S-Matrix for AdS from General Boundary QFT

Daniele Colosi, Max Dohse, Robert Oeckl

Centro de Ciencias Matematicas
UNAM Campus Morelia

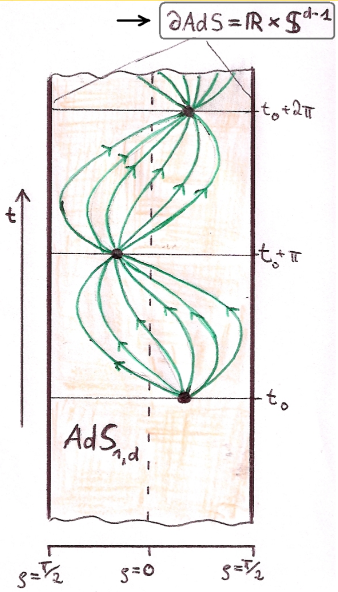
Outline

1. Anti de Sitter Spacetime (AdS)
2. General Boundary Formulation (GBF)
3. Holomorphic Quantization (HQ)
4. HQ on AdS
5. Invariance under AdS isometry actions

Outline

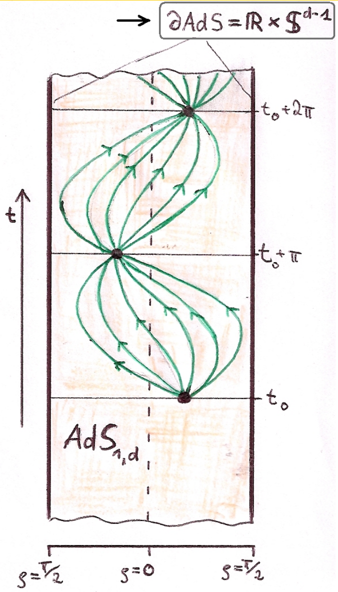
Anti-deSitter spacetime (AdS)

- constant negative curvature
- global coordinates:
  time $t \in [-\infty, +\infty]$
  radius $\rho \in [0, \frac{\pi}{2})$
  angles $\Omega = (\theta, \varphi)$ on $\mathbb{S}^2$
- d = spatial dimension
- static metric:
  $\mathrm{d}s^2_{\mathrm{AdS}} = \frac{R^2_{\mathrm{AdS}}}{\cos^2\rho}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sin^2\rho\, \mathrm{d}s^2_{\mathbb{S}^{d-1}}\right)$
- Penrose diagram
  with timelike geodesics
- no (temporally) asymptotically
  free states, no standard S-matrix



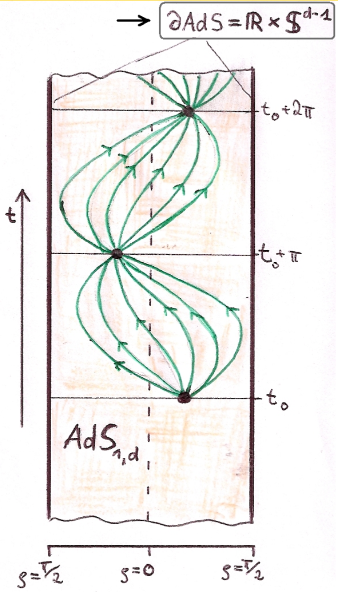
Anti-deSitter spacetime (AdS)

- constant negative curvature
- global coordinates:
  time $t \in [-\infty, +\infty]$
  radius $\rho \in [0, \frac{\pi}{2})$
  angles $\Omega = (\theta, \varphi)$ on $\mathbb{S}^2$
- d = spatial dimension
- static metric:
  $\mathrm{d}s^2_{\mathrm{AdS}} = \frac{R^2_{\mathrm{AdS}}}{\cos^2\rho}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sin^2\rho\,\mathrm{d}s^2_{\mathbb{S}^{d-1}}\right)$
- Penrose diagram
  with timelike geodesics
- no (temporally) asymptotically
  free states, no standard S-matrix



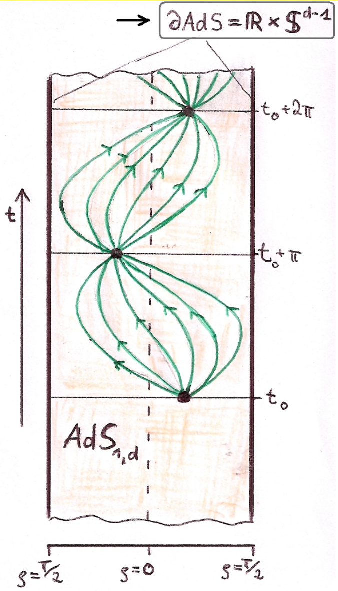
Anti-deSitter spacetime (AdS)

- constant negative curvature
- global coordinates:
  time $t \in [-\infty, +\infty]$
  radius $\rho \in [0, \frac{\pi}{2})$
  angles $\Omega = (\theta, \varphi)$ on $\mathbb{S}^2$
- d = spatial dimension
- static metric:
  $ds^2_{\text{AdS}} = \frac{R^2_{\text{AdS}}}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho \, ds^2_{\mathbb{S}^{d-1}}\right)$
- Penrose diagram
  with timelike geodesics
- no (temporally) asymptotically
  free states, no standard S-matrix



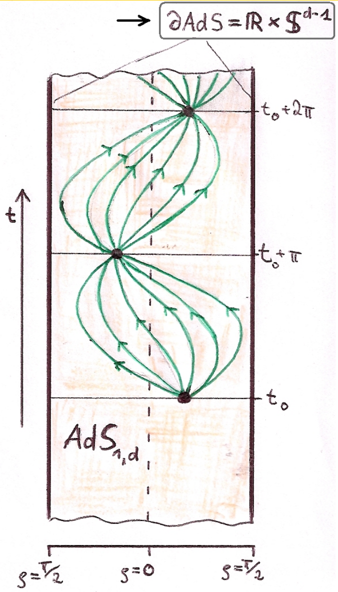
Anti-deSitter spacetime (AdS)

- constant negative curvature
- global coordinates:
  time $t \in [-\infty, +\infty]$
  radius $\rho \in [0, \frac{\pi}{2})$
  angles $\Omega = (\theta, \varphi)$ on $\mathbb{S}^2$
- d = spatial dimension
- static metric:
  $\mathrm{d}s^2_{\mathrm{AdS}} = \frac{R^2_{\mathrm{AdS}}}{\cos^2\rho}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sin^2\rho\,\mathrm{d}s^2_{\mathbb{S}^{d-1}}\right)$
- Penrose diagram
  with timelike geodesics
- no (temporally) asymptotically
  free states, no standard S-matrix



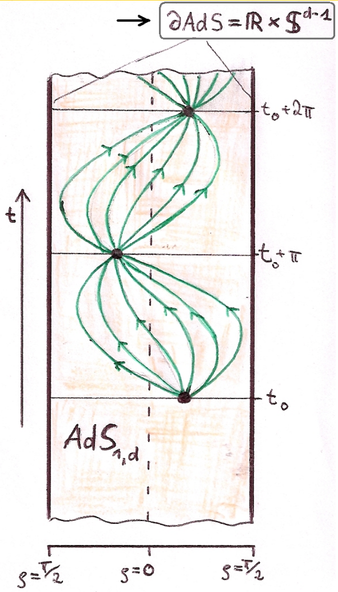
Anti-deSitter spacetime (AdS)

