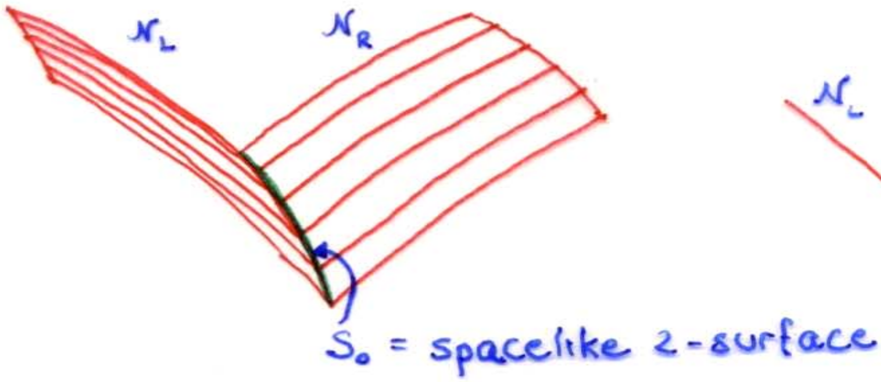
CANONICAL GRAVITY WITH FREE NULL INITIAL DATA

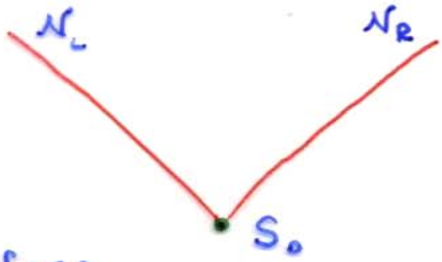
MICHAEL REISENBERGER
UNIVERSIDAD DE LA REPÚBLICA
URUGUAY

gr-qc 0703134

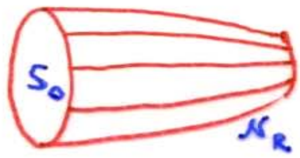
WHAT IS IT?

- Canonical GR using initial data on a piecewise $\underbrace{\text{null hypersurface}}_{\text{normal vector lightlike}}$
- could be a lightcone 
- we will use two intersecting null hypersurfaces
  - "open book"

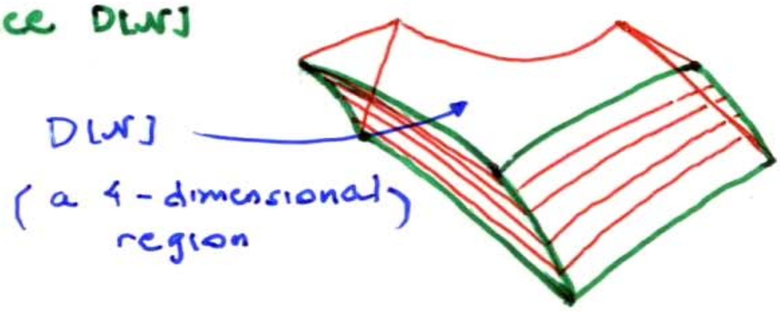




$\mathcal{N}_L, \mathcal{N}_R$ = null 3-surfaces swept out by null geodesics emerging normally from the two sides of S_0.



- initial data on $\mathcal{N} = \mathcal{N}_L \cup \mathcal{N}_R$ specifies solution to field equations in domain of dependence $D[\mathcal{N}]$



WHY STUDY THIS?

1. No constraints - can identify free, complete data (~1962 Sachs, Bondi, Penrose Dautcourt ...)

2. Lorentzian

3. Observables - free initial data has direct interpretation in terms of test light rays = observables

4. No problem of time

5. Holography Beckenstein → Susskind → Bousso bound:

If generators of a branch ($\mathcal{N}_R$ say) of $\mathcal{N}$ are non-expanding at S_0 then they argue

$$\text{Entropy on } \mathcal{N}_R \leq \frac{\text{Area}[S_0]}{4 \, \text{Area}_{\text{Planck}}}$$

