The Raychaudhuri Equation for the Interior of Effective Loop Quantum Black Holes

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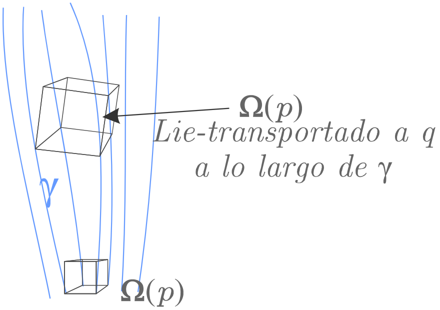


Raychaudhury identity:

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\underset{\text{expansion}}{\underset{\downarrow}{\theta^2}} - \underset{\text{shear}}{\underset{\downarrow}{\sigma^{ab}}}\sigma_{ab} + \underset{\text{twist}}{\underset{\downarrow}{\omega^{ab}}}\omega_{ab} - \underset{\text{Ricci}}{\underset{\downarrow}{R_{ab}}}\gamma^a \underset{\text{tangent to }\gamma}{\underset{\downarrow}{\gamma^b}}$$

Evolution of Ω :

$$\mathcal{L}_\gamma \Omega = \theta\Omega \iff \{H, \Omega\} = \theta\Omega$$

Expansion factor and singularities.

- Atractive or Repulsive interactio?

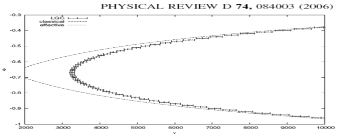
$$\theta > 0 \qquad \text{or} \quad \theta < 0$$

- Classical GR+the strong energy condition ($R_{ab}\gamma^a\gamma^b \geq 0$) implies:

$$\theta \to -\infty$$

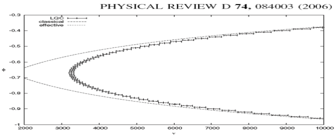
- Divergence of θ is a necessary condition in singularity theorems

- Ashtekar, Pawlowski and Singh:



Friedmann-Robertson-Walker singularity resolution

- Ashtekar, Pawlowski and Singh:



- θ is bounded:

$$H_{eff}^2 = \frac{\theta_{eff}^2}{9} = \frac{1}{4\underset{\substack{\downarrow \\ 4\sqrt{3}\pi\gamma l_{Pl}^2}}{\lambda^2}\underset{\substack{\downarrow \\ \text{Immirzi}}}{\gamma}} \sin^2(2\lambda \underset{\substack{\downarrow \\ \textit{canonical coordinate}}}{\beta}) \approx \frac{8\pi G}{3}\rho(1 - \frac{\rho}{\underset{\substack{\downarrow \\ 0.41\rho_{\text{Planck}}}}{\rho_{crit}}}) \underset{\rho << \rho_{crit}}{\longrightarrow} \frac{8\pi G}{3}\rho$$

$$\rho \approx \frac{3}{8\pi G\gamma^2}\frac{\sin^2(\lambda\beta)}{\lambda^2}$$

Black Hole Interior (Kantowski-Sachs)

- Symmetry group: $\mathbb{R} \times SU(2)$

$$[\xi_i, \xi_j] = \epsilon_{ij}^{k} \underset{\in \mathfrak{su}(2)}{\underset{\downarrow}{\xi_k}} \qquad [\xi_i, \underset{\in \mathfrak{R}}{\underset{\downarrow}{\zeta}}] = 0$$

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- +No matter
- + Einstein Field Equations

$\Longrightarrow$ Schwarzschild Interior:

$$ds^2 = -\left(1 - \frac{2m}{T}\right) dR^2 + \frac{1}{\left(1 - \frac{2m}{T}\right)} dT^2 + T^2 d\Omega^2$$

Hamiltonian formulation in Ashtekar variables

- Kantowski-Sachs ($\mathbb{R} \times SO(3)$) symmetric connection and triads:

$$A = A_a{}^i \underset{-i\sigma_i/2}{\underset{\downarrow}{\tau_i}} dx^a = L^{-1} c\tau_3 dx + b\tau_2 d\theta - b\tau_1 \sin\theta d\phi + \tau_3 \cos\theta d\phi,$$

$$\tilde{E} = p_c \tau_3 \sin\theta\, \partial_x + L^{-1} p_b \tau_2 \sin\theta\, \partial_\theta - L^{-1} p_b \tau_1\, \partial_\phi, \tag{1}$$

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$$qq^{ab} = \delta^{ij} \tilde{E}^a{}_i \tilde{E}^b{}_j, \quad (3)$$

