Recent results on BF gravity

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Coupling of matter fields to BF gravity

Action principle

- BF: $S[B,A,\phi,\mu]=\int_M\left(B^{IJ}\wedge F_{IJ}[A]-\phi_{IJKL}B^{IJ}\wedge B^{KL}+\mu H(\phi)\right)$; $H(\phi)=a_1\phi^{IJ}{}_{IJ}+a_2\epsilon^{IJKL}\phi_{IJKL}$.

PRD 81, 044033 (2010); PRD 85 064011 (2012) [Mercedes Velázquez's PhD thesis]

- Cosmological constant
- Scalar field
- Maxwell field
- Yang-Mills field
- fermions (in progress, Mariano Celada)

Geometrical meaning of Krasnov's modification

$$
\begin{aligned}
S &= \int \left(B_i \wedge F^i[A] - \frac{1}{2} C_{ij} B^i \wedge B^i - \frac{1}{6} (\Lambda + 3\Psi)\, \delta_{ij} B^i \wedge B^j \right), \\
\Psi &= \Psi\left(\mathrm{tr}C^2, \mathrm{tr}C^3 \right).
\end{aligned}
$$

- *Perturbative* analysis in CQG **27**, 145011 (2010). All the fields are expanded around a point that represents Plebański theory.
- *Exact* analysis in Mercedes Velázquez's PhD thesis and Diego González's MSc thesis.
- Exact analysis is simpler and clearer than the perturbative one.
- B's: The splitting of the self-dual subspaces is a "kind of rotation" with respect to the splitting in the Plebański case.
- A^i: The internal connection is linked to a space-time connection exactly like in the Plebański theory.

From gauge theories to gravity theories

- Plebański theory, Krasnov's modification, pure-spin connection formulation, etc ... are examples of *gauge theories* that are also diffeomorphism invariant.
- Space-time notions such as spacetime metric, spacetime connection, etc simply do *not* exist.
- The link between internal or gauge geometrical structures and spacetime concepts is *not* natural, it does *not* come out from only handling the original equations of motion; it is imposed by *hand*.
- Plebański theory has been re-analysed in detail (Diego González's talk)

Hamiltonian analysis of BF gravity

- Equivalence between the two forms the Immirzi parameter is introduced at Lagrangian level [Merceds Velázquez's PhD thesis and SIGMA **7**, 103 (2011)].
- The point above was explored at Hamiltonian level in Mariano Celada's MSc thesis and CGQ **29**, 205010 (2012) [Mariano Celada's talk]
- Relationship with other approaches, for instance, with the work of Alejandro Perez (coming living review 2012) [in progress].
- Same thing for the couplings of matter fields to BF gravity.

BF gravity boundaries

- Boundaries are natural in general relativity.
- Our approach is based on the covariant canonical formalism.
- Previous works by Momen, Husain-Major, etc ...
- Bering idea ...

Lagrangian approach to the physical dfc

- Alternative method to Dirac's one that allows us to count the number of physical degrees of freedom.
- It is combination of Noether's theorem and "Lagrangian constraints".
- Particle systems and non-explicitly covariant field theory (Daniel Higuita's MSc thesis).
- The method has been extended recently and now it is explicitly covariant. It works!
- The covariant method has been applied to BF theory with and without cosmological constant.
- The method has been applied to "Holst term": $De^I = 0$ and $R^I{}_J[\omega] \wedge e^J = 0$. It is much easier than PRD **81** 064033 (2010).