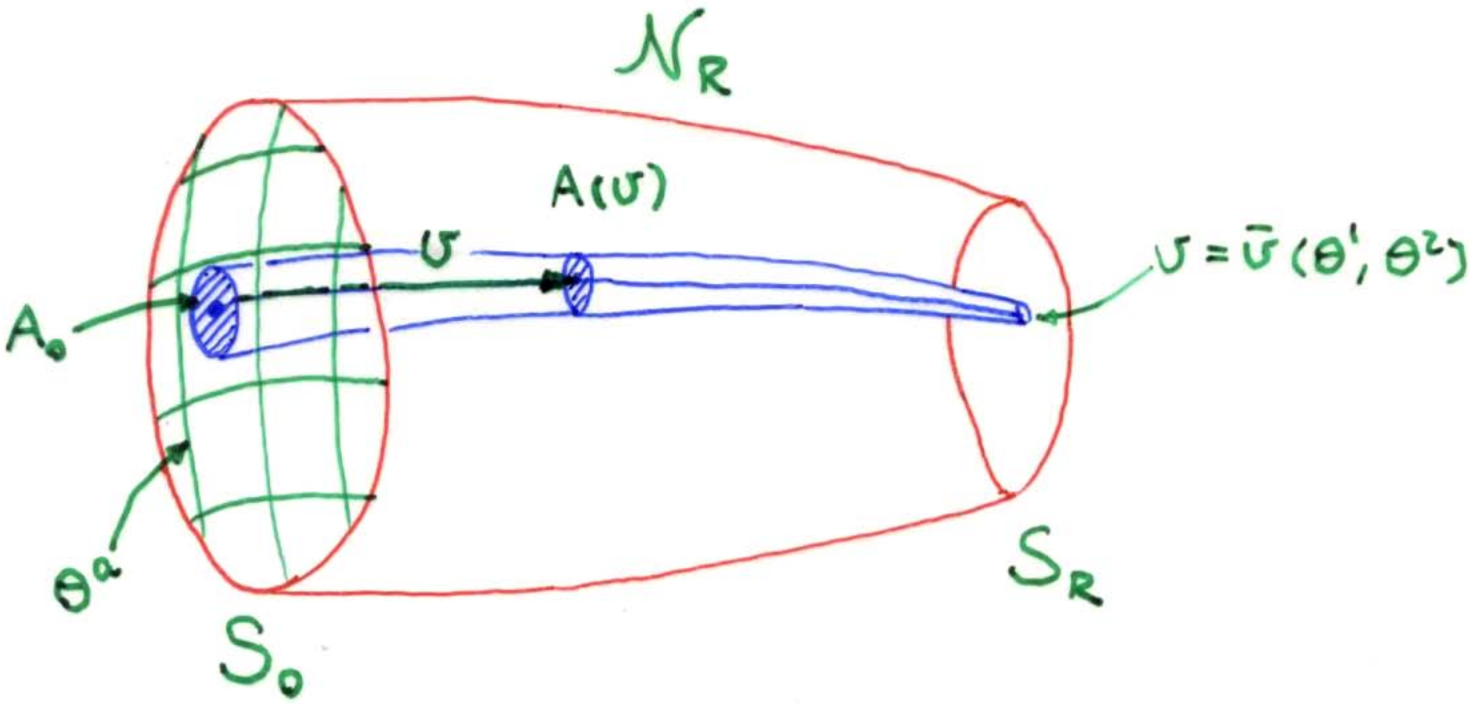
- usually highest entropy thermodynamic macrostate of a system has essentially all microstates.

→ suggests $\dim H = e^{A[S_0]/4 A_{\text{Planck}}}$

H = Hilbert space of gravitational field associated with data on $\mathcal{N}_R$

Perhaps gravity can be described by finite dimensional Hilbert spaces associated with regions of spacetime!
Null canonical GR ideal framework to check this.

THE FREE DATA 1: Coordinates adapted to $\mathcal{N}$



θ^1, θ^2 = coordinates on S_0

- held constant along generators

v = parameter along each generator

= "area parameter"

Def^n — cross sectional area of infinitesimal bundle of neighboring generators

$$A(v) = A_0 \, v^2$$

A_0 ↑ cross section at S_0

THE FREE DATA 2: The data

- data on all of $\mathcal{N}$ and data on S_0

On $\mathcal{N}_R$ ($\mathcal{N}_L$):

- "conformal 2-metric" $e_{ab}(\theta^1, \theta^2, v)$

Def^n: metric on $\mathcal{N}_R$ degenerate because $\mathcal{N}_R$ null

$$ds^2 = h_{ab}\, d\theta^a\, d\theta^b \longleftarrow \text{no } dv\text{s}$$

$$e_{ab} = h_{ab}/\sqrt{\det h} \longleftarrow \text{makes } \det e = 1$$

On S_0:

- $\rho_0 = \sqrt{\det h}$ = "area density"
- $\lambda = -\log(-\vec{n}_L \cdot \vec{n}_R)$ $\qquad \vec{n} = \frac{\partial}{\partial v}$ = tangent to generators
- $\omega_a = \dfrac{\vec{n}_R \cdot \nabla \vec{n}_L - \vec{n}_L \cdot \nabla \vec{n}_R}{\vec{n}_R \cdot \vec{n}_L}$
- $\phi : S_L \to S_R$ = diffeo defined by following generators from S_L to S_0 and then from S_0 to S_R

FREE DATA 3 : Comments

- Exclusion of caustics automatic:

  On $\mathcal{N}_R(\mathcal{N}_L)$ υ ranges over $[1, \bar{\upsilon}]$ $\bar{\upsilon} > 0$

  $\Rightarrow$ $\upsilon > 0$ ← no caustic

- If no caustics on $\mathcal{N}$ can eliminate all crossings of generators by going to covering spacetime.