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- Geometrical quantities

$$S^2 - \text{area: } A = 4\pi p_c \;; \qquad (I \times S^2) - \text{volume: } \Omega = 4\pi p_b \sqrt{p_c}\,. \tag{6}$$

$$\text{Expansion Factor: } \theta = \frac{\dot{\Omega}}{\Omega} \tag{7}$$

$$\text{Shear: } \sigma = \frac{1}{\sqrt{3}} \left(\frac{1}{p_b} \dot{p}_b - \frac{1}{p_c} \dot{p}_c \right) \tag{8}$$

$$\text{Twist: } \omega = 0 \tag{9}$$

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- Hamiltonian "al gusto"

Classical dynamics

- $$H_{cl} = -\frac{N}{2G\gamma^2}\left[2bc\sqrt{p_c} + (b^2+\gamma^2)\frac{p_b}{\sqrt{p_c}}\right].$$

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- On the constraint surface:

$$\theta \approx \frac{1}{\gamma}\frac{1}{b\ \underset{\text{divergences?}}{\underset{\downarrow}{\sqrt{p_c}}}}\left(3b^2-\gamma^2\right)$$

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- $$\begin{aligned} R_{00} &= -\frac{1}{3}\theta^2 - \sigma^{ab}\sigma_{ab} + \omega^{ab}\omega_{ab} - \frac{\mathrm{d}\theta}{\mathrm{d}\tau} \\ &= \frac{1}{2\gamma^2 p_b p_c}\left(b^2 p_b + 2bcp_c + p_b^2\right) \\ &\approx 0 \rightarrow \text{Eintein Field Eq. in vac} \end{aligned}$$

Effective Semiclassical Black Hole InteriorS

- "Holonomize" classical connection::

$$b \longrightarrow \frac{\sin(\mu_b b)}{\mu_b}, \qquad c \longrightarrow \frac{\sin(\mu_c c)}{\mu_c}$$

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  μ_0-scheme:

$$\mu_b = cte, \qquad \mu_c = cte$$

  $\bar{\mu}$-scheme:

$$\bar{\mu}_b = \sqrt{\frac{\Delta}{p_b}}, \qquad \bar{\mu}_c = \sqrt{\frac{\Delta}{p_c}},$$

  Very succesfull $\bar{\mu}'$-scheme:

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- Hamiltonian

$$H_{eff} = -\frac{N}{2G\gamma^2}\left[2\frac{\sin(\mu_b b)}{\mu_b}\frac{\sin(\mu_c c)}{\mu_c}\sqrt{p_c} + (\left(\frac{\sin(\mu_b b)}{\mu_b}\right)^2 + \gamma^2)\frac{p_b}{\sqrt{p_c}}\right]$$

Effective expansion factors

- μ_0 :

$$\theta_0 = +\frac{1}{p_b}\frac{1}{\gamma}\sqrt{p_c}\frac{\sin(\mu_c c)}{\mu_c}\cos\left(\mu_b b\right) + \frac{1}{\sqrt{p_c}}\frac{1}{\gamma}\frac{\sin\left(\mu_b b\right)}{\mu_b}\cos\left(\mu_b b\right) + \frac{1}{\sqrt{p_c}}\frac{1}{\gamma}\frac{\sin(\mu_b b)}{\mu_b}\cos(\mu_c c)$$

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- $\bar{\mu}$:

$$\bar{\theta} = +\frac{1}{p_b}\frac{1}{\gamma}\frac{1}{\sqrt{\Delta}}p_c \sin\left(\sqrt{\frac{\Delta}{p_c}}c\right)\cos\left(\sqrt{\frac{\Delta}{p_b}}b\right) + \frac{1}{\sqrt{p_c}}\frac{1}{\gamma}\frac{1}{\sqrt{\Delta}}p_b^{\frac{1}{2}}\sin\left(\sqrt{\frac{\Delta}{p_b}}b\right)\cos\left(\sqrt{\frac{\Delta}{p_b}}b\right) + \frac{1}{\sqrt{p_c}}\frac{1}{\gamma}\frac{1}{\sqrt{\Delta}}p_b^{\frac{1}{2}}\sin\left(\sqrt{\frac{\Delta}{p_b}}b\right)\cos\left(\sqrt{\frac{\Delta}{p_c}}c\right)$$