- constant negative curvature
- global coordinates:
  time $t \in [-\infty, +\infty]$
  radius $\rho \in [0, \frac{\pi}{2})$
  angles $\Omega = (\theta, \varphi)$ on $\mathbb{S}^2$
- d = spatial dimension
- static metric:
  $\mathrm{d}s^2_{\mathrm{AdS}} = \frac{R^2_{\mathrm{AdS}}}{\cos^2\rho}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sin^2\rho\, \mathrm{d}s^2_{\mathbb{S}^{d-1}}\right)$
- Penrose diagram
  with timelike geodesics
- no (temporally) asymptotically
  free states, no standard S-matrix



Anti-deSitter spacetime (AdS)

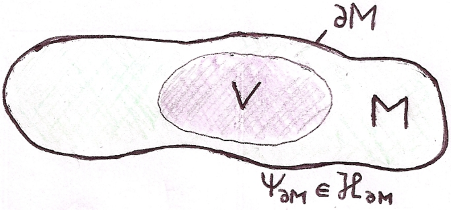
- constant negative curvature
- global coordinates:
  time $t \in [-\infty, +\infty]$
  radius $\rho \in [0, \frac{\pi}{2})$
  angles $\Omega = (\theta, \varphi)$ on $\mathbb{S}^2$
- d = spatial dimension
- static metric:
  $\mathrm{d}s^2_{\mathrm{AdS}} = \frac{R^2_{\mathrm{AdS}}}{\cos^2\rho}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sin^2\rho \, \mathrm{d}s^2_{\mathbb{S}^{d-1}}\right)$
- Penrose diagram
  with timelike geodesics
- no (temporally) asymptotically
  free states, no standard S-matrix



Outline

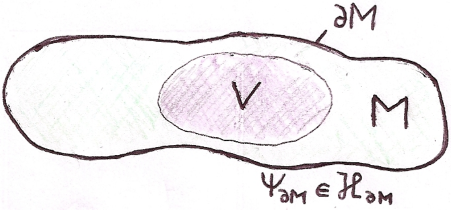
General Boundary Formulation (GBF)

- quantum states $\psi_{\partial M}$ live in Hilbert spaces associated to boundaries of spacetime regions
- amplitudes $\rho_M(\psi_{\partial M})$ depend on action S of theory on region's interior
- probability interpretation: probability of observing outcoming particles of $\psi_{\partial M}$ on condition that incoming particles were prepared



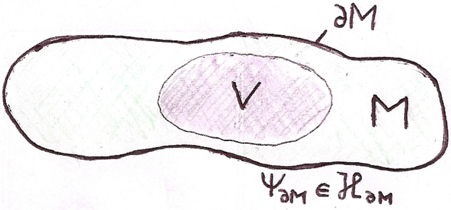
General Boundary Formulation (GBF)

- quantum states $\psi_{\partial \mathrm{M}}$ live in Hilbert spaces associated to boundaries of spacetime regions
- amplitudes $\rho_{\mathrm{M}}(\psi_{\partial \mathrm{M}})$ depend on action S of theory on region's interior
- probability interpretation: probability of observing outcoming particles of $\psi_{\partial \mathrm{M}}$ on condition that incoming particles were prepared



General Boundary Formulation (GBF)

- quantum states $\psi_{\partial\mathrm{M}}$ live in Hilbert spaces associated to boundaries of spacetime regions
- amplitudes $\rho_{\mathrm{M}}(\psi_{\partial\mathrm{M}})$ depend on action S of theory on region's interior
- probability interpretation: probability of observing outcoming particles of $\psi_{\partial\mathrm{M}}$ on condition that incoming particles were prepared

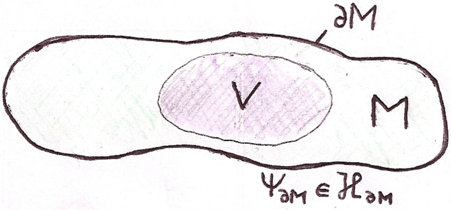


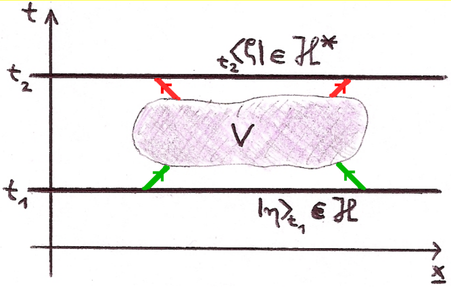
General Boundary Formulation (GBF)

- quantum states $\psi_{\partial \mathrm{M}}$ live in Hilbert spaces associated to boundaries of spacetime regions
- amplitudes $\rho_{\mathrm{M}}(\psi_{\partial \mathrm{M}})$ depend on action S of theory on region's interior
- probability interpretation: probability of observing outcoming particles of $\psi_{\partial \mathrm{M}}$ on condition that incoming particles were prepared
- standard QT recovered: slice

  amplitude
  $\rho_{[t_1,t_2]}(\eta_1 \otimes \zeta_2) = \langle \zeta_2 \,|\, \mathcal{U}_{[t_1,t_2]} \,|\, \eta_1 \rangle$

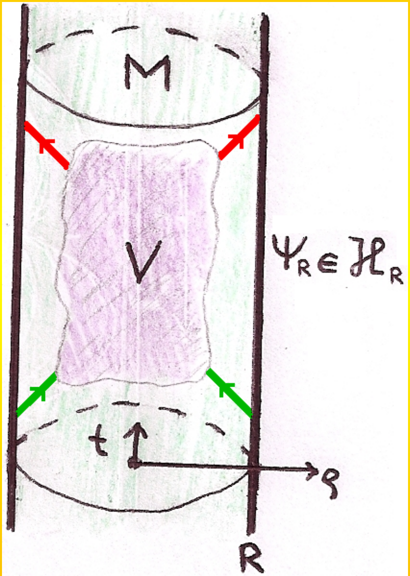
  probability
  $P(\zeta_2|\eta_1) = \left| \langle \zeta_2 \,|\, \mathcal{U}_{[t_1,t_2]} \,|\, \eta_1 \rangle \right|^2$





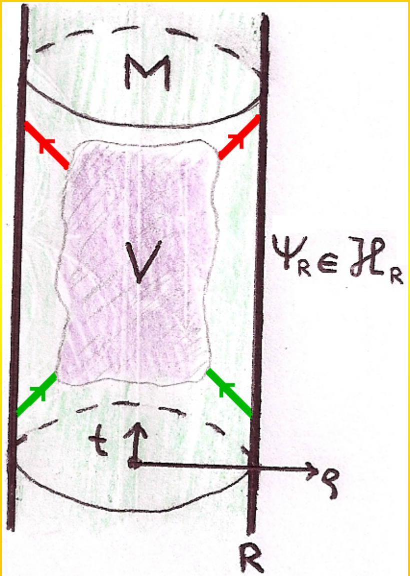
General Boundary Formulation (GBF)

- quantum states $\psi_{\partial\mathrm{M}}$ live in Hilbert spaces associated to boundaries of spacetime regions
- amplitudes $\rho_{\mathrm{M}}(\psi_{\partial\mathrm{M}})$ depend on action S of theory on region's interior
- probability interpretation: probability of observing outcoming particles of $\psi_{\partial\mathrm{M}}$ on condition that incoming particles were prepared
- **for AdS: rod hypercylinder**
- metric $\Rightarrow$ (radially) asymptotically free states near AdS boundary!
- thus limit of amplitude: S-matrix
- on Minkowski: standard and radial S-matrizes equivalent



