for sets dual to triangulations of regions of R^3)

Isolated braids stable.
Braids propagate, propagation is chiral
Some combine with adjacent braids, hence interact
Interactions are chiral.
Both framed and unframed braids propagate and interact
Correspondence with preons etc not yet established.

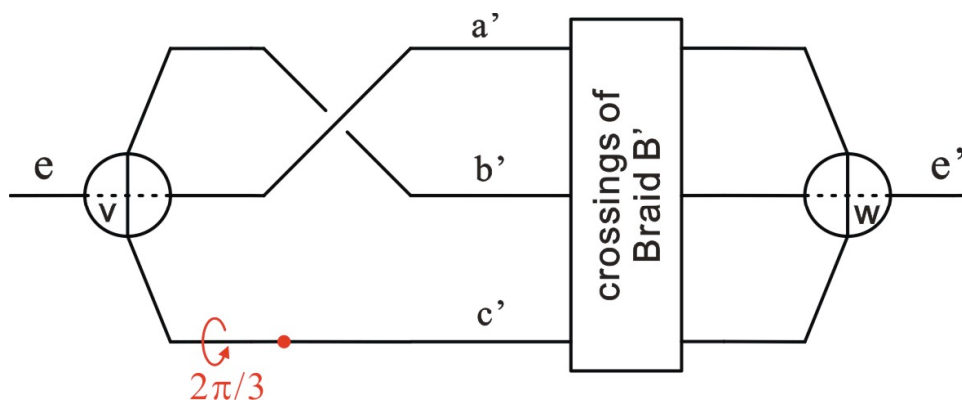
Right interaction, example, with twists, details:



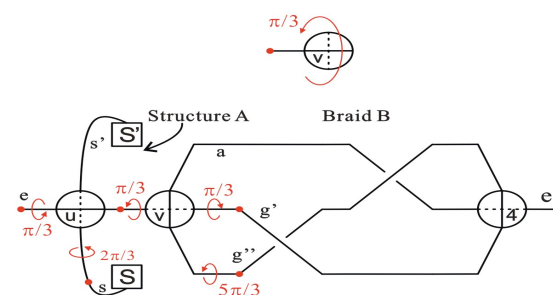
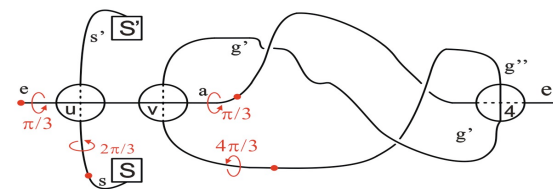
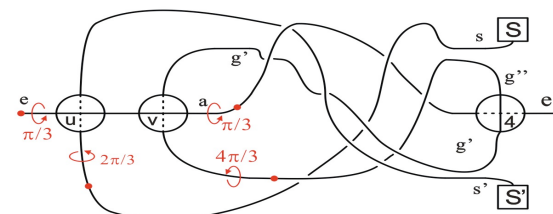
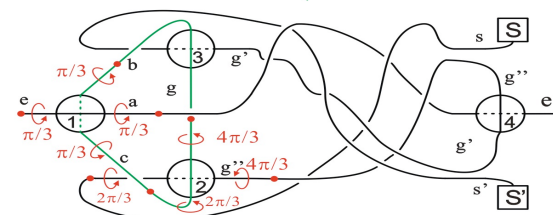
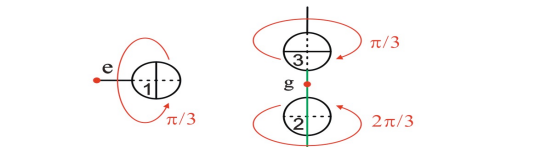
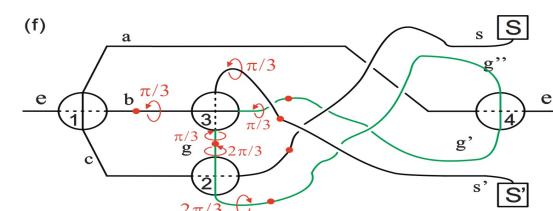
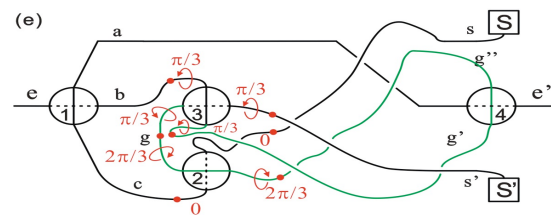
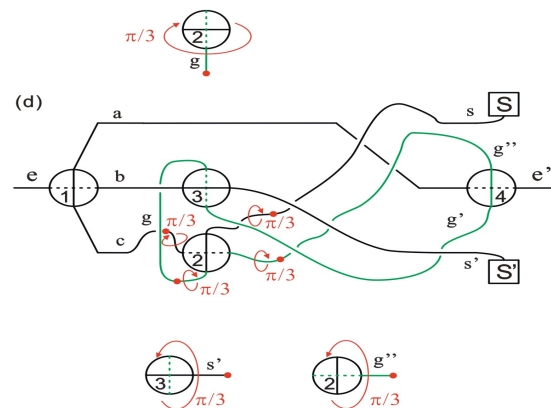
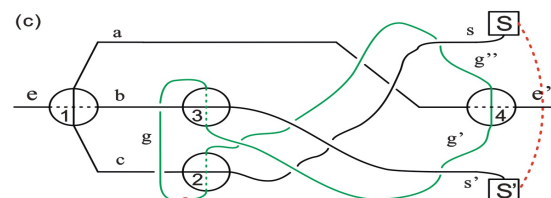
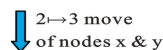
Right interaction, example, with twists, details:

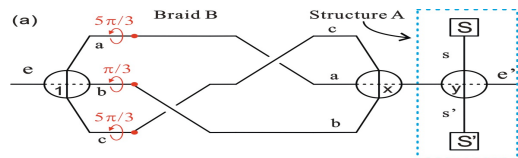


4 to 1
move

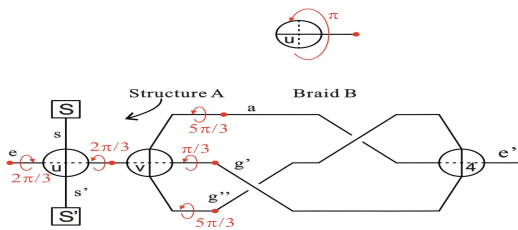
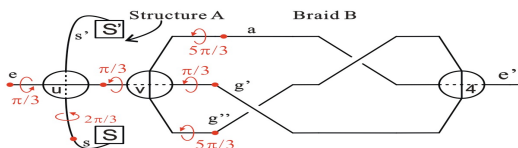
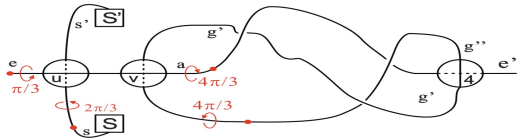
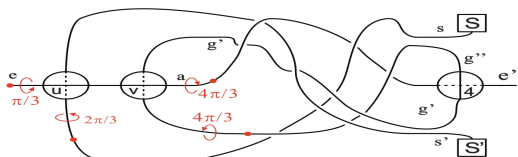
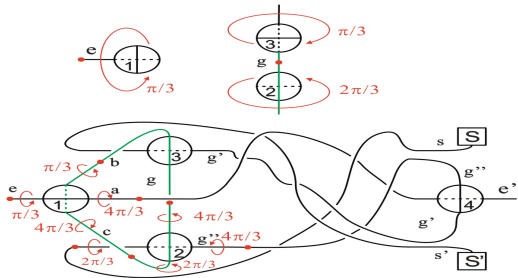
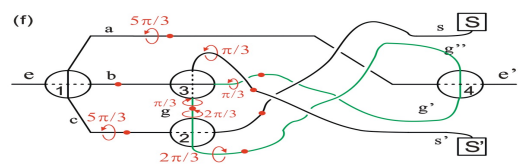
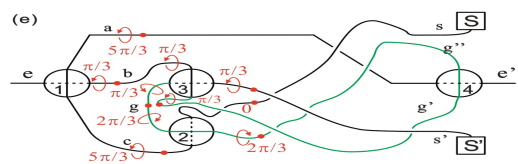
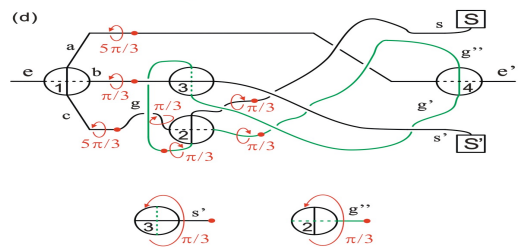
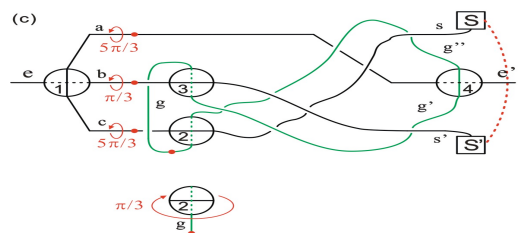
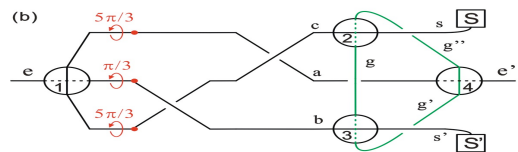