- $\bar{\mu}'$

$$\bar{\theta}' = \frac{1}{\Omega}\left[\Omega, H_{eff}\right]$$
$$= \frac{1}{\gamma}\frac{1}{\sqrt{\Delta}}\left[\sin\left(\frac{\sqrt{p_c\Delta}}{p_b}c\right)\cos\left(\sqrt{\frac{\Delta}{p_c}}b\right) + \right.$$
$$\left. + \sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(\frac{\sqrt{p_c\Delta}}{p_b}c\right) + \sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(\sqrt{\frac{\Delta}{p_c}}b\right)\right]$$

$$\theta_{fis} = \frac{p_c\sin\left(b\mu_b\right)\left(\sqrt{-\frac{\gamma^4\Delta^2\csc^2(b\mu_b)}{p_c^2} - \sin^2\left(b\mu_b\right) - \frac{2\gamma^2\Delta}{p_c} + 4} + \cos\left(b\mu_b\right)\right) - \gamma^2\Delta\cot\left(b\mu_b\right)}{2\gamma\sqrt{\Delta}p_c}$$

BOUNDED ON ALL PHASE SPACE

Gravitational repulsion

without energy conditions violations

Shear is bounded:

$$\bar{\sigma}' = \frac{1}{\sqrt{3}}\left(\frac{1}{p_b}\dot{p}_b - \frac{1}{p_c}\dot{p}_c\right)$$

$$= \frac{1}{\sqrt{3}}\frac{1}{\gamma}\frac{1}{\sqrt{\Delta}}\left(\sin\left(\frac{\sqrt{p_c\Delta}}{p_b}c\right)\cos\left(\sqrt{\frac{\Delta}{p_c}}b\right)\right.$$

$$\left. + \sin\left(\sqrt{\frac{\Delta}{p_c}}b\right) - 2\sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(\frac{\sqrt{p_c\Delta}}{p_b}c\right)\right)$$

- $$\begin{aligned}
\dot{\theta} = &\ (\text{bounded terms on all phase space}) \\
&+ (\text{bounded terms on the constraint surface}) \\
&+ \frac{1}{\gamma^2}\frac{1}{\Delta}\left[\frac{\sqrt{p_c\Delta}}{p_b}c - \sqrt{\frac{\Delta}{p_c}}b\right]\cos\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right)\sin\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right) \\
&+ \frac{1}{\gamma^2}\frac{1}{\Delta}\left(\frac{\sqrt{p_c\Delta}}{p_b}c - \sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(2\sqrt{\frac{\Delta}{p_c}}b\right)\sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(\frac{\sqrt{p_c\Delta}}{p_b}c\right) \\
&+ \frac{1}{\gamma^2}\frac{1}{\Delta}\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right)\sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(\sqrt{\frac{\Delta}{p_c}}b + \frac{\sqrt{p_c\Delta}}{p_b}c\right)
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- Need to know something about $\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right)$

Expansion factor evolution

- $$\begin{aligned}\dot{\theta} &= (\text{bounded terms on all phase space}) \\ &+ (\text{bounded terms on the constraint surface}) \\ &+ \frac{1}{\gamma^2}\frac{1}{\Delta}\left[\frac{\sqrt{p_c\Delta}}{p_b}c - \sqrt{\frac{\Delta}{p_c}}b\right]\cos\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right)\sin\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right) \\ &+ \frac{1}{\gamma^2}\frac{1}{\Delta}\left(\frac{\sqrt{p_c\Delta}}{p_b}c - \sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(2\sqrt{\frac{\Delta}{p_c}}b\right)\sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(\frac{\sqrt{p_c\Delta}}{p_b}c\right) \\ &+ \frac{1}{\gamma^2}\frac{1}{\Delta}\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right)\sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\cos\left(\sqrt{\frac{\Delta}{p_c}}b + \frac{\sqrt{p_c\Delta}}{p_b}c\right)\end{aligned}$$
- Need to know something about $\left(\sqrt{\frac{\Delta}{p_c}}b - \frac{\sqrt{p_c\Delta}}{p_b}c\right)$
- Qualitative analysis+theory of differential equations+numerical investigations

Qualitative analysis of the constraint

$$H = 2\sin\left(\sqrt{\frac{\Delta}{p_c}}b\right)\sin\left(\frac{\sqrt{p_c\Delta}}{p_b}c\right) + \sin^2\left(\sqrt{\frac{\Delta}{p_c}}b\right) + \frac{\Delta\gamma^2}{p_c} \approx 0$$

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- For finite p_c, p_b is bounded above onshell.
- As p_c is bounded below and $p_b \neq 0$, $\mathbf{V} = 4\pi p_b \sqrt{p_c} \neq \mathbf{0}$. (Clasically $\mathbf{V} \to \mathbf{0}$ on the physical singularity and on the horizon).

Numerical analysis is on work...

Conclusions

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- *THANKS!*