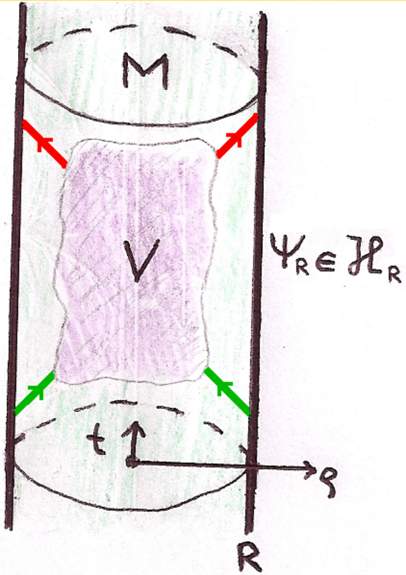
General Boundary Formulation (GBF)

- quantum states $\psi_{\partial \mathrm{M}}$ live in Hilbert spaces associated to boundaries of spacetime regions
- amplitudes $\rho_{\mathrm{M}}(\psi_{\partial \mathrm{M}})$ depend on action S of theory on region's interior
- probability interpretation: probability of observing outcoming particles of $\psi_{\partial \mathrm{M}}$ on condition that incoming particles were prepared
- **for AdS: rod hypercylinder**
- metric $\Rightarrow$ (radially) asymptotically free states near AdS boundary!
- thus limit of amplitude: S-matrix
- on Minkowski: standard and radial S-matrizes equivalent



General Boundary Formulation (GBF)

- quantum states $\psi_{\partial \mathrm{M}}$ live in Hilbert spaces associated to boundaries of spacetime regions
- amplitudes $\rho_{\mathrm{M}}(\psi_{\partial \mathrm{M}})$ depend on action S of theory on region's interior
- probability interpretation: probability of observing outcoming particles of $\psi_{\partial \mathrm{M}}$ on condition that incoming particles were prepared
- **for AdS: rod hypercylinder**
- metric $\Rightarrow$ (radially) asymptotically free states near AdS boundary!
- thus limit of amplitude: S-matrix
- on Minkowski: standard and radial S-matrizes equivalent

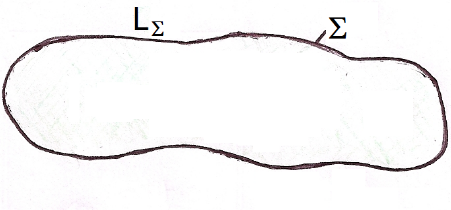


Outline

Holomorphic Quantization (HQ)

INGREDIENTS:

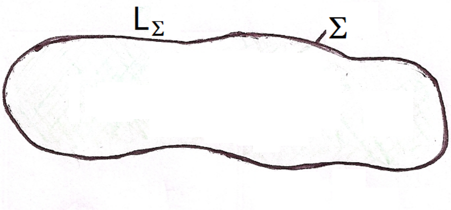
- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \; L_\Sigma \times L_\Sigma \; \to \; \mathbb{R}$
- complex structure: $J_\Sigma : \; L_\Sigma \; \to \; L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \;=\; 2\omega_\Sigma(\cdot, J_\Sigma\,\cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \;=\; g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \;=\; L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \;=\; \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



Holomorphic Quantization (HQ)

INGREDIENTS:

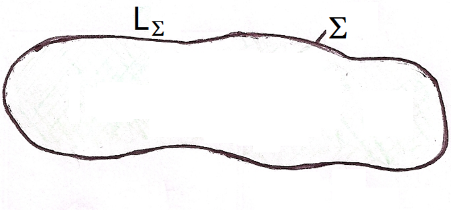
- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \ L_\Sigma \times L_\Sigma \ \to \ \mathbb{R}$
- complex structure: $J_\Sigma : \ L_\Sigma \ \to \ L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \ = \ 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \ = \ g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \ = \ L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \ = \ \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



Holomorphic Quantization (HQ)

INGREDIENTS:

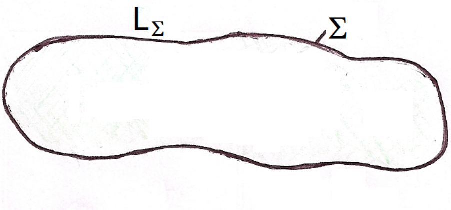
- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \; L_\Sigma \times L_\Sigma \; \to \; \mathbb{R}$
- complex structure: $J_\Sigma : \; L_\Sigma \; \to \; L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \;=\; 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \;=\; g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \;=\; L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \;=\; \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



Holomorphic Quantization (HQ)

INGREDIENTS:

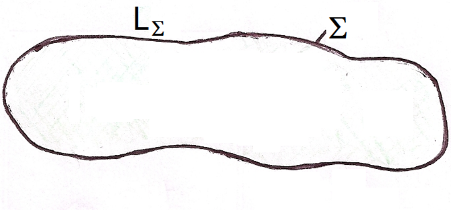
- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \; L_\Sigma \times L_\Sigma \; \to \; \mathbb{R}$
- complex structure: $J_\Sigma : \; L_\Sigma \; \to \; L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \; = \; 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \; = \; g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \; = \; L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \; = \; \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



Holomorphic Quantization (HQ)

<u>INGREDIENTS:</u>

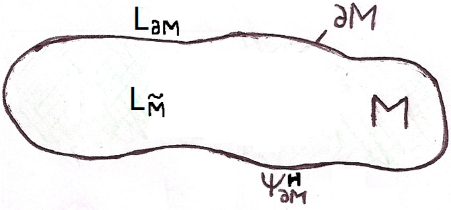
- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \ L_\Sigma \times L_\Sigma \ \to \ \mathbb{R}$
- complex structure: $J_\Sigma : \ L_\Sigma \ \to \ L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \ = \ 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \ = \ g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \ = \ L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \ = \ \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



Holomorphic Quantization (HQ)

<u>INGREDIENTS:</u>

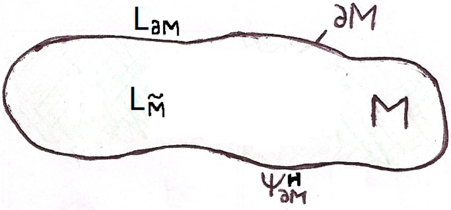
- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure:
  $\omega_\Sigma : \ L_\Sigma \times L_\Sigma \ \to \ \mathbb{R}$
- complex structure:
  $J_\Sigma : \ L_\Sigma \ \to \ L_\Sigma$
- field metric:
  $g_\Sigma(\cdot,\cdot) \ = \ 2\omega_\Sigma(\cdot, J_\Sigma \, \cdot)$
- inner product:
  $\{\cdot,\cdot\}_\Sigma \ = \ g_\Sigma(\cdot,\cdot) + 2\mathrm{i}\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M:
  solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$
  then $L_{\partial M} \ = \ L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \ = \ \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$
  with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



Holomorphic Quantization (HQ)

INGREDIENTS:

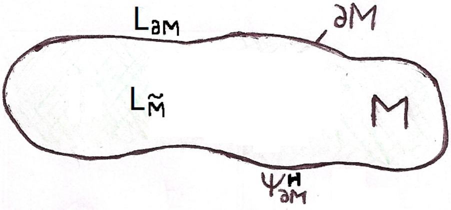
- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \; L_\Sigma \times L_\Sigma \;\to\; \mathbb{R}$
- complex structure: $J_\Sigma : \; L_\Sigma \;\to\; L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \;=\; 2\omega_\Sigma(\cdot, J_\Sigma\,\cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \;=\; g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \;=\; L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \;=\; \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



Holomorphic Quantization (HQ)

INGREDIENTS:

- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \; L_\Sigma \times L_\Sigma \; \to \; \mathbb{R}$
- complex structure: $J_\Sigma : \; L_\Sigma \; \to \; L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \; = \; 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \; = \; g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \; = \; L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \; = \; \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$

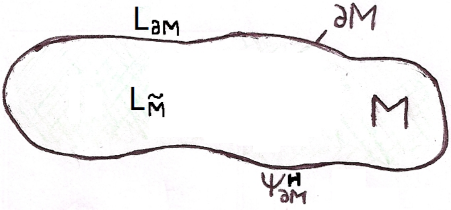


- states are holomorphic function(al)s: $\psi^{H}_\Sigma : \; L_\Sigma \; \to \; \mathbb{C}$
- normalized coherent states: $\phi \in L_\Sigma$ $\psi^{H,\phi}_\Sigma(\lambda) \; = \; \mathcal{N}^{H}_\phi \; \exp \tfrac{1}{2}\{\phi,\lambda\}_\Sigma$ with $\mathcal{N}^{H}_\phi \; = \; \exp -\tfrac{1}{4}\{\phi,\phi\}_\Sigma$
- amplitude: region M, boundary ∂M $\rho^{H,0}_M(\psi^{H,\phi}_{\partial M})$ $= \exp\left(-\tfrac{i}{2} g_{\partial M}(\phi^{\mathbb{R}},\phi^{\mathbb{I}}) - \tfrac{1}{2} g_{\partial M}(\phi^{\mathbb{I}},\phi^{\mathbb{I}})\right)$

Holomorphic Quantization (HQ)

INGREDIENTS:

- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \; L_\Sigma \times L_\Sigma \;\to\; \mathbb{R}$
- complex structure: $J_\Sigma : \; L_\Sigma \;\to\; L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \;=\; 2\omega_\Sigma(\cdot, J_\Sigma\,\cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \;=\; g_\Sigma(\cdot,\cdot) + 2\mathrm{i}\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \;=\; L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \;=\; \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$

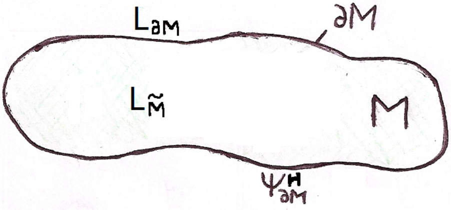


- states are holomorphic function(al)s: $\psi^{H}_\Sigma : \; L_\Sigma \;\to\; \mathbb{C}$
- normalized coherent states: $\phi \in L_\Sigma$ $\psi^{H,\phi}_\Sigma(\lambda) \;=\; \mathcal{N}^{H}_\phi \; \exp \tfrac{1}{2}\{\phi,\lambda\}_\Sigma$ with $\mathcal{N}^{H}_\phi \;=\; \exp -\tfrac{1}{4}\{\phi,\phi\}_\Sigma$
- amplitude: region M, boundary ∂M $\rho^{H,0}_M(\psi^{H,\phi}_{\partial M})$ $= \exp\left(-\tfrac{\mathrm{i}}{2} g_{\partial M}(\phi^{\mathbb{R}},\phi^{\mathbb{I}}) - \tfrac{1}{2} g_{\partial M}(\phi^{\mathbb{I}},\phi^{\mathbb{I}})\right)$

Holomorphic Quantization (HQ)

INGREDIENTS:

- associate to any hypersurface Σ space L_Σ of solutions near Σ
- symplectic structure: $\omega_\Sigma : \; L_\Sigma \times L_\Sigma \; \to \; \mathbb{R}$
- complex structure: $J_\Sigma : \; L_\Sigma \; \to \; L_\Sigma$
- field metric: $g_\Sigma(\cdot,\cdot) \; = \; 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$
- inner product: $\{\cdot,\cdot\}_\Sigma \; = \; g_\Sigma(\cdot,\cdot) + 2i\,\omega_\Sigma(\cdot,\cdot)$
- **Lemma:** region M, boundary ∂M: solutions on M: $L_{\tilde{M}} \subset L_{\partial M}$ then $L_{\partial M} \; = \; L_{\tilde{M}} \oplus J_{\partial M} L_{\tilde{M}}$
- for $\phi \in L_{\partial M}$ thus $\phi \; = \; \phi^{\mathbb{R}} + J_{\partial M}\phi^{\mathbb{I}}$ with $\phi^{\mathbb{R}}, \phi^{\mathbb{I}} \in L_{\tilde{M}}$



- states are holomorphic function(al)s: $\psi^{H}_{\Sigma} : \; L_\Sigma \; \to \; \mathbb{C}$
- normalized coherent states: $\phi \in L_\Sigma$
  $\psi^{H,\phi}_{\Sigma}(\lambda) \; = \; \mathcal{N}^{H}_{\phi} \; \exp \frac{1}{2}\{\phi, \lambda\}_\Sigma$
  with $\mathcal{N}^{H}_{\phi} \; = \; \exp -\frac{1}{4}\{\phi, \phi\}_\Sigma$
- amplitude: region M, boundary ∂M
  $\rho^{H,0}_{M}(\psi^{H,\phi}_{\partial M})$
  $= \exp\left(-\frac{i}{2} g_{\partial M}(\phi^{\mathbb{R}}, \phi^{\mathbb{I}}) - \frac{1}{2} g_{\partial M}(\phi^{\mathbb{I}}, \phi^{\mathbb{I}})\right)$

Outline

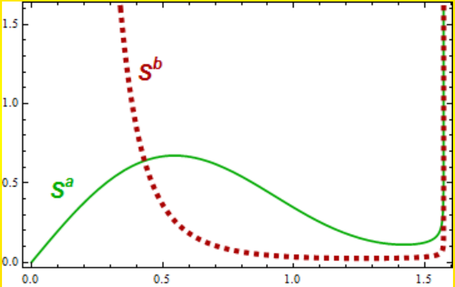
Klein-Gordon Modes

- free Klein-Gordon equation:
  $(\Box_{\mathrm{AdS}} - m^2)\phi(t,\rho,\Omega) = 0$
- two types of Klein-Gordon modes:
  both diverge approaching boundary

  S^a-modes:
  $\mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\; Y_l^{m_l}(\Omega)\; S^a_{\omega l}(\rho)$
  is regular on time axis $\rho = 0$

  S^b-modes:
  $\mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\; Y_l^{m_l}(\Omega)\; S^b_{\omega l}(\rho)$
  is divergent on time axis $\rho = 0$

  ω = frequency
  l, m_l = angular momentum numbers
  $Y_l^{m_l}(\Omega)$ = spherical harmonics
  $S^{a,b}_{\omega l}(\rho) \approx$ "$F^{a,b}(m,\omega,l,\sin^2\rho)$"
  hypergeometric functions



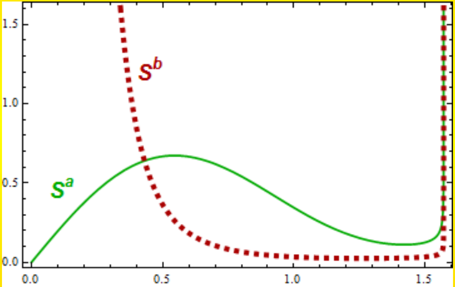
Klein-Gordon Modes

- free Klein-Gordon equation: $(\Box_{\text{AdS}} - m^2)\phi(t,\rho,\Omega) = 0$
- two types of Klein-Gordon modes: both diverge approaching boundary

S^a-modes:
$\mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^a_{\omega l}(\rho)$
is regular on time axis $\rho = 0$

S^b-modes:
$\mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^b_{\omega l}(\rho)$
is divergent on time axis $\rho = 0$

ω = frequency
l, m_l = angular momentum numbers
$Y_l^{m_l}(\Omega)$ = spherical harmonics
$S^{a,b}_{\omega l}(\rho) \approx$ "$F^{a,b}(m,\omega,l,\sin^2\rho)$" hypergeometric functions