**Requires modified
3-->2 move to allow
 2π twist around
closed loop.**



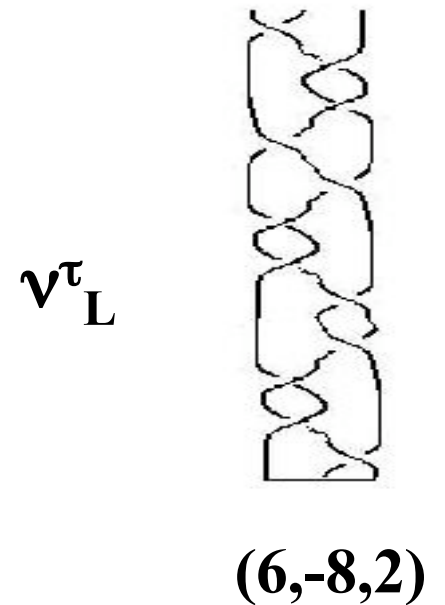
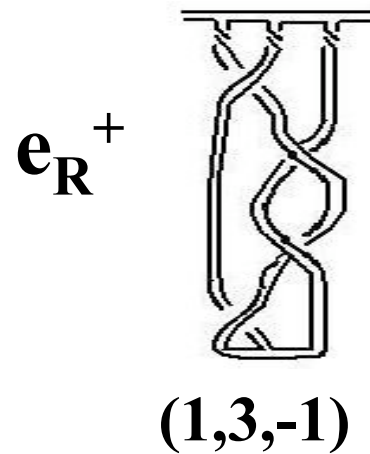
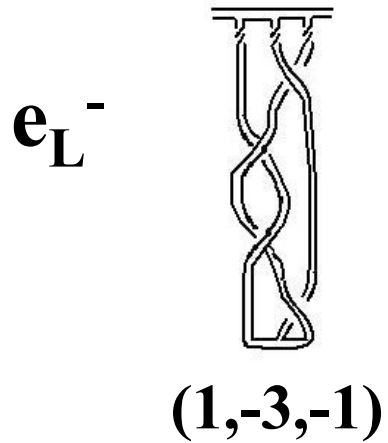
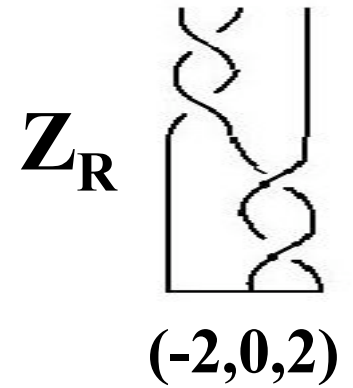
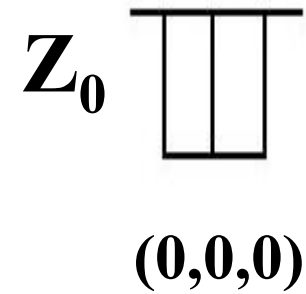
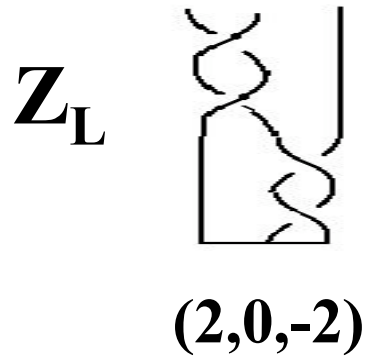
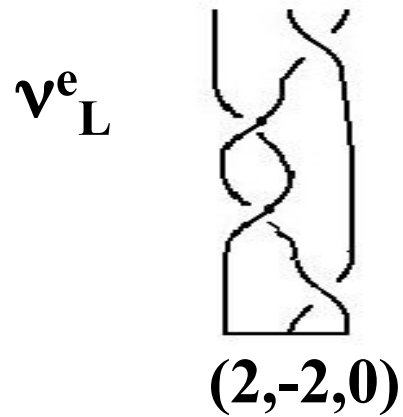


$2 \rightarrow 3$ move
of nodes x & y



Multigenerational scheme (Hackett):

- Based on Lou's number's (a,b,c)