- $e_{ab}(\theta, \upsilon)$ smooth + Einstein equation

  $\Rightarrow$ υ monotonic $\Rightarrow$ good parameter

- data free and complete but has gauge freedom: coordinate choices on S_0 and S_L, S_R (to define ϕ)

  → gauge fix coords on S_L, S_R to e_{ab} there

  → work with slightly larger data set subject to one constraint generating diffeos on S_0 — could also gauge fix here.

- ϕ neglected in past (though implicit in Sach's data)

  - essentially canonically conjugate to ω

FREE DATA 3 : Comments

- Exclusion of caustics automatic:

  On $\mathcal{N}_R (\mathcal{N}_L)$ v ranges over $[1, \bar{v}]$ $\bar{v} > 0$

  $\Rightarrow v > 0 \leftarrow$ no caustic

- If no caustics on $\mathcal{N}$ can eliminate all crossings of generators by going to covering spacetime.

- $e_{ab}(\theta, v)$ smooth + Einstein equation

  $\Rightarrow v$ monotonic $\Rightarrow$ good parameter

- data free and complete but has gauge freedom: coordinate choices on S_0 and S_L, S_R (to define ϕ)

  $\rightarrow$ gauge fix coords on S_L, S_R to e_{ab} there

  $\rightarrow$ work with slightly larger data set subject to one constraint generating diffeos on S_0 — could also gauge fix here.

- ϕ neglected in past (though implicit in Sach's data)

  - essentially canonically conjugate to ω

POISSON BRACKET 1

- PB corresponds to Einstein - Hilbert action
- will talk about true Poisson bracket on initial data (satisfies Jacobi relation) - not pre-Poisson bracket of gr-qc 0703134

Method for obtaining Poisson bracket

"observables"

1. Define a class of "nice" diffeo invariant functions of geometry in interior of domain of dependence
2. Take Poisson bracket on observables to be Peierls bracket (a good fundamental definition of Poisson brackets)
3. Look for bracket on initial data which reproduces bracket on observables when these are expressed as functions of the initial data

↑

Sufficient condition:

P.B. on initial data a "generalized inverse" to symplectic 2-form

POISSON BRACKET 2

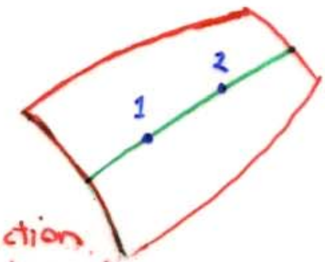
Brackets of e_{ab}: (see also Gambini & Restuccia '77)

Parametrize e_{ab} by a complex scalar μ:

$$ds^2 = \rho\, e_{ab}\, d\theta^a d\theta^b$$
$$= \frac{\rho}{1-\mu\bar{\mu}}(dz + \mu d\bar{z})(d\bar{z} + \bar{\mu} dz)$$

with

$$z = \theta^1 + i\theta^2 \qquad \rho = \sqrt{\det h_{ab}}$$



step function

$$\{\mu(1), \bar{\mu}(2)\} = 4\pi G \frac{\delta^2(\theta_1 - \theta_2)}{\sqrt{\rho_1}\sqrt{\rho_2}}\, H(1,2)$$
$$\times [1-\mu\bar{\mu}]_1 [1-\mu\bar{\mu}]_2\, e^{\int_1^2 \frac{\bar{\mu} d\mu - \mu d\bar{\mu}}{1-\mu\bar{\mu}}}$$

$$H(1,2) = \begin{cases} 1 & \text{if 2 follows 1 along generator starting from } S_0 \\ 0 & \text{if 1 follows 2} \end{cases}$$

- Only data on same generator have non-zero bracket
  - ← from causality: points on distinct generators are spacelike separated
- First line Minkowski space bracket for complex Klein-G
- Bracket covariant under transformations of θ coordinates

POISSON BRACKET 3

A polarization

$\{\mu(1), \mu(2)\} = 0$, indeed μ, ρ, ϕ all commute

so in a quantization could use wavefunctions $\Psi(\rho, \phi, \mu)$ depending only on ρ, ϕ and holomorphically on μ.

A loopy speculation

$$A \equiv \mathrm{Area}[S_0] = \int_{S_0} \rho_0 \, d^2\theta$$

$$\{\lambda(1), \rho_0(2)\} = -8\pi G\, \delta^2(\theta_1 - \theta_2)$$

$$\Rightarrow \{\lambda(1), A\} = -\frac{8\pi A_{planck}}{\hbar}$$

$\Rightarrow$ In "loop quantization" of scalar λ and density ρ_0

$\exists\ \widehat{e^{iK\lambda(1)}}$ unitary which acts by adding a quantum of area $8\pi K A_{Planck}$ ($K \in \mathbb{R}$ a constant

$\Rightarrow$ discrete area spectrum

- Don't know that this is correct quantization in context of whole theory.

CURRENT WORK

- Quantization
- (with Gonzalo Amiano) classical evolution of free data when S_0 moved
  - quantizations should be equivalent for different choices of S_0