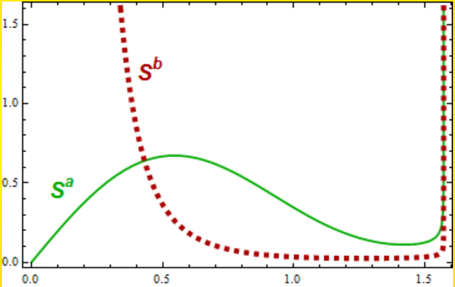
Klein-Gordon Modes

- free Klein-Gordon equation:
  $(\Box_{\text{AdS}} - m^2)\phi(t,\rho,\Omega) = 0$
- two types of Klein-Gordon modes:
  both diverge approaching boundary

  S^a-modes:
  $\mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^a_{\omega l}(\rho)$
  is regular on time axis $\rho = 0$

  S^b-modes:
  $\mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^b_{\omega l}(\rho)$
  is divergent on time axis $\rho = 0$

  ω = frequency
  l, m_l = angular momentum numbers
  $Y_l^{m_l}(\Omega)$ = spherical harmonics
  $S^{a,b}_{\omega l}(\rho) \approx$ "$F^{a,b}(m,\omega,l,\sin^2\rho)$"
  hypergeometric functions



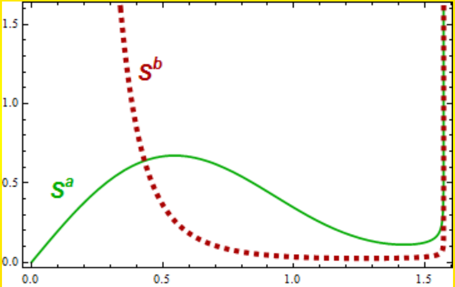
Klein-Gordon Modes

- free Klein-Gordon equation:
  $(\Box_{\mathrm{AdS}} - m^2)\phi(t,\rho,\Omega) = 0$
- two types of Klein-Gordon modes: both diverge approaching boundary

  S^a-modes:
  $\mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^a_{\omega l}(\rho)$
  is regular on time axis $\rho = 0$

  S^b-modes:
  $\mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^b_{\omega l}(\rho)$
  is divergent on time axis $\rho = 0$

  ω = frequency
  l, m_l = angular momentum numbers
  $Y_l^{m_l}(\Omega)$ = spherical harmonics
  $S^{a,b}_{\omega l}(\rho) \approx$ "$F^{a,b}(m,\omega,l,\sin^2\rho)$"
  hypergeometric functions



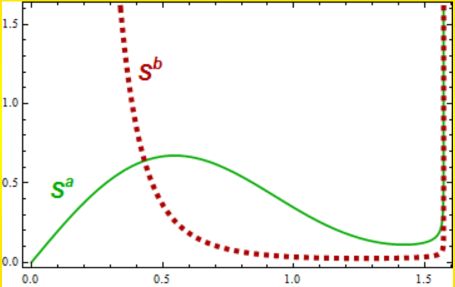
Klein-Gordon Modes

- free Klein-Gordon equation:
  $(\Box_{\text{AdS}} - m^2)\phi(t,\rho,\Omega) = 0$
- two types of Klein-Gordon modes:
  both diverge approaching boundary

  S^a-modes:
  $\mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^a_{\omega l}(\rho)$
  is regular on time axis $\rho = 0$

  S^b-modes:
  $\mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) = \mathrm{e}^{-\mathrm{i}\omega t}\ Y_l^{m_l}(\Omega)\ S^b_{\omega l}(\rho)$
  is divergent on time axis $\rho = 0$

  ω = frequency
  l, m_l = angular momentum numbers
  $Y_l^{m_l}(\Omega)$ = spherical harmonics
  $S^{a,b}_{\omega l}(\rho) \approx$ "$F^{a,b}(m,\omega,l,\sin^2\rho)$"
  hypergeometric functions



HQ ingredients for AdS

- field expansion near Σ_ρ with $\phi^{a,b}_{\omega l m_l}$ = momentum representation

$$\phi(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^{a}_{\omega l m_l}\; c^{j}_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^{b}_{\omega l m_l}\; c^{n}_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- complex structure: ρ-independent!

$$(\mathrm{J}_{\Sigma_\rho}\phi)(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ -\phi^{b}_{\omega l m_l}\; c^{j}_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^{a}_{\omega l m_l}\; c^{n}_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- $\phi \;=\; \phi^{\mathbb{R}} + \mathrm{J}_{\Sigma_{\rho_0}} \phi^{\mathbb{I}}$

$$\phi^{\mathbb{R}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^{a}_{\omega l m_l}\; c^{j}_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

$$\phi^{\mathbb{I}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^{b}_{\omega l m_l}\; c^{j}_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- symplectic structure: ρ-independent! $\qquad c^{j}_{\omega l}\, c^{n}_{\omega l} = \frac{-1}{2l+1}$

$$\omega_{\Sigma_\rho}(\eta,\zeta) \;=\; \tfrac{1}{2} \int\!\mathrm{d}t\,\mathrm{d}^{d-1}\Omega\; R^2_{\mathrm{AdS}} \tan^2\rho\; \left(\eta\,\partial_\rho\zeta - \zeta\,\partial_\rho\eta\right)$$
$$\;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{R^2_{\mathrm{AdS}}}{16\pi} \left\{ \eta^{a}_{\omega l m_l}\, \zeta^{b}_{-\omega,l,-m_l} - \eta^{b}_{\omega l m_l}\, \zeta^{a}_{-\omega,l,-m_l} \right\}$$

- field metric: ρ-independent!

$$\mathrm{g}_{\Sigma_\rho}(\eta,\zeta) \;=\; 2\omega_{\Sigma_\rho}(\eta, \mathrm{J}_{\Sigma_\rho}\zeta) \;=\; R^2_{\mathrm{AdS}} \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{8\pi} \left\{ \eta^{a}_{\omega l m_l}\, \zeta^{a}_{-\omega,l,-m_l} + \eta^{b}_{\omega l m_l}\, \zeta^{b}_{-\omega,l,-m_l} \right\}$$

HQ ingredients for AdS

- field expansion near Σ_ρ with $\phi^{a,b}_{\omega l m_l}$ = momentum representation

$$\phi(t,\rho,\Omega) = \int \mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\ c^j_{\omega l}\ \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^b_{\omega l m_l}\ c^n_{\omega l}\ \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- complex structure: ρ-independent!

$$(\mathrm{J}_{\Sigma_\rho}\phi)(t,\rho,\Omega) = \int \mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ -\phi^b_{\omega l m_l}\ c^j_{\omega l}\ \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^a_{\omega l m_l}\ c^n_{\omega l}\ \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- $\phi = \phi^{\mathbb{R}} + \mathrm{J}_{\Sigma_{\rho_0}}\phi^{\mathbb{I}}$

$$\phi^{\mathbb{R}}(t,\rho,\Omega) = \int \mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\ c^j_{\omega l}\ \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

$$\phi^{\mathbb{I}}(t,\rho,\Omega) = \int \mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^b_{\omega l m_l}\ c^j_{\omega l}\ \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- symplectic structure: ρ-independent! $\quad c^j_{\omega l} c^n_{\omega l} = \frac{-1}{2l+1}$

$$\omega_{\Sigma_\rho}(\eta,\zeta) = \frac{1}{2} \int \mathrm{d}t\, \mathrm{d}^{d-1}\Omega\ R^2_{\mathrm{AdS}} \tan^2\rho\ \left(\eta\, \partial_\rho \zeta - \zeta\, \partial_\rho \eta \right)$$

$$= \int \mathrm{d}\omega \sum_{l,m_l} \frac{R^2_{\mathrm{AdS}}}{16\pi} \left\{ \eta^a_{\omega l m_l}\ \zeta^b_{-\omega,l,-m_l} - \eta^b_{\omega l m_l}\ \zeta^a_{-\omega,l,-m_l} \right\}$$

- field metric: ρ-independent!

$$\mathrm{g}_{\Sigma_\rho}(\eta,\zeta) = 2\omega_{\Sigma_\rho}(\eta, \mathrm{J}_{\Sigma_\rho}\zeta) = R^2_{\mathrm{AdS}} \int \mathrm{d}\omega \sum_{l,m_l} \frac{1}{8\pi} \left\{ \eta^a_{\omega l m_l}\ \zeta^a_{-\omega,l,-m_l} + \eta^b_{\omega l m_l}\ \zeta^b_{-\omega,l,-m_l} \right\}$$

HQ ingredients for AdS

- field expansion near Σ_ρ with $\phi^{a,b}_{\omega l m_l}$ = momentum representation

$$\phi(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^b_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- complex structure: ρ-independent!

$$(\mathrm{J}_{\Sigma_\rho}\phi)(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ -\phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^a_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- $\phi \;=\; \phi^{\mathbb{R}} + \mathrm{J}_{\Sigma_{\rho_0}} \phi^{\mathbb{I}}$

$$\phi^{\mathbb{R}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

$$\phi^{\mathbb{I}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- symplectic structure: ρ-independent! $\qquad c^j_{\omega l} c^n_{\omega l} = \frac{-1}{2l+1}$

$$\omega_{\Sigma_\rho}(\eta,\zeta) \;=\; \frac{1}{2} \int\!\mathrm{d}t\,\mathrm{d}^{d-1}\Omega\; R^2_{\mathrm{AdS}} \tan^2\!\rho\; \left(\eta\,\partial_\rho\zeta - \zeta\,\partial_\rho\eta\right)$$
$$\;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{R^2_{\mathrm{AdS}}}{16\pi} \left\{ \eta^a_{\omega l m_l}\,\zeta^b_{-\omega,l,-m_l} - \eta^b_{\omega l m_l}\,\zeta^a_{-\omega,l,-m_l} \right\}$$

- field metric: ρ-independent!

$$\mathrm{g}_{\Sigma_\rho}(\eta,\zeta) \;=\; 2\omega_{\Sigma_\rho}(\eta,\mathrm{J}_{\Sigma_\rho}\zeta) \;=\; R^2_{\mathrm{AdS}} \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{8\pi} \left\{ \eta^a_{\omega l m_l}\,\zeta^a_{-\omega,l,-m_l} + \eta^b_{\omega l m_l}\,\zeta^b_{-\omega,l,-m_l} \right\}$$

HQ ingredients for AdS

- field expansion near Σ_ρ with $\phi^{a,b}_{\omega l m_l}$ = momentum representation

$$\phi(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \tfrac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^b_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- complex structure: ρ-independent!

$$(\mathrm{J}_{\Sigma_\rho}\phi)(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \tfrac{1}{4\pi} \left\{ -\phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^a_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- $\phi \;=\; \phi^{\mathbb{R}} + \mathrm{J}_{\Sigma_{\rho_0}} \phi^{\mathbb{I}}$

$$\phi^{\mathbb{R}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \tfrac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

$$\phi^{\mathbb{I}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \tfrac{1}{4\pi} \left\{ \phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- symplectic structure: ρ-independent! $\qquad c^j_{\omega l} c^n_{\omega l} = \frac{-1}{2l+1}$

$$\omega_{\Sigma_\rho}(\eta,\zeta) \;=\; \tfrac{1}{2} \int\!\mathrm{d}t\, \mathrm{d}^{d-1}\Omega\; R^2_{\mathrm{AdS}} \tan^2\rho\; \left(\eta\,\partial_\rho\zeta - \zeta\,\partial_\rho\eta\right)$$
$$\;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{R^2_{\mathrm{AdS}}}{16\pi} \left\{ \eta^a_{\omega l m_l}\, \zeta^b_{-\omega,l,-m_l} - \eta^b_{\omega l m_l}\, \zeta^a_{-\omega,l,-m_l} \right\}$$

- field metric: ρ-independent!

$$\mathrm{g}_{\Sigma_\rho}(\eta,\zeta) \;=\; 2\omega_{\Sigma_\rho}(\eta, \mathrm{J}_{\Sigma_\rho}\zeta) \;=\; R^2_{\mathrm{AdS}} \int\!\mathrm{d}\omega \sum_{l,m_l} \tfrac{1}{8\pi} \left\{ \eta^a_{\omega l m_l}\, \zeta^a_{-\omega,l,-m_l} + \eta^b_{\omega l m_l}\, \zeta^b_{-\omega,l,-m_l} \right\}$$

HQ ingredients for AdS

- field expansion near Σ_ρ with $\phi^{a,b}_{\omega l m_l}$ = momentum representation

$$\phi(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l} \; c^j_{\omega l} \; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^b_{\omega l m_l} \; c^n_{\omega l} \; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- complex structure: ρ-independent!

$$(\mathrm{J}_{\Sigma_\rho}\phi)(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ -\phi^b_{\omega l m_l} \; c^j_{\omega l} \; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^a_{\omega l m_l} \; c^n_{\omega l} \; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- $\phi = \phi^{\mathbb{R}} + \mathrm{J}_{\Sigma_{\rho_0}} \phi^{\mathbb{I}}$

$$\phi^{\mathbb{R}}(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l} \; c^j_{\omega l} \; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

$$\phi^{\mathbb{I}}(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^b_{\omega l m_l} \; c^j_{\omega l} \; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- symplectic structure: ρ-independent! $\qquad c^j_{\omega l} c^n_{\omega l} = \frac{-1}{2l+1}$

$$\omega_{\Sigma_\rho}(\eta,\zeta) = \frac{1}{2} \int dt \, d^{d-1}\Omega \, R^2_{\mathrm{AdS}} \tan^2\rho \, \left(\eta \, \partial_\rho \zeta - \zeta \, \partial_\rho \eta \right)$$
$$= \int d\omega \sum_{l,m_l} \frac{R^2_{\mathrm{AdS}}}{16\pi} \left\{ \eta^a_{\omega l m_l} \zeta^b_{-\omega,l,-m_l} - \eta^b_{\omega l m_l} \zeta^a_{-\omega,l,-m_l} \right\}$$

- field metric: ρ-independent!

$$\mathrm{g}_{\Sigma_\rho}(\eta,\zeta) = 2\omega_{\Sigma_\rho}(\eta, \mathrm{J}_{\Sigma_\rho}\zeta) = R^2_{\mathrm{AdS}} \int d\omega \sum_{l,m_l} \frac{1}{8\pi} \left\{ \eta^a_{\omega l m_l} \zeta^a_{-\omega,l,-m_l} + \eta^b_{\omega l m_l} \zeta^b_{-\omega,l,-m_l} \right\}$$

HQ ingredients for AdS

- field expansion near Σ_ρ with $\phi^{a,b}_{\omega l m_l}$ = momentum representation

$$\phi(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^b_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- complex structure: ρ-independent!

$$(\mathrm{J}_{\Sigma_\rho}\phi)(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ -\phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^a_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- $\phi \;=\; \phi^{\mathbb{R}} + \mathrm{J}_{\Sigma_{\rho_0}}\phi^{\mathbb{I}}$

$$\phi^{\mathbb{R}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

$$\phi^{\mathbb{I}}(t,\rho,\Omega) \;=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$

- symplectic structure: ρ-independent!

$$c^j_{\omega l}\, c^n_{\omega l} \;=\; \frac{-1}{2l+1}$$

$$\begin{aligned}\omega_{\Sigma_\rho}(\eta,\zeta) \;&=\; \tfrac{1}{2}\int\!\mathrm{d}t\,\mathrm{d}^{d-1}\Omega\; R^2_{\mathrm{AdS}} \tan^2\rho\; \left(\eta\,\partial_\rho\zeta - \zeta\,\partial_\rho\eta\right) \\ &=\; \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{R^2_{\mathrm{AdS}}}{16\pi} \left\{ \eta^a_{\omega l m_l}\; \zeta^b_{-\omega,l,-m_l} - \eta^b_{\omega l m_l}\; \zeta^a_{-\omega,l,-m_l} \right\}\end{aligned}$$

- field metric: ρ-independent!

$$\mathrm{g}_{\Sigma_\rho}(\eta,\zeta) \;=\; 2\omega_{\Sigma_\rho}(\eta,\mathrm{J}_{\Sigma_\rho}\zeta) \;=\; R^2_{\mathrm{AdS}} \int\!\mathrm{d}\omega \sum_{l,m_l} \frac{1}{8\pi} \left\{ \eta^a_{\omega l m_l}\; \zeta^a_{-\omega,l,-m_l} + \eta^b_{\omega l m_l}\; \zeta^b_{-\omega,l,-m_l} \right\}$$

HQ ingredients for AdS

- field expansion near Σ_ρ with $\phi^{a,b}_{\omega l m_l}$ = momentum representation
$$\phi(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^b_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$
- complex structure: ρ-independent!
$$(\mathrm{J}_{\Sigma_\rho}\phi)(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ -\phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) + \phi^a_{\omega l m_l}\; c^n_{\omega l}\; \mu^{(b)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$
- $\phi = \phi^{\mathbb{R}} + \mathrm{J}_{\Sigma_{\rho_0}}\phi^{\mathbb{I}}$
$$\phi^{\mathbb{R}}(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^a_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$
$$\phi^{\mathbb{I}}(t,\rho,\Omega) = \int d\omega \sum_{l,m_l} \frac{1}{4\pi} \left\{ \phi^b_{\omega l m_l}\; c^j_{\omega l}\; \mu^{(a)}_{\omega l m_l}(t,\rho,\Omega) \right\}$$
- symplectic structure: ρ-independent! $\qquad c^j_{\omega l} c^n_{\omega l} = \frac{-1}{2l+1}$
$$\omega_{\Sigma_\rho}(\eta,\zeta) = \frac{1}{2}\int dt\, d^{d-1}\Omega\, R^2_{\text{AdS}} \tan^2\rho \, \left(\eta\,\partial_\rho\zeta - \zeta\,\partial_\rho\eta\right)$$
$$= \int d\omega \sum_{l,m_l} \frac{R^2_{\text{AdS}}}{16\pi} \left\{ \eta^a_{\omega l m_l}\, \zeta^b_{-\omega,l,-m_l} - \eta^b_{\omega l m_l}\, \zeta^a_{-\omega,l,-m_l} \right\}$$
- field metric: ρ-independent!
$$\mathrm{g}_{\Sigma_\rho}(\eta,\zeta) = 2\omega_{\Sigma_\rho}(\eta,\mathrm{J}_{\Sigma_\rho}\zeta) = R^2_{\text{AdS}} \int d\omega \sum_{l,m_l} \frac{1}{8\pi} \left\{ \eta^a_{\omega l m_l}\, \zeta^a_{-\omega,l,-m_l} + \eta^b_{\omega l m_l}\, \zeta^b_{-\omega,l,-m_l} \right\}$$

HQ amplitude for AdS

- amplitude: AdS rod region M_{ρ_0}, boundary hypercylinder Σ_{ρ_0}

$$\begin{aligned} \rho^{\mathrm{H},0}_{\mathrm{M}_{\rho_0}}(\psi^{\mathrm{H},\phi}_{\Sigma_{\rho_0}}) &= \exp\Big(-\tfrac{\mathrm{i}}{2}\mathrm{g}_{\Sigma_{\rho_0}}(\phi^{\mathbb{R}},\phi^{\mathbb{I}}) - \tfrac{1}{2}\mathrm{g}_{\Sigma_{\rho_0}}(\phi^{\mathbb{I}},\phi^{\mathbb{I}})\Big) \\ &= \exp\int \mathrm{d}\omega \sum_{l,m_l} \frac{R^2_{\mathrm{AdS}}}{8\pi}\Big\{-\frac{\mathrm{i}}{2}\phi^a_{\omega l m_l}\,\phi^b_{-\omega,l,-m_l} - \frac{1}{2}\phi^b_{\omega l m_l}\,\phi^b_{-\omega,l,-m_l}\Big\} \end{aligned}$$

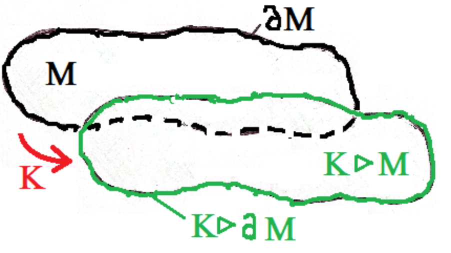
Outline

Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \to K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \to K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}\,(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\,\lambda)$

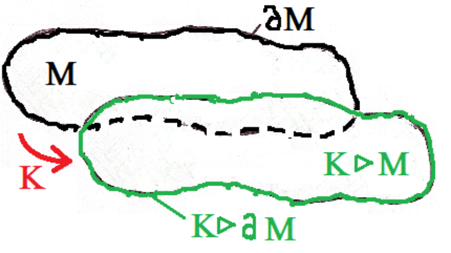
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

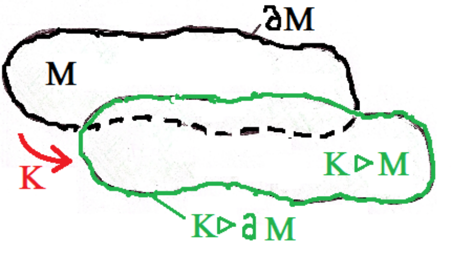
1. symplectic structure K-invariant:
   $\omega_{K \triangleright \partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K \triangleright \partial\mathrm{M}}(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\, \lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

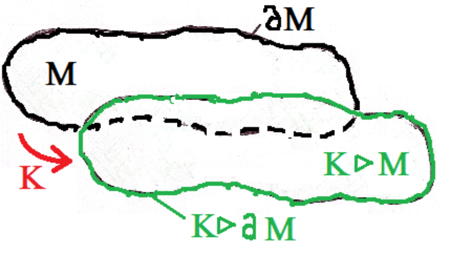
1. symplectic structure K-invariant:
   $\omega_{K \triangleright \partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K \triangleright \partial\mathrm{M}}(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}} \lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \to K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \to K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

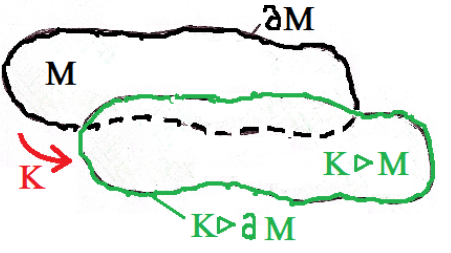
1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}\,(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\,\lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \to K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \to K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}\,(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\,\lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$

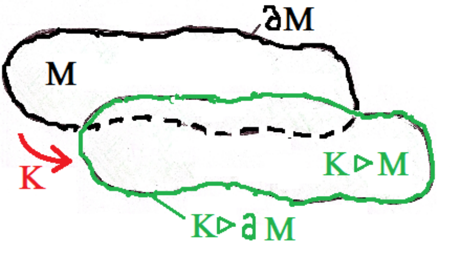


- then we have:
  $\mathrm{g}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \lambda)$
  $= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, \mathrm{J}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda)\big)$
  $= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\lambda)\big)$
  $= \omega_{\partial\mathrm{M}}(\lambda, \mathrm{J}_{\partial\mathrm{M}}\lambda)$
  $= \mathrm{g}_{\partial\mathrm{M}}(\lambda, \lambda)$

Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}\,(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\,\lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$

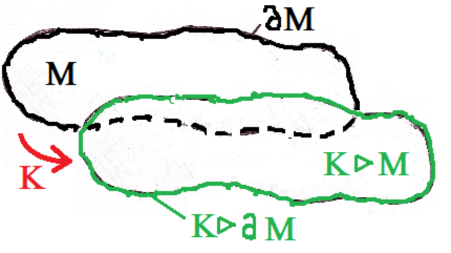


- then we have:
  $\mathrm{g}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \lambda)$
  $= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, \mathrm{J}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda)\big)$
  $= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\lambda)\big)$
  $= \omega_{\partial\mathrm{M}}(\lambda, \mathrm{J}_{\partial\mathrm{M}}\lambda)$
  $= \mathrm{g}_{\partial\mathrm{M}}(\lambda, \lambda)$

Invariance under isometry actions

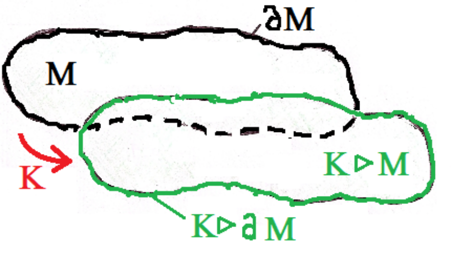
- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \overset{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}\,(K \triangleright \lambda) \overset{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\,\lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



- then we have:
  $\mathrm{g}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \lambda)$
  $= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, \mathrm{J}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda)\big)$
  $= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\lambda)\big)$
  $= \omega_{\partial\mathrm{M}}(\lambda, \mathrm{J}_{\partial\mathrm{M}}\lambda)$
  $= \mathrm{g}_{\partial\mathrm{M}}(\lambda, \lambda)$

Invariance under isometry actions



- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

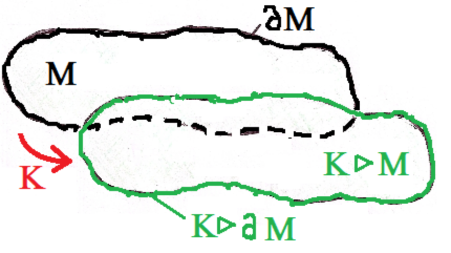
1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \overset{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda) \overset{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\, \lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$

- then we have:
  $$\begin{aligned} &\mathrm{g}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \lambda) \\ &= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, \mathrm{J}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda)\big) \\ &= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\lambda)\big) \\ &= \omega_{\partial\mathrm{M}}(\lambda, \mathrm{J}_{\partial\mathrm{M}}\lambda) \\ &= \mathrm{g}_{\partial\mathrm{M}}(\lambda, \lambda) \end{aligned}$$

Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}\,(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\,\lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



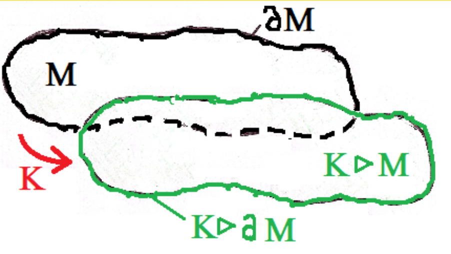
- then we have:

$$
\begin{aligned}
&\mathrm{g}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \lambda) \\
&= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, \mathrm{J}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda)\big) \\
&= \omega_{K\triangleright\partial\mathrm{M}}\big(K \triangleright \lambda, K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\lambda)\big) \\
&= \omega_{\partial\mathrm{M}}(\lambda, \mathrm{J}_{\partial\mathrm{M}}\lambda) \\
&= \mathrm{g}_{\partial\mathrm{M}}(\lambda, \lambda)
\end{aligned}
$$

Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

1. symplectic structure K-invariant:
   $\omega_{K \triangleright \partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K \triangleright \partial\mathrm{M}}\,(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\,\lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



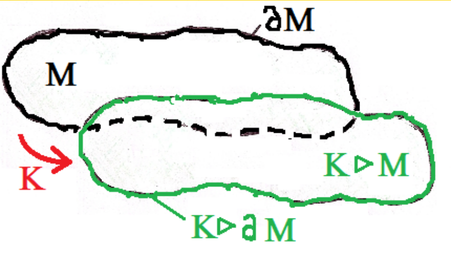
AdS: fix $c_{\omega l}^{j,n}$

- K-$\mathrm{J}_{\Sigma_{\rho_0}}$ commutation:
  time translation, rotations, boosts: DONE.
- ω-invariance:
  time translation, rotations: DONE.
  boosts: fix complex phase!

Invariance under isometry actions

- isometry K:
  $K: \quad \mathrm{M} \rightarrow K \triangleright \mathrm{M}$
  $K: \quad \partial\mathrm{M} \rightarrow K \triangleright \partial\mathrm{M} = \partial(K \triangleright \mathrm{M})$
- isometry invariance of amplitude requires two properties:

1. symplectic structure K-invariant:
   $\omega_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda, K \triangleright \phi) \stackrel{!}{=} \omega_{\partial\mathrm{M}}(\lambda, \phi)$
2. complex structure commutes with K:
   $\mathrm{J}_{K\triangleright\partial\mathrm{M}}(K \triangleright \lambda) \stackrel{!}{=} K \triangleright (\mathrm{J}_{\partial\mathrm{M}}\, \lambda)$
   for all $\lambda, \phi \in \mathrm{L}_{\partial\mathrm{M}}$



AdS: fix $c_{\omega l}^{j,n}$

- K-$\mathrm{J}_{\Sigma_{\rho_0}}$ commutation:
  time translation, rotations, boosts: DONE.
- ω-invariance:
  time translation, rotations: DONE.
  boosts: fix complex phase!

Outlook

- fix $c_{\omega l}^{j,n}$ completely for isometry invariance
- include interactions
- compare with radial S-matrix of Giddings

Outlook

- fix $c_{\omega l}^{j,n}$ completely for isometry invariance
- include interactions
- compare with radial S-matrix of Giddings

Outlook

- fix $c_{\omega l}^{j,n}$ completely for isometry invariance
- include interactions
- compare with radial S-matrix of Giddings

THANK YOU VERY MUCH FOR YOUR ATTENTION!

Related Literature:

- R. Oeckl: Holomorphic quantization of linear field theory in the GBF,
  SIGMA 8:050, [http://arxiv.org/abs/1104.5527], (2012)
- D. Colosi, R.Oeckl and M. Dohse:
  S-Matrix for AdS from General Boundary QFT,
  JPCS (IOP Science) 360, [http://iopscience.iop.org/1742-6596/360/1/012012], (2012)
- D. Colosi and R. Oeckl: $\mathcal{S}$-matrix at spatial infinity,
  Phys. Lett. B665, [http://arxiv.org/abs/0710.5203], (2007)
- S. Giddings: The boundary $\mathcal{S}$-matrix and the AdS to CFT dictionary,
  Phys.Rev.Lett. 83:2707-2010, [http://arxiv.org/abs/hep-th/9903048], (1999)