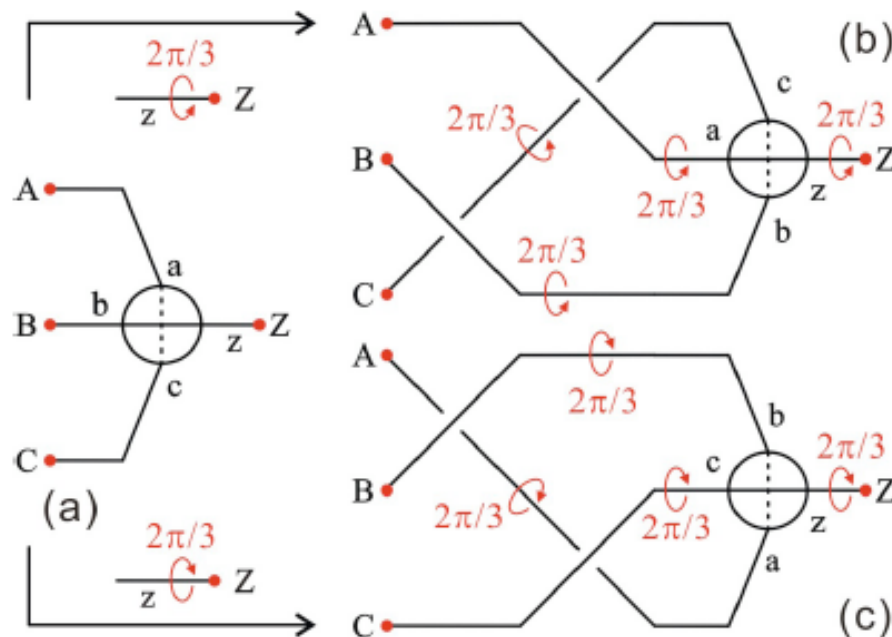


Equivalence moves for graph projections:

- Reidemeister moves
- Rotations of nodes

These rotations relate equivalent projections of nodes and their edges

$2\pi/3$ rotations:

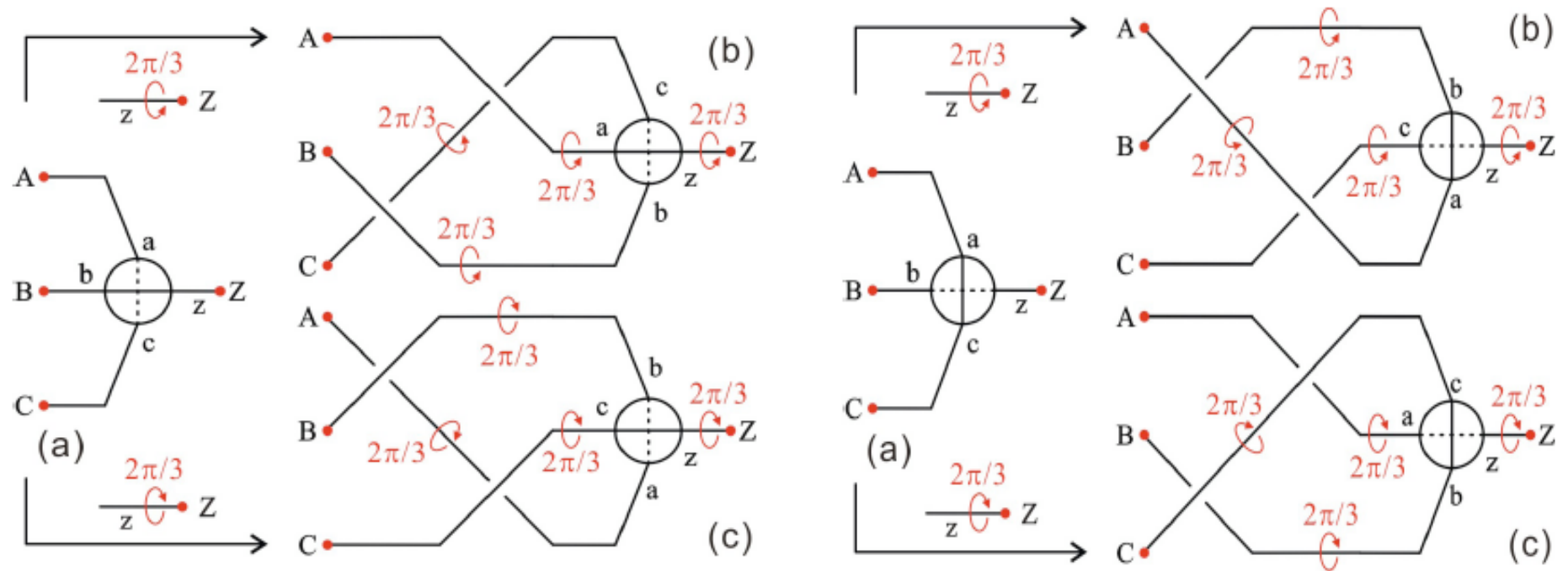


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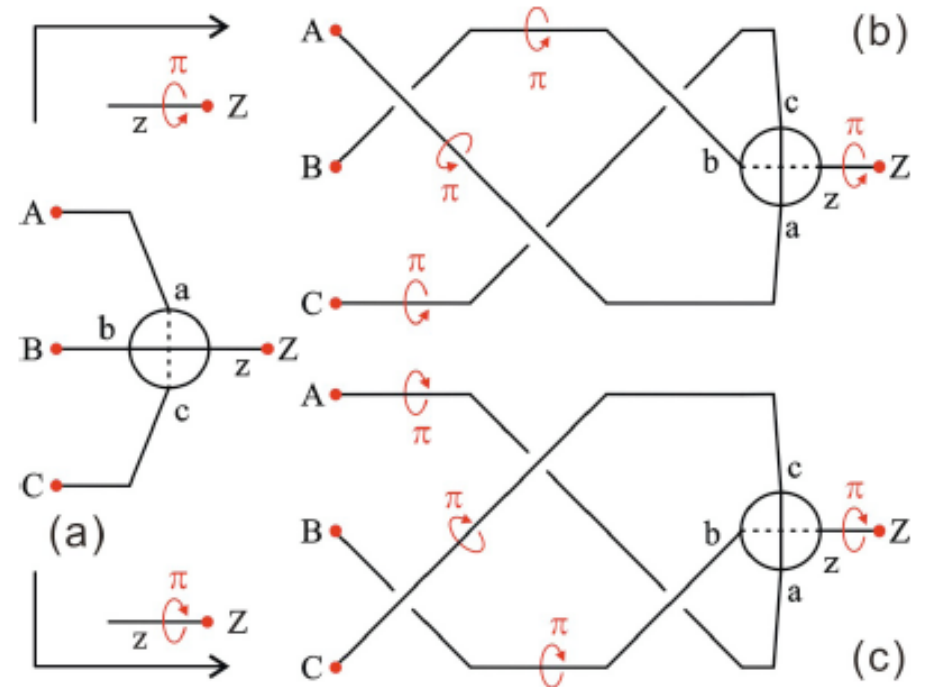
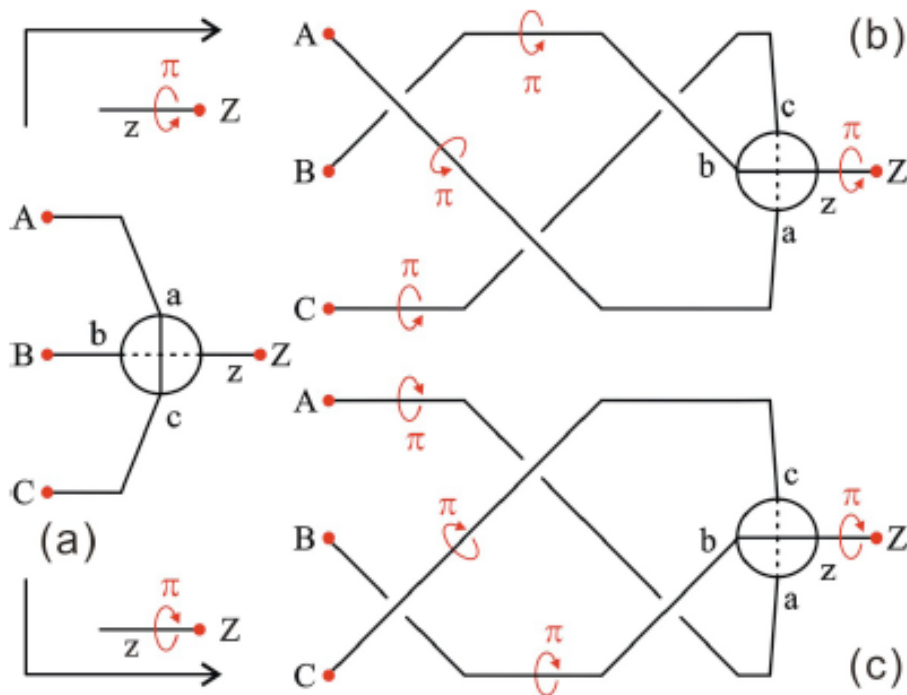
- Reidemeister moves
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π rotations:

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- Reidemeister moves
- Rotations of nodes

π rotations:



Detail: right propagation for unframed graphs, no